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FOR σ - MODELS WITH TORSION

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$O(\alpha')$ Conformal Anomaly and String Theory Effective Action for σ -Models with Torsion

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We present the generalization of the discussion on the integrability condition given by Callan et al. and apply it to the calculation of the two-loop dilaton β -function and effective action in the σ -model with the background g_{ij} , b_{ij} and ϕ .

Strings propagating in background fields are described by two-dimensional non-linear σ -models. The conditions for conformal invariance of the σ -model are identified with the equations of motion for the massless string modes.^{1)~7)} These equations can be derived from a single spacetime low-energy effective action, which reproduces the classical S-matrix elements for the massless states of the string theory.^{8),9)}

The above-mentioned ideas have been successfully tested by explicit calculations for the purely metric bosonic σ -model up to two loops in perturbation theory,⁸⁾ as well as for the $N=1$ and $N=2$ supersymmetric σ -models up to four loops.^{10)~13)} The two-loop β -functions β^g_{ij} and β^b_{ij} governing the renormalization of the target space metric tensor g_{ij} and torsion potential b_{ij} of the bosonic two-dimensional non-linear σ -model with Wess-Zumino-Witten-like terms have been recalculated recently,^{14),15)} correcting an error in the previous computation of Ref. 16).

In this work we present a consistency condition for β^g_{ij} and β^b_{ij} . The latter, together with the requirement of vanishing conformal anomaly, suffices to establish the existence of an integrability condition for the generalized conformal invariance equations of Ref. 8). Starting from the two-loop β -functions β^g_{ij} and β^b_{ij} ,^{14),15)} we show that the consistency condition is verified and derive the dilaton β -function β^ϕ and the effective action, including $O(\alpha')$ terms.

We start from the generalized conformal invariance equations⁸⁾

$$\beta^g_{ij} = R_{ij} - \frac{1}{4} H_{ikl} H_{jkl} + T_{ij} - D_i D_j \phi = 0, \quad (1)$$

$$\beta^b_{ij} = -\frac{1}{2} D_k H_{ijk} + A_{ij} - \frac{1}{2} H_{ijk} D_k \phi = 0, \quad (2)$$

where $H_{ijk} = 3D_{[i} b_{jk]}$. We have introduced in the β -functions the tensors T_{ij} and A_{ij} , which represent corrections to terms of leading order in α' . In (1) and (2) we set $V_i = \frac{1}{2} D_i \phi$ (in the notation of Ref. 8)), i.e., we restricted ourselves to consider only field redefinitions which guarantee that the conformal anomaly vanishes.¹⁷⁾ A discussion of the conditions for ultraviolet finiteness of the σ -model in the case of a generic V_i

is presented in Ref. 18). In order to achieve conformal invariance on the flat world-sheet, one must restrict the form of the field redefinitions allowed for the σ -model with torsion. Such restriction coincides with the one which ensures that conformal invariance is maintained in the curved world-sheet calculations.¹⁷⁾

Taking the divergence of (1), and using the Bianchi identity, as well as (1) and (2) themselves, one gets

$$D_i \left[-\frac{1}{2}R + D^2\phi + \frac{1}{2}(D\phi)^2 + \frac{1}{24}H^2 \right] = D_j T_{ij} + T_{ij} D_j - \frac{1}{2} H_{ijl} A_{jl}. \quad (3)$$

Thus, checking the validity of the following consistency condition,

$$D_j T_{ij} + T_{ij} D_j - \frac{1}{2} H_{ijl} A_{jl} = D_i S \quad (4)$$

for some scalar function S on the manifold, ensures that (1) and (2) admit an integrability condition

$$D_i \left[-\frac{1}{2}R + D^2\phi + \frac{1}{2}(D\phi)^2 + \frac{1}{24}H^2 - S \right] = 0. \quad (5)$$

Notice that (4) is completely general, as long as T_{ij} and A_{ij} remain unspecified. When considering loop corrections to the β -functions satisfying (4), one can integrate (5) to obtain

$$-R + 2(D^2\phi) + (D\phi)^2 + (1/12)H^2 - 2S = \text{constant}. \quad (6)$$

Equation (6) with S defined by (4) represents the generalization to higher loops of the equation of motion of the dilaton field, for the σ -model with torsion. For the existence of the flat spacetime solution we must set the constant to zero. We identify the conformal anomaly, including higher orders in α' , as

$$\beta^\phi = (\alpha'/4) [-R + 2D^2\phi + (D\phi)^2 + (1/12)H^2 - 2S]. \quad (7)$$

This generalizes the arguments of Ref. 8) to the case in which a background antisymmetric field is present. For a justification of the identification (7), see the note added below.

Next, we focus on the two-loop corrections. In this case the forms of the tensors in (1) and (2) are^{14),16)}

$$\begin{aligned} T_{ij} = & -\frac{1}{4}\alpha' \left[-2R_{iek}R_{jek} - \frac{1}{2}H_{skj}H_{sem}R_{ikem} - R_{kije}H_{kst}H_{est} \right. \\ & + (D_e H_{jmk})(D_m H_{iek}) - \frac{1}{2}H_{ski}H_{sem}R_{jkem} - \frac{1}{4}H_{jme}H_{imk}H_{kst}H_{est} \\ & \left. - \frac{1}{4}H_{sie}H_{smk}H_{rjk}H_{rme} \right], \quad (8) \end{aligned}$$

$$A_{ij} = -\frac{1}{4} \alpha' \left[(D_e H_{st(i)} R_{j)est} - \frac{1}{4} H_{em(i)} (D_j H_{esk}) H_{msk} + \frac{1}{4} H_{em(i)} (D_{|e} H_{j)sk}) H_{msk} - \frac{1}{2} (D_e H_{ijk}) H_{kst} H_{est} \right]. \tag{9}$$

Performing an algebraic exercise, whose details can be found in Ref. 18), one can show that the consistency condition (4) is satisfied by (8) and (9):

$$D_j T_{ij} + T_{ij} D_j - \frac{1}{2} H_{ijl} A_{jl} = -\frac{1}{4} \alpha' D_i \left\{ -\frac{1}{2} R_{abcd} R_{abcd} + \frac{1}{4} H_{skj} H_{sem} R_{kjem} + \frac{1}{16} \left[H_{kst} H_{est} H_{kab} H_{eab} - \frac{1}{3} H_{sje} H_{rjk} H_{smk} H_{rme} \right] \right\}. \tag{10}$$

Thus, we conclude that the conformal invariance equations (1) and (2) admit indeed to $O(\alpha')$ an integrability condition requiring the constancy of a scalar function, which is identified with the conformal anomaly of the σ -model and the dilaton equation of motion. Comparing (10) with (6), we have

$$S = -\frac{1}{4} \alpha' \left\{ -\frac{1}{2} R_{abcd} R_{abcd} + \frac{1}{4} H_{skj} H_{sem} R_{kjem} + \frac{1}{16} \left[H_{kst} H_{est} H_{kab} H_{eab} - \frac{1}{3} H_{sje} H_{rjk} H_{smk} H_{rme} \right] \right\}. \tag{11}$$

Substituting (11) into (7), we obtain the two-loop expression of the conformal anomaly

$$\beta^\phi = -R + 2D^2 \phi + (D\phi)^2 + \frac{1}{12} H^2 + \frac{1}{2} \alpha' \left\{ -\frac{1}{2} R_{abcd} R_{abcd} + \frac{1}{4} H_{skj} H_{sem} R_{kjem} + \frac{1}{16} \left[H_{kst} H_{est} H_{kab} H_{eab} - \frac{1}{3} H_{sje} H_{rjk} H_{smk} H_{rme} \right] \right\}. \tag{12}$$

The spacetime equations of motions (1), (2) and (12) for the massless string modes may be derived from the following spacetime effective action:

$$I_{\text{spacetime}} = - \int d^{26} X \sqrt{g} e^\phi \beta^\phi = \int d^{26} X \sqrt{g} e^\phi \left\{ R + (D\phi)^2 - \frac{1}{12} H^2 - \frac{1}{2} \alpha' \left[-\frac{1}{2} R_{abcd} R_{abcd} + \frac{1}{4} H_{skj} H_{sem} R_{kjem} + \frac{1}{16} H_{kst} H_{est} H_{kab} H_{eab} - \frac{1}{48} H_{sje} H_{rjk} H_{smk} H_{rme} \right] \right\}. \tag{13}$$

This action is the 1PI generating functional for the massless particle tree scattering amplitudes. In fact, (13) agrees on-shell with the action of Ref. 9) which generates all

bosonic string tree-level three-particle amplitudes correct to $O(\alpha')$. Four-particle scattering amplitudes, which correspond to quartic H -field interactions, are computed for the heterotic string in Ref. 19). A low-energy effective action, restricted to the case $D_i\phi=0$, appears in Ref. 15), but the coefficient of the term $(R_{abcd})^2$ differs from the one in (13), as well as from the result holding for the pure metric σ -model.⁸⁾ We believe that (13) is correct because it reproduces the results from the low-energy scattering of strings.⁹⁾

A few remarks are in order.

- i) We have generalized Callan et al.'s idea, including the effect of the antisymmetric tensor b_{ij} , while they only consider the metric tensor g_{ij} .
- ii) The proof of the consistency condition is not so straightforward, due to the complicated algebraic structure of the counterterms (8) and (9). Such a proof is crucial, in order to show that the σ -model description is consistent with the conformal invariance of the bosonic string.
- iii) The results (12) and (13) are far from obvious, because terms involving spacetime derivatives of the dilaton field do not appear in the corrections of higher order in α' to Eqs. (12) and (13). So far, a cause (e.g., some symmetry of the massless particle string tree S -matrix) has not been found. We believe it interesting to show that this feature is present in the model in study.

In concluding, we wish to remark that our result shows that the identification of the on-shell effective action with the conformal anomaly suggested in Ref. 8) is correct including $O(\alpha')$ terms. In the context of the $N=1$ and $N=2$ supersymmetric σ -models it was shown that no terms containing the dilaton field appear in the $O(\alpha'^3)$ corrections to the spacetime effective action.^{12),13)} This feature is present in our result (13) as well, showing that the explicit coupling to the dilaton field does not contribute to the $O(\alpha')$ corrections. Thus, in computing the two-loop corrections to all the β -functions of the bosonic σ -model with torsion, one can assume that the world-sheet is flat. It is interesting to conjecture that a general proof of this statement could be derived for all conformally invariant two-dimensional σ -models describing classical string physics.

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Note added: Well after the completion of the present work, we received a preprint²⁰⁾ where the two-loop calculation is carried out, according to the effective action method suggested in Ref. 3), based on the covariant Polyakov path integral. The result (3·24) of Ref. 20) agrees with (13) of the present work. In Ref. 20) the expression of β^g is also indirectly obtained, by varying the effective action with respect to ϕ . The result of Ref. 20) for the central charge coefficient supports the identification of the conformal anomaly with (7) above. The latter is in agreement with (6·9) of Ref. 20), giving the averaged Weyl anomaly coefficient.