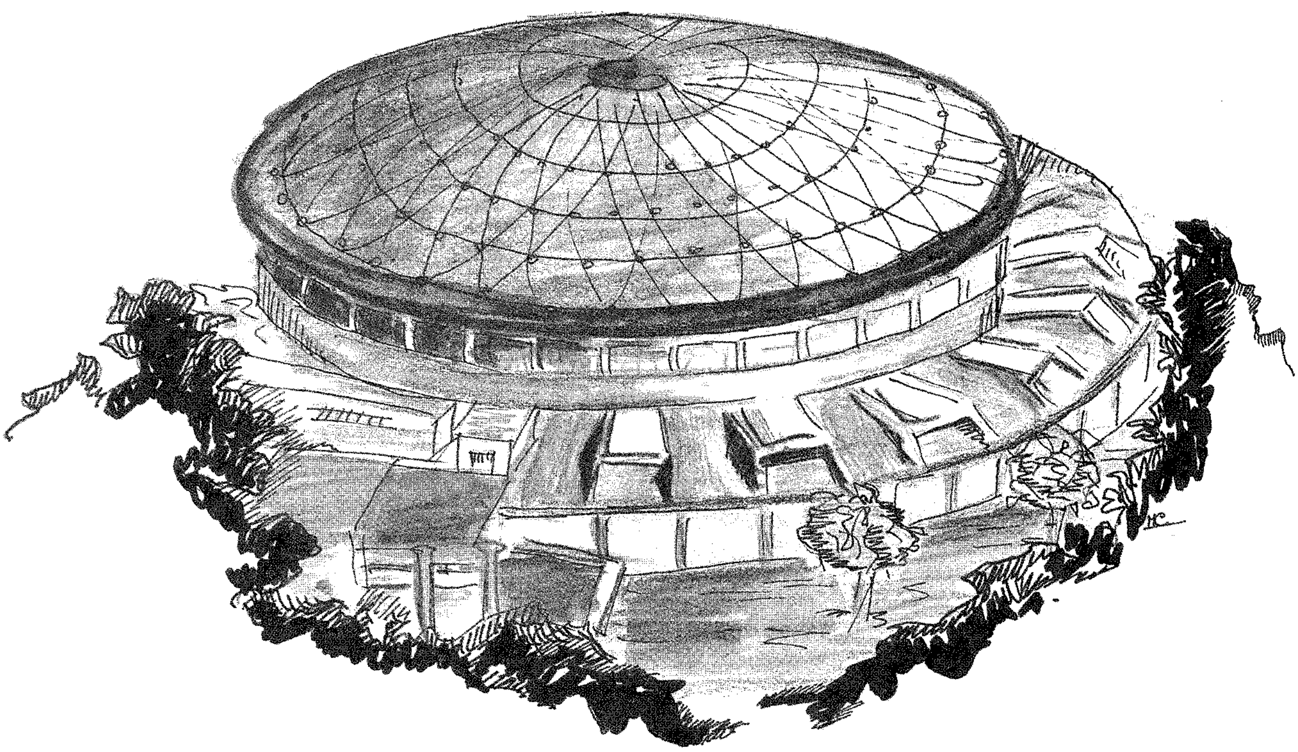
Laboratori Nazionali di Frascati

Submitted to Rev. Sci. Instr.

LNF-88/64(P)
11 Novembre 1988

R. Boni, A. Savoia, B. Spataro, F. Tazzioli, P. Fabbricatore, R. Parodi, P. Fernandes:

A SUPERCONDUCTING MICROWAVE UNDULATOR



Servizio Documentazione
dei Laboratori Nazionali di Frascati
P.O. Box, 13 - 00044 Frascati (Italy)

INFN - Laboratori Nazionali di Frascati
Servizio Documentazione

LNF-88/64(P)
11 Novembre 1988

A SUPERCONDUCTING MICROWAVE UNDULATOR

R. Boni, A. Savoia, B. Spataro, F. Tazzioli,
INFN-Laboratori Nazionali di Frascati, Via E. Fermi 40, 00044-Frascati, Italy

P. Fabbricatore, R. Parodi,
INFN-Sezione di Genova, Via Dodecaneso 33, 16146-Genova, Italy

P. Fernandes,
CNR-Istituto di Matematica Applicata, Via L. B. Alberti 4, 16146-Genova, Italy

ABSTRACT

We have studied the possibility of using a superconducting microwave cavity resonator as an undulator. A small cavity, made from bulk niobium, has been built and tested. The cavity has an elliptical cross section in order to force a linearly polarized oscillation mode. The TE_{118} mode has been chosen as the most suitable. Fields of about 300 Gauss have been obtained with 10 W of microwave power. Fields up to 1kG are reachable with niobium of higher thermal conductivity and better surface treatment.

A circular cross section cavity, where left- or right-handed helical fields can be excited, is also briefly studied. If used as an undulator this cavity produces circularly polarized radiation which can be switched in short times from left- to right-handed polarization.

1. - INTRODUCTION

The emission of electromagnetic radiation from charged particle beams produced in an accelerating structure is stimulated by the insertion of devices, like undulators, along the path of the particles. One of the interesting features of the emitted radiation is its polarization, which can be linear or circular, depending on the undulator structure. The fields used to date are static magnetic fields generated by conventional electromagnets, superconducting magnets or permanent magnets. The linear undulators, which produce essentially plane polarized light, are made of arrays of dipole magnets with the field alternating in polarity and transverse to the electron orbit. They are currently used in storage rings and in linacs or microtrons as synchrotron radiation sources and for free electron lasers.

Helical undulators, which produce circularly polarized light, are far less common. A double-helix bifilar solenoid magnet has been built and used on a linac by Elias and coworkers(1). A superconducting version is being studied for the Trieste machine(2). Several other schemes have been proposed which utilize two crossed permanent magnet planar undulators(3) or a double array of permanent magnet dipoles(4),(5) whose fields lie in two perpendicular planes and are shifted by λ/4. As source of circularly polarized light the radiation emitted by bending magnets viewed slightly below or above the orbit plane is currently used. By means of a mirror both left- and right-handed components can be alternately collected. To increase the obtainable flux an asymmetric multipole wiggler has been proposed(6).

We now report a different type of undulator, which makes use of a superconducting microwave cavity resonator oscillating on a harmonic of the fundamental frequency.

2. - EM FIELDS AND EQUATION OF MOTION

The most suitable working mode is the transverse electric TE mode because it has no longitudinal electric field which could interfere adversely with the particle beam. The case of a rectangular resonator oscillating in a TE_{10} mode has been worked out by Shintake et al.(7) and by K. Batchelor(8). Here we will consider the case of a cylindrical resonator with circular cross section. This case can be solved exactly(9). There are two sets of solutions, degenerate in energy, displaced by 90°.

The lowest possible mode having a transverse magnetic field component is TE_{11n}. For this mode we express the fields in cartesian coordinates (see Fig. 1) and retaining only the first-order terms we obtain for the first set of solutions:

$$E_x \approx 0 \tag{1a}$$

$$E_y \approx -E_o \sin \omega t \sin \frac{2\pi z}{\lambda_g} \tag{1b}$$

$$H_x \approx \frac{E_o}{Z_g} \cos \omega t \cos \frac{2\pi z}{\lambda_g} \tag{1c}$$

$$H_y \approx 0 \tag{1d}$$

$$H_z \approx \frac{E_o \lambda_o}{Z_o \lambda_c} k_c x \cos \omega t \sin \frac{2\pi z}{\lambda_g} \tag{1e}$$

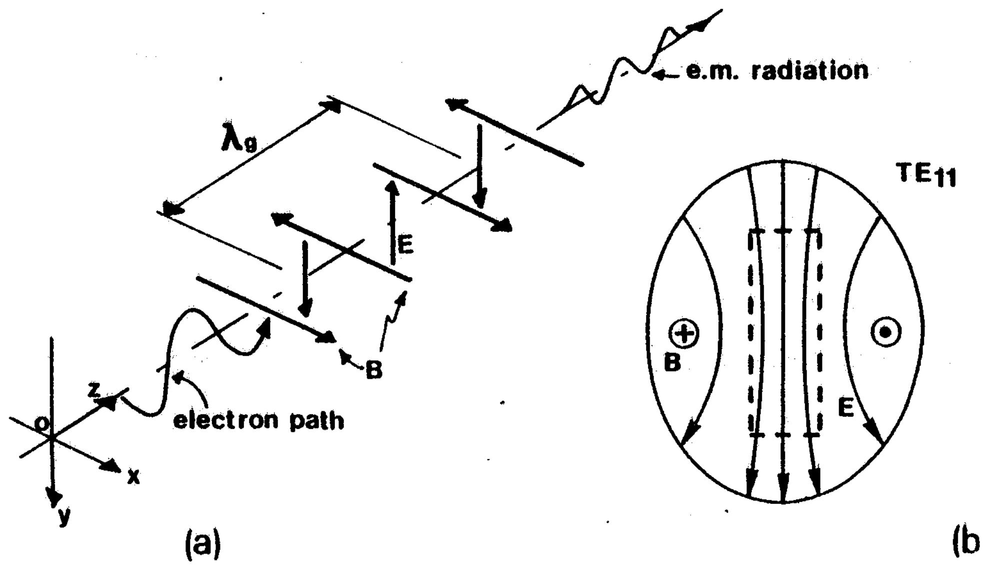


FIG. 1 - a) Pattern of the standing electromagnetic fields in the microwave undulator. The wave is plane polarized and the electron undulates in the yz plane. b) Field pattern for the TE_{11p} mode in the transverse section. The dashed line shows how the entrance and exit beam ports can be inserted.

Z_o is the vacuum impedance (≈377 Ω), $Z_g = Z_o[1-(\lambda_o/\lambda_c)^2]^{-1/2}$ is the guide impedance and $\lambda_c = 2\pi/k_c$ is the cut-off wavelength in the guide. The guide resonant wavelength is $\lambda_g = 2d/n$, where d is the length of the cavity and n is an integer, which gives the number of half wavelengths contained axially in the cavity.

λ_g is connected to the free-space wavelength λ_o by:

$$\lambda_g = \lambda_o[1-(\lambda_o/\lambda_c)^2]^{-1/2} \tag{2}$$

If we neglect the effect of the longitudinal component of the magnetic field, the equations of motion for a relativistic electron

$$\frac{d\vec{P}}{dt} = -e\left(\vec{E} + \vec{v} \times \vec{B}\right) \tag{3}$$

have a transverse component as in Ref. 8:

$$\frac{d\beta_y}{dt} = \frac{-eE_o}{2m_o c\gamma}\left[\left(1+\frac{\lambda_o}{\lambda_g}\right)\cos 2\pi z\left(\frac{1}{\lambda_o}+\frac{1}{\lambda_g}\right) - \left(1-\frac{\lambda_o}{\lambda_g}\right)\cos 2\pi z\left(\frac{1}{\lambda_o}-\frac{1}{\lambda_g}\right)\right] \tag{4}$$

where β_y is the electron velocity in the y direction divided by the velocity of light c, m_o is the rest mass of the electron, γ its energy divided by $m_o c^2$ and the substitution $\omega = 2\pi z/\lambda_o$ has been performed. In equation (4) the effect of forward and backward components of the standing microwave field is represented by the two terms in square brackets. By comparing this equation with the corresponding for a static undulator [10]

$$\frac{d\beta_y}{dt} = \frac{eB_u}{m_o \gamma} \cos \frac{2\pi z}{\lambda_u} \tag{5}$$

we can define an equivalent undulator field B_{uf}, wavelength λ_{uf} and field factor K_{uf} for the forward component:

$$B_{uf} = \frac{E_o}{2c}\left(\frac{\lambda_g - \lambda_o}{\lambda_g}\right); \quad \lambda_{uf} = \frac{\lambda_g \lambda_o}{\lambda_g - \lambda_o}; \quad K_{uf} = \frac{e}{2\pi m_o c} B_{uf} \lambda_{uf} \tag{6}$$

and for the backward

$$B_{ub} = \frac{E_o}{2c}\left(\frac{\lambda_g - \lambda_o}{\lambda_g}\right); \quad \lambda_{ub} = \frac{\lambda_u \lambda_o}{\lambda_g + \lambda_o}; \quad K_{ub} = \frac{e}{2\pi m_o c} B_{ub} \lambda_{ub} \tag{7}$$

It is evident that, since $\lambda_g \approx \lambda_o$, the spectrum emitted by the forward component lies at long wavelengths, whereas $\lambda_{ub} \approx \lambda_o/2$. In either case however we have $B\,\lambda = E_o\,\lambda_o/2c$ and $K_{uf} = K_{ub}$.

In the case of elliptical resonant cavity the fields are still given to the first order by eqs. (1a) to (1e) but now the degeneracy of the modes is removed. This case has been studied using the computer code OSCAR2D[11]. Table I reports the characteristics of some modes, whose frequency is close to the working frequency. The last three columns give the maximum transverse magnetic field H_{Tmax} on the axis corresponding to a quality factor Q_o for an RF input power P. Q_o is computed for a residual surface resistance of $2\text{x}10^{-7}\ \Omega$ at 1.8 K . The actual field can be evaluated using the formula:

$$H = A \times \sqrt{P\,Q_o} \tag{8}$$

where the constant A depends only on geometrical factors for a given working mode. It can be computed inserting in (8) the data of Table I.

In the Table the letter s or p denotes the modes having the electric field component $E_r(r,0)$ parallel or perpendicular to the ϕ=0 axis, which has been chosen coincident with the ellipse minor axis and with the x axis (see Fig. 1).

TABLE I - Characteristics of some resonant modes of the elliptical cavity, whose frequency is close to the TE_{118p} mode. $f_c = c/\lambda_c$ is the cut-off frequency and n is the number of half wavelengths along the z axis. H_{Tmax} is the peak transverse magnetic field obtained with the computed quality factor Q_o for an input power P.

MODE	fc (MHz)	fo (MHz)	n	H_T(G)	Q_ox(10^9)	P (mW)
TE 11s	1770	5851	8	19,0	5,42	6,77
TE 11p	2172	5985	8	27,0	6,19	6,07
TM 01s	2593	6151	8	24,5	3,89	10,90
TE 12s	2993	5725	7	25,8	4,44	8,09
TE 21p	3280	5880	7	22,6	3,91	9,40
TE 12s	3709	6130	7	27,0	4,56	8,40
TM 11s	3894	6243	7	31,9	3,90	10,00
TM 11p	4347	6033	6	35,6	3,51	10,80
TE 31s	4382	6058	6	20,5	3,16	12,00
TE 31p	4433	6095	6	20,6	3,07	12,40
TM 22s	5231	5928	4	32,6	3,81	9,76
TE 22p	5479	5864	3	12,7	4,91	7,50
TM 21p	5516	5899	3	30,7	3,63	10,20
TE 41s	5572	5952	3	11,5	2,47	15,00
TE 41p	5587	5966	3	11,5	2,42	15,50

3. - TEST CAVITY AND MEASUREMENTS

The test cavity is a waveguide 215 mm long, of elliptical cross section, 80 mm of minor and 100 mm of major axis, shorted at the ends by two plates. On each plate there are small tubes used to feed the RF power through a loop, to pick up a signal with an antenna and to pump the cavity. The choice of a superconducting cavity allows to obtain high electromagnetic fields with a modest microwave power. The material used is bulk niobium of reactor grade with a residual resistivity ratio RRR=68. The most suitable mode for our purpose is the TE_{118p}, since it has the highest Q_o. The resonant frequency is derived from Table I. The actual frequency, 6006 MHz, however is slightly higher; it depends also on the cleaning cycles to which the cavity was subjected. The cavity was pumped at room temperature down to less than 10^{-8} Torr.

The cryostat used is a commercial vertical dewar shielded from the earth magnetic field by several sheets of μ-metal. The residual field measured at the cavity position was <30 mGauss. The pumping system on liquid helium bath was able to mantain 1.8 K with a thermal input of 20 W.

The circuit used to drive the cavity and to measure its characteristics is shown in Fig. 2.

The RF power is fed into the cavity by a small loop and the field is sampled with an antenna. The cavity is kept in resonance by a feedback system in which a mixer compares the phases of the input and output cavity signals and produces a dc signal to drive the frequency of the generator.

The position of the feeding loop is vertically adjusted to match the output impedance of the power amplifier to the input impedance of the resonator, in order to minimize the power reflected from the cavity. The waveform of a pulse of reflected power is observed on an oscilloscope. The waveform corresponding to a well matched coupling is shown in Fig. 3.

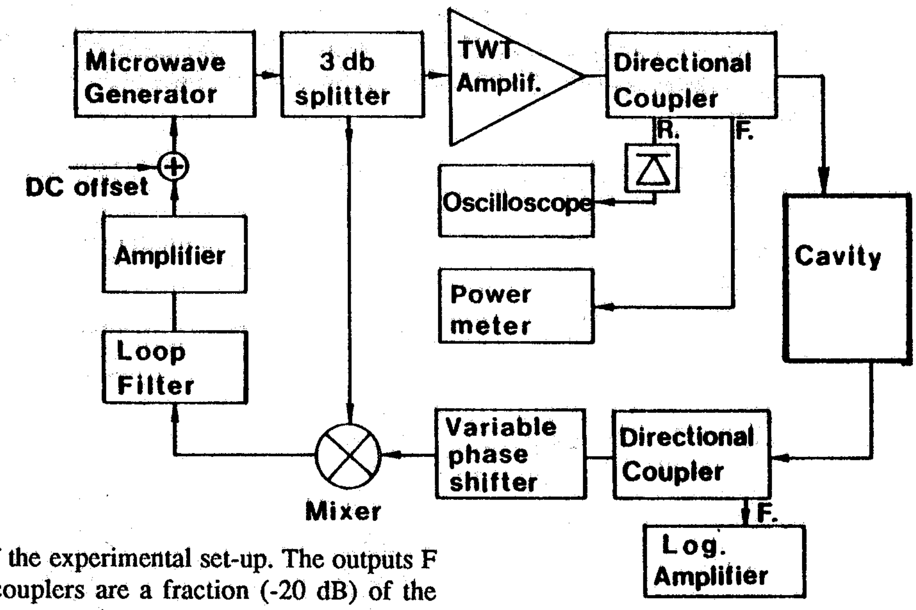
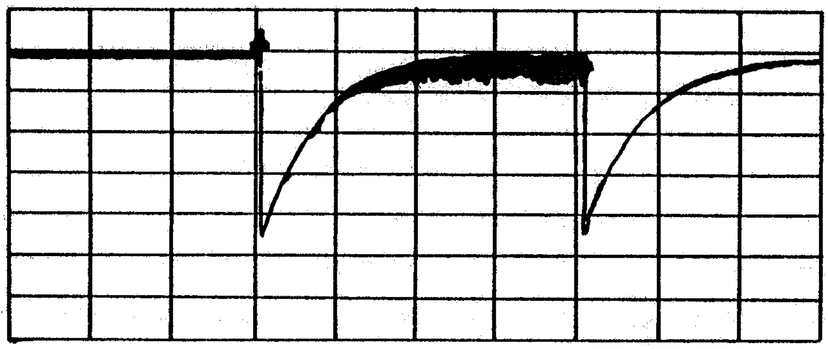


FIG. 2 - Block diagram of the experimental set-up. The outputs F and R of the directional couplers are a fraction (-20 dB) of the forward and reverse microwave signals.



Time/div. 20 msec

FIG. 3 - Waveform of the signal reflected from the cavity in a well matched condition.

The decay of the fields excited in the cavity is observed with a logarithmic amplifier and the quality factor Q_o of the cavity is computed from the slope of the straight line that corresponds to an exponential decay .

A factor $Q_o \approx 9 \times 10^7$ at 4.2 K and $Q_o \approx 6 \times 10^8$ at 1.5 K has been measured. The cavity was then driven in continuous wave and a magnetic field > 300 Gauss was reached, with an input power of ≈10 W.

4. - THE HELICAL UNDULATOR

If one adds two linearly polarized modes in space and time quadrature in an undulator cavity of circular cross section, one obtains a helical distribution of the fields along z axis, with x y components in a transverse section of the resonator given, to the first order, by:

$$E_x \approx E_o \cos \omega t \sin \frac{2\pi z}{\lambda_g} \tag{9a}$$

$$E_y \approx -E_o \sin \omega t \sin \frac{2\pi z}{\lambda_g} \tag{9b}$$

$$H_y \approx \frac{E_o}{Z_g} \cos \omega t \sin \frac{2\pi z}{\lambda_g} \tag{9c}$$

$$H_x \approx -\frac{E_o}{Z_g} \sin \omega t \cos \frac{2\pi z}{\lambda_g} \tag{9d}$$

where E_0 is the peak transverse electric field. The space quadrature is obtained by feeding the cavity with two antennas at 90° in space and the phase quadrature is obtained by suitable phase shifters (see Fig. 4).

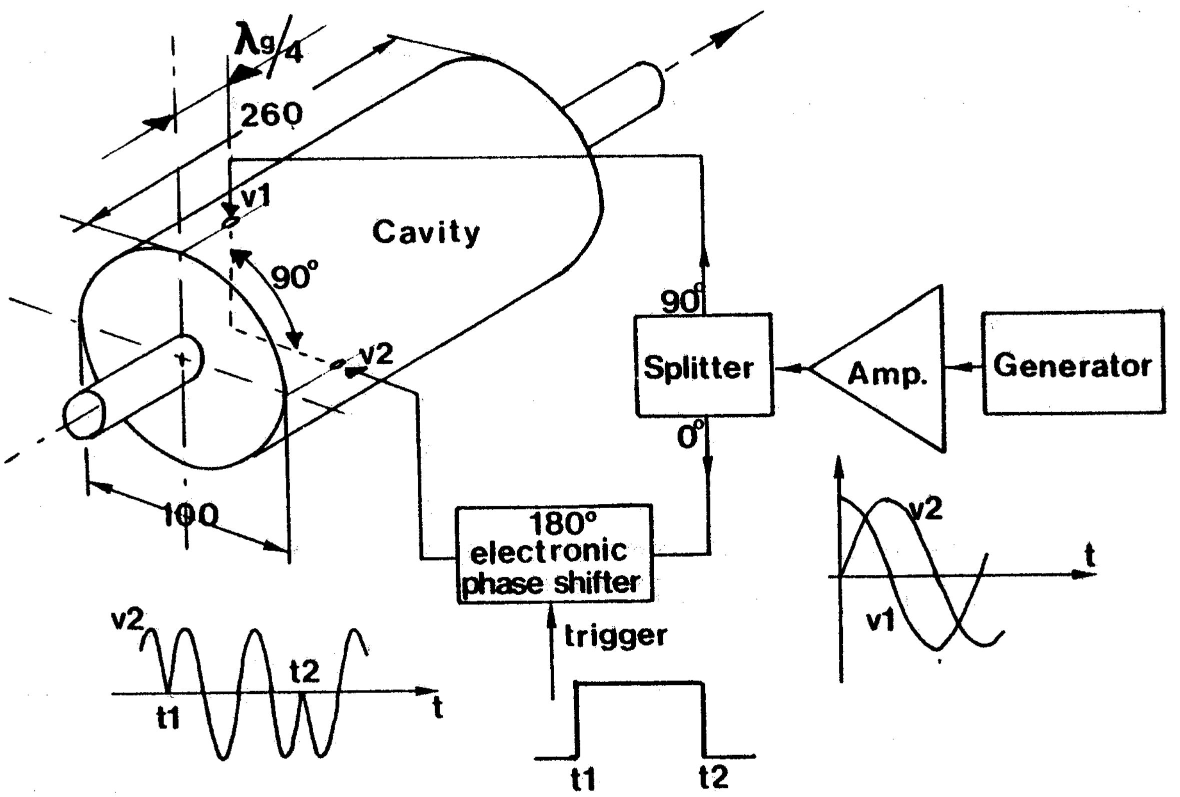


FIG. 4 - Experimental set-up for testing the behaviour of a circular cavity resonating in a circularly polarized mode. V1 and V2 are the RF voltages applied to two antennas placed at 90° from each other and at $\lambda_g/4$ from one end-plate, where the electric field has its maximum value. Cavity dimensions are in mm.

In the same way as for the planar case, eq. (4), we can derive the equation of motion for the transverse plane:

$$\frac{d\beta_x}{dt} = \frac{-eE_o}{2\,m_o c\gamma}\left[\left(1+\frac{\lambda_o}{\lambda_g}\right)\sin 2\pi z\left(\frac{1}{\lambda_o}+\frac{1}{\lambda_g}\right)+\left(1-\frac{\lambda_o}{\lambda_g}\right)\sin 2\pi z\left(\frac{1}{\lambda_o}-\frac{1}{\lambda_g}\right)\right]$$

$$\frac{d\beta_x}{dt} = \frac{-eE_o}{2\,m_o c\gamma}\left[\left(1+\frac{\lambda_o}{\lambda_g}\right)\cos 2\pi z\left(\frac{1}{\lambda_o}+\frac{1}{\lambda_g}\right)-\left(1-\frac{\lambda_o}{\lambda_g}\right)\cos 2\pi z\left(\frac{1}{\lambda_o}-\frac{1}{\lambda_g}\right)\right]$$

and again the effect of backward and forward wave is given by the two terms in the square brackets. The effect of these fields is to produce a helical motion of the electron.

The polarization of the fields can be changed from right to left- handed by changing the phase of one of the components by 180°. The time needed to pass from one polarization to the other one is determined by the forced build-up of the fields in the cavity and can be of the order of a few milliseconds.

A cylindrical cavity made of electrolitic copper whose dimensions are shown in Fig. 4 has been used to test the possibility of exciting a circularly polarized oscillation mode. Also in this case the working frequency was chosen close to 6 GHz, which corresponds to the $TE_{1,1,10}$ mode. Two RF signals were picked up by two small probes placed like the feeding antennas but on the opposite side of the cavity. The phase relation between them was compared with a mixer, whose output was zero when the two signals were in time quadrature.

5. - DISCUSSION

The maximum obtained field was limited by the available RF power. With a better treatment of the cavity surface a quality factor greater then 10^9 can certainly be obtained, thus permitting to approach with only 10 W of RF power the maximum achievable magnetic field which, at 6 GHz and 1.8K, has been estimated(12) to be 1000 Gauss for bulk niobium with RRR=62 and 1500 Gauss for RRR=620. These limits seem to be due to a thermal instability depending also on the thermal conductivity of niobium. It is likely that using niobium with higher RRR or niobium sputtered on copper would permit to reach higher fields. Therefore a field of 1000 Gauss seems to be well within the limits of actual technology. With such field and an equivalent undulator wavelength of 2.5 cm the field factor is $K_{ub} = 0.23$ and the photon flux emitted in the fundamental frequency is ≈ 1/10 of the maximum achievable at K ≈ 1. Of course a prototype undulator would longer than the present test cavity. A length of ≈1m is conceivable, which will require ≈ 40 W of

RF power and of refrigerating capability. At 6 GHz of working frequency one would get a 40 period undulator.

To install a microwave undulator on an accelerator requires an entrance and exit beam port on the cavity. These ports behave like a waveguide and their dimensions must be such that the wave cannot propagate outside the cavity, that is the cut-off frequency of the ports must be higher than the operating frequency. This is a problem for a storage ring, in which the transverse aperture of the vacuum chamber cannot be reduced at will. This sets a limit to the highest frequency which can be used. If a vertical aperture of 1 cm can be tolerated, the planar cavity allows to use a rectangular port 1cm high and several centimeters wide (see Fig. 1).

The helical cavity is more critical because the ports now must be cylindrical. If a diameter of 3 cm can be accepted for a length of say 10 cm for each port, the lowest usable wavelength is ≈7.5 cm corresponding to 4 GHz. The undulator equivalent wavelength would be ≈3.8 cm and K≈0.35. If used on a 1.5 GeV machine the energy of the fundamental harmonic would be 600 eV. On a 6 GeV machine this energy would be ≈10 KeV. It is possible to change the operating frequency in steps of the fundamental, thus changing correspondingly the emitted wavelength.

A more serious problem arises on a storage ring because of the interaction with the beam. It turns out that, integrating once the equation of motion (4) and calculating the result at the exit of the cavity (z=d), one gets zero deflection, which is a well known result for pure TE fields[13]. With a second integration we get at the exit the maximum orbit displacement y(d):

$$y\,(d) \leq \frac{K\,\lambda_{uf}}{2\,\pi\,\gamma} \approx 14\ \mu m$$

with K=0.23, $\lambda_{uf} \approx 114$ cm and $\gamma = 3000$. This displacement seems tolerable. In the case of the helical cavity there is also a strong coupling between horizontal and vertical emittances which increases the vertical beam dimension[14].

6. - CONCLUSIONS

A cylindrical cavity resonator of elliptical or circular cross section is shown to behave like a plane or helical undulator whose equivalent wavelength can be as short as 2.5 cm at 6 GHz. The use of superconducting niobium allows to obtain, with the actual technology, magnetic fields of 1 kG. In this case the equivalent undulator field factor K is about 0.23 and the photon flux thus obtainable on the fundamental frequency is ≈1/10 of that for K≈1, where the maximum occurs. In storage ring the lowest possible wavelength is determined by the minimum transverse dimension acceptable in the vacuum chamber. For the planar cavity, the horizontal aperture can be rather large, whilst the vertical, if reduced to 1 cm, would allow a maximum frequency of 15 GHz, thus shortening the undulator wavelength to ≈ 1 cm still keeping the magnetic field at 1 kG. Of course the field factor decreases proportionally to the wavelength and the photon flux decreases quadratically. The circular cavity, which can behave like a helical undulator, is interesting because it could provide a source if quasi-monochromatic circularly polarized radiation whose polarization direction can be changed from left- to right-handed in few milliseconds. If a beam entrance and exit port of 3 cm in diameter can be inserted, the minimum undulator wavelength is ≈ 3.8 cm.

During this conference we became aware from E.S. Gluskin that two double-helix bifilar solenoid magnets have been installed on the storage ring VEPP2M in Novosibirsk and successfully used to produce circularly polarized light.

ACKNOWLEDGEMENTS

This work would not have been possible without the collaboration of several people: R. Bolli, F. Campolungo and M. Capozi who have taken care of the cryogenics and vacuum, P. Baldini, F. Lucibello and S. Quaglia for the microwave equipments and circuitry. Special thanks are also due to S. Faini and to the staff of Cryogenic laboratory of Laboratori Nazionali di Frascati for the continuous assistance in all cryogenic operations. We are indebted with Dr E. Chiaveri of CERN for the e-beam welding of the cavity.

REFERENCES

1) L. R. Elias, W. M. Fairbank, J. M. Madey, H. A. Schwettman, T. I. Smith, Phys. Rev. Letters 36, (1976) 717.
2) C. Rubbia, Sincrotrone di Trieste, Report ST/M-87/19.
3) K.-J. Kim, Nucl. Instr. and Meth. 219, (1984) 425.
4) M. B. Moiseev, M. M. Nikitin and N. I. Fedosov, Izv. VUZ Fiz. (USSR) 3, (1978) 76.
5) H. Onuki, Nucl. Instr. and Meth. A246, (1986) 94.
6) J. Goulon, P. Elleaume and D. Raoux, Nucl. Instr. and Meth., A254,(1987)192.
7) T. Shintake, K. Huke, J. Tanaka, I. Sato and I. Kumabe, Jpn. J. Appl. Phys., 22, (1983) 844.
8) K. Batchelor, 1986 Linear Accelerator Conf. Proc. (SLAC-303), Stanford, CA, USA, (1986), pg. 272.
9) G. Goubau: Electromagnetic waveguides and cavities, Pergamon Press, Oxford, (1961).
10) S. Krinsky, M. L. Perlman and R. E. Watson, Handbook on Synchrotron Radiation, E.E. Koch Ed., North Holland Publishing Co., 1983, Vol. 1A, Chap.2, pg.140.
11) P. Fernandes and R. Parodi, IEEE Trans. on Magnetics, Mag 24, Vol 1, Jan 1988, pg. 154
12) P. Fernandes and R. Parodi, Cryogenics, Aug. 1984, pg. 433.
13) G. Lambertson: "Physics of Particle Accelerators", AIP-153, (1985) pg. 1416.
14) J. P. Blewett and R. Chasman, J. Appl. Phys., 48, (1977), 2692.