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F. AVERSA [1], P. CHIAPPETTA, M. GRECO [1] and J.Ph. GUILLET [2]
Centre de Physique Théorique [3], CNRS, Case 907, Luminy, F-13288 Marseille Cedex 9, France

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Motivated by recent calculations of matrix elements for $O(\alpha_s^3)$ parton–parton scattering processes, we evaluate the K-factors and the relative transverse-momentum distributions for jet production at high p_T. We present results for a set of parton–parton subprocesses involving quarks of different flavours, in the framework of a general analysis of all one-loop QCD corrections.

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HIGHER-ORDER CORRECTIONS TO QCD JETS

F. AVERSA [1], P. CHIAPPETTA, M. GRECO [1] and J.Ph. GUILLET [2]
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The new generation of hadron colliders will provide decisive tests of the standard model and eventually give evidence for new physics only if QCD radiative corrections are under reasonable control. Indeed preliminary evaluations [1–3] of $O(\alpha_s^3)$ parton–parton subprocesses indicate large corrections to jet production at high p_T. In particular Ellis et al. [1] and Slominski and Furmanski [2] have calculated the radiative corrections to the scattering of non-identical quarks and found a K-factor of order of two at present energies.

More recently Ellis and Sexton [4] have computed the $O(\alpha_s^3)$ matrix elements for all $(2\to2)$ and $(2\to3)$ parton subprocesses in n dimensions. Real processes were already calculated in four dimensions in ref. [5]. Motivated by the results of ref. [4] we have made a further step towards a full evaluation of all $O(\alpha_s^3)$ cross sections.

In the present letter we present our results for the real and virtual subprocesses:

$$\text{I. } q_j+q_k\to q_j+q_k+(g), \quad \text{II. } q_j+\bar{q}_k\to q_j+\bar{q}_k+(g), \quad \text{III. } q_j+\bar{q}_j\to q_k+\bar{q}_k+(g)\,. \tag{1}$$

More precisely we evaluate the K-factors and the relative transverse momentum distributions of the two final jets. We agree with the results of refs. [1,2] for the subprocess I.

We shortly describe the method followed in our analysis. A more detailed discussion, including all $O(\alpha_s^3)$ subprocesses, will be given soon elsewhere. Starting from the expressions of the matrix elements in n dimensions we perform the phase space integration of the real processes, then we cancel the $(1/\varepsilon^2)$ divergences by adding the virtual corrections. The left-over $(1/\varepsilon)$ terms, corresponding to mass singularities, are then absorbed into the structure and fragmentation functions beyond the leading order. The algebraic manipulations are done in parallel with two independent programmes, one using REDUCE and the other one MACSYMA.

The inclusive cross section for the one-hadron inclusive production at large transverse momentum

$$H_1(K_1)+H_2(K_2)\to H_3(K_3)+X \tag{2}$$

is given [1] by

[1] INFN, Laboratori Nazionali di Frascati, I-00044 Frascati, Italy.
[2] Also at Institut de Physique Nucléaire, Université de Lausanne, CH-1015 Lausanne, Switzerland.
[3] Laboratoire propre du CNRS LP 7061.

$$E_3\frac{d\sigma}{d^3k_3}=\frac{1}{\pi S}\sum_{i,j,l}\int\limits_{1-V+VW}^{1}\frac{dx_3}{x_3^2}\int\limits_{VW/x_3}^{1-(1-V)/x_3}\frac{dv}{1-v}\int\limits_{VW/x_3v}^{1}\frac{dw}{w}D_{p_l}^{H_3}(x_3,M^2)F_{p_i}^{H_1}(x_1,M^2)F_{p_j}^{H_2}(x_2,M^2)$$

$$\times\left(\frac{1}{v}\frac{d}{dv}\sigma^0_{p_ip_j\to p_l}(s,v)\delta(1-w)+\frac{\alpha_s(\mu^2)}{2\pi}K_{p_ip_j\to p_l}(s,v,w)\right),\tag{3}$$

where the hadronic variables V and W are related to the Mandelstam variables as $V=1+T/S$, $W=-U/(S+T)$, and similarly for the partonic variables v, w and s, t, u. Furthermore $x_1=VW/x_3vw$, $x_2=(1-V)/x_3(1-v)$ and $s=Sx_1x_2$. $F^{H}_{p_i}(x_1,M^2)$ are the structure functions of a parton p_i inside the hadron H at the factorization mass scale M^2, and $D^{H_3}_{p_l}(x_3,M^2)$, similarly, are the fragmentation functions. The Born cross sections $d\sigma^0_{p_ip_j\to p_l}(s,v)$ for the three subprocesses (1) are

$$\frac{d\sigma^0_{I}(s,v)}{dv}=\frac{d\sigma^0_{II}(s,v)}{dv}=\frac{\pi C_F\alpha_s^2(\mu^2)}{Ns}\frac{1+v^2}{(1-v)^2},\quad\frac{d\sigma^0_{III}(s,v)}{dv}=\frac{\pi C_F\alpha_s^2(\mu^2)}{Ns}[(1-v)^2+v^2].\tag{4}$$

Finally $K_{p_ip_j\to p_l}(s,v,w)$ are the partonic K-factors and $\alpha_s(\mu^2)$ is the running coupling constant evaluated to the renormalization scale μ^2:

$$\alpha_s(\mu^2)=\frac{2\pi}{\beta_0\ln(\mu^2/\Lambda^2)}\left(1-\frac{\beta_1\ln[\ln(\mu^2/\Lambda^2)]}{\beta_0^2\ln(\mu^2/\Lambda^2)}\right),\tag{5}$$

with $\beta_0=\frac{11}{6}N-\frac{1}{3}N_F$, $\beta_1=\frac{17}{6}N^2-\frac{5}{6}N\,N_F-\frac{1}{2}C_FN_F$, and $N=3$ (N_F) being the number of colours (flavours) with $C_F=(N^2-1)/2N$.

As explicitly shown in ref. [1], the partonic K-factors $K_{p_ip_j\to p_l}(s,v,w)$ are obtained after factoring the mass singularities out of the partonic cross sections and into the distribution functions defined beyond the leading order in deep inelastic scattering. In addition to collinear gluon radiation from initial and final legs, this procedure involves the following subprocesses:

$$g+\overset{(-)}{q}_j\to g+\overset{(-)}{q}_j\quad\text{for I, II},\qquad q_j+\bar q_j\to g+g\quad\text{for III}.\tag{6a,b}$$

The reaction (6a) [respectively (6b)] needs the knowledge of the gluon structure [respectively fragmentation] function beyond the leading order.

The corresponding $(1/\varepsilon)$ singular parts only are known [6]. The finite terms $f_{p_ip_j}$ or $d_{p_ip_j}$ in fact are only available for quarks [7] ($p_i=q,\bar q$). In the following f_{gq} and d_{qg} will not be specified.

Following the notations of ref. [1] and working in the $\overline{MS}$ scheme we find the following results.

$$K^{(p)}(s,v,w)=\frac{1}{v}\frac{d\sigma^0_{(p)}(s,v)}{dv}\Big([c_1^{(p)}+\tilde c_1^{(p)}\ln(s/M^2)+\hat c_1^{(p)}\ln(s/\mu^2)]\delta(1-w)$$

$$+[c_2^{(p)}+\tilde c_2^{(p)}\ln(s/M^2)]\frac{1}{(1-w)_+}+c_3^{(p)}\left[\frac{\ln(1-w)}{1-w}\right]_+\Big)+\frac{\pi C_F\alpha_s^2(\mu^2)}{sNv}K^{1(p)}(s,v,w),\tag{7}$$

with

$$K^{1(p)}(s,v,w)=c_5^{(p)}\ln v+c_6^{(p)}\ln(1-vw)+c_7^{(p)}\ln(1-v+vw)$$

$$+c_8^{(p)}\ln(1-v)+c_9^{(p)}\ln w+c_{10}^{(p)}\ln(1-w)+c_{11}^{(p)}+\tilde c_{11}^{(p)}\ln(s/M^2)$$

$$+c_{12}^{(p)}\frac{\ln(1-v+vw)}{1-w}+c_{13}^{(p)}\frac{\ln w}{1-w}+c_{14}^{(p)}\frac{\ln[(1-vw)/(1-v)]}{1-w}+f_g^{(p)},\tag{8}$$

where p=I, II, III.

The coefficients c are given in tables 1–3. We agree with the results of ref. [1] for the process I, when the

Table 1
For process I. $C_F=(N^2-1)/2N$, $T_r=\frac{1}{2}N_F$. We choose $N=3$.

$$\hat{c}_1=4\,T_r/3-11\,N/3$$

$$\tilde{c}_1=C_F[8\log v-4\log(1-v)+9]/2$$

$$\begin{aligned}c_1=&C_F[108\,v^2\log^2 v+84\log^2 v-96\log(1-v)\,v^2\log v+9\,v^2\log v-24\,v\log v-48\log(1-v)\log v+33\log v\\&+12\log^2(1-v)\,v^2+51\log(1-v)\,v^2+10\,\pi^2v^2+6\,v^2-36\log^2(1-v)+3\log(1-v)-14\,\pi^2+6]/6\,(v^2+1)\\&-\mathrm{N}[81\,\mathrm{v}^2\log^2 v+63\log^2 v-54\log(1-v)\,v^2\log v-18\,v\log v-18\log(1-v)\log v+18\log v-9\log^2(1-v)\,v^2+102\log(1-v)\,v^2\\&-9\,\pi^2v^2-170\,v^2-18\log(1-v)\,v-63\log^2(1-v)+48\log(1-v)-27\,\pi^2-170]/18(v^2+1)+(4/9)\,T_r[3\log(1-v)-5]\end{aligned}$$

$$\tilde{c}_2=6\,C_F$$

$$c_2=3\,C_F[6\log v+2\log(1-v)+1]-4\,N[\log v+\log(1-v)]$$

$$c_3=2^2C_F$$

$$\begin{aligned}c_5=&-C_F(2\,v^4w^4-2\,v^4w^3-2\,v^3w^3+v^4w^2+25\,v^3w^2-21\,v^2w^2+vw^2-9\,v^3w+5\,v^2w-35\,vw-w+2\,v^2+2)/(v-1)^2w(vw-1)\\&-Nv(2\,v^4w^4-8\,v^4w^3+4\,v^3w^3+3v^4w^2+10\,v^3w^2-v^2w^2-4\,v^3w-4\,v^2w-12\,vw+3\,v^2-2\,v+9)/(v-1)^2(vw-1)^3\end{aligned}$$

$$c_6=\frac{2\,C_F(vw-1)(2\,v^2w^2-2\,v^2w+2\,vw+v^2+1)}{(v-1)^2w}+\frac{Nv(4\,vw-v+5)}{(v-1)^2}$$

$$c_7=\frac{2\,C_Fv(2\,v^2w^2-2\,v^2w+4\,vw+v^2+3)}{(v-1)^2}-\frac{Nv}{v-1}$$

$$\begin{aligned}c_8=&-C_F(4\,v^6w^6-4\,v^6w^5-8\,v^5w^5+2\,v^6w^4+9\,v^5w^4+9\,v^4w^4-9\,v^5w^3-19\,v^4w^3-8\,v^3w^3\\&+12\,v^4w^2+29\,v^3w^2+3\,v^2w^2-9\,v^3w-11\,v^2w-4\,vw+2\,v^2+2)/(v-1)^2w(vw-1)^3\\&-\frac{Nv(2\,v^2w^2+v^2+4\,v-3)}{(v-1)^2(vw-1)}\end{aligned}$$

$$\begin{aligned}c_9=&C_F(4\,v^3w^3-21\,v^3w^2+16\,v^2w^2-vw^2+11\,v^3w+5\,v^2w+23\,vw+w-3\,v^2-3)/(v-1)^2w(vw-1)\\&-\frac{Nv(2\,v^2w^2-5\,v^2w+5\,vw+3\,v^2+v+6)}{(v-1)^2(vw-1)}\end{aligned}$$

$$\begin{aligned}c_{10}=&\frac{Nv(3\,v^3w^3+v^2w^2-2\,v^2w-5\,vw+3)}{(v-1)(vw-1)^3}\\&-C_F(2\,v^4w^4-2\,v^4w^3+2\,v^3w^3+v^4w^2+8\,v^3w^2-5\,v^2w^2+6\,v^2w-14\,vw+v^2+1)/(v-1)^2w(vw-1)\end{aligned}$$

$$\begin{aligned}\tilde{c}_{11}=&-C_F(4\,v^7w^7-8\,v^7w^6-4\,v^6w^6+6\,v^7w^5+14\,v^6w^5-v^5w^5+v^4w^5-2\,v^7w^4-16\,v^6w^4-5\,v^5w^4-11\,v^4w^4\\&-2\,v^3w^4+7\,v^6w^3+14\,v^5w^3+8\,v^4w^3+15\,v^3w^3-8\,v^5w^2-10\,v^4w^2-11\,v^3w^2+3\,v^2w^2+2\,vw^2\\&+7\,v^4w-4\,v^3w+14\,v^2w-12\,vw-w-2\,v^3+2\,v^2-2\,v+2)/(v-1)^2w(vw-1)^3(vw-v+1)\\&-\frac{Nv(v^2w^2+1)(2\,v^2w^2-2\,v^2w-2\,vw+v^2+1)}{(v-1)^2(vw-1)^3}\end{aligned}$$

$$\begin{aligned}c_{11}=&C_F(2\,v^6w^6-4\,v^6w^5-2\,v^5w^5+v^6w^4+10\,v^5w^4-25\,v^4w^4+2\,v^3w^4+v^6w^3\\&-12\,v^5w^3+27\,v^4w^3+18\,v^3w^3-2\,v^2w^3+13\,v^5w^2-70\,v^4w^2+71\,v^3w^2-26\,v^2w^2-2\,vw^2\\&+19\,v^4w-52\,v^3w+61\,v^2w-32\,vw+2\,w-v^3+5\,v^2-5\,v+1)/2\,(v-1)^2w(vw-1)^2(vw-v+1)\\&-N(2\,v^5w^4-v^4w^4-3\,v^5w^3+2\,v^3w^3+3\,v^5w^2-12\,v^4w^2+10\,v^3w^2\\&+4\,v^4w-3\,v^3w-2\,vw-3\,v^3+6\,v^2-4\,v+1)/(v-1)(vw-1)^3(vw-v+1)\end{aligned}$$

$$c_{12}=\frac{2\,N(v+1)}{v-1}-\frac{8\,C_Fv^2}{(v-1)^2}$$

$$c_{13}=\frac{2\,C_F(11\,v^2+7)}{(v-1)^2}-\frac{N(5\,v^2+3)}{(v-1)^2}$$

$$c_{14}=-\frac{N(v^2+7)}{(v-1)^2}-\frac{8\,C_F(v+1)}{v-1}$$

$$f_g=\frac{Nv(v^2w^2+1)f_{gq}[(v-1)/(vw-1)]}{C_F(v-1)(vw-1)^2}+\frac{(v^2w^2+1)f_{gq}[(v-1)/(vw-1)]}{(v-1)w}$$

Table 2
For process II.

$\hat{c}_1 = 4\,T_r/3 - 11\,N/3$

$\tilde{c}_1 = C_F[8\log(v) + 4\log(1-v) + 9]/2$

$c_1 = N[162\,v^2\log^2 v + 126\log^2 v - 216\log(1-v)\,v^2\log v - 36\,v\log v - 144\log(1-v)\log v + 36\log v + 63\log^2(1-v)\,v^2$
$\quad - 48\log(1-v)\,v^2 + 36\,\pi^2 v^2 + 170\,v^2 + 18\log(1-v)\,v + 9\log^2(1-v) - 102\log(1-v) + 170]/18\,(v^2+1)$
$\quad - C_F[108\,v^2\log^2 v + 84\log^2 v - 144\log(1-v)\,v^2\log v - 9\,v^2\log v - 24\,v\log v - 96\log(1-v)\log v + 15\log v + 36\log^2(1-v)\,v^2$
$\quad - 3\log(1-v)v^2 + 14\,\pi^2 v^2 - 6v^2 - 12\log^2(1-v) - 51\log(1-v) - 10\,\pi^2 - 6]/6\,(v^2+1) + (4/9)T_r[3\log(1-v) - 5]$

$\tilde{c}_2 = 6\,C_F$

$c_2 = 4\,N[2\log v - \log(1-v)] - C_F[14\log v - 6\log(1-v) - 3]$

$c_3 = 2^2 C_F$

$c_5 = -Nv(2v^4w^4 + 10v^4w^3 - 14\,v^3w^3 - 3\,v^4w^2 - 20\,v^3w^2 + 11\,v^2w^2 + 8\,v^3w + 2\,v^2w + 18\,vw - 3\,v^2 + 4\,v - 15)/(v-1)^2(vw-1)^3$
$\quad - C_F(2\,v^4w^4 - 2\,v^4w^3 - 2\,v^3w^3 + v^4w^2 - 23\,v^3w^2 + 27\,v^2w^2 + vw^2 + 7\,v^3w - 11\,v^2w + 29\,vw - w + 2\,v^2 + 2)/(v-1)^2 w(vw-1)$

$$c_6 = \frac{2\,C_F(2\,v^3w^3 - 2\,v^3w^2 + v^3w - 6\,v^2w + 7\,vw - v^2 - 1)}{(v-1)^2 w} + \frac{Nv(4\,vw + 5\,v - 1)}{(v-1)^2}$$

$$c_7 = \frac{2\,C_F v(2\,v^2w^2 - 2\,v^2w + 4\,vw + v^2 - 4\,v + 7)}{(v-1)^2} + \frac{2\,Nv}{v-1}$$

$c_8 = -C_F(4\,v^6w^6 - 4\,v^6w^5 - 8\,v^5w^5 + 2\,v^6w^4 + 9\,v^5w^4 + 9\,v^4w^4 - 9\,v^5w^3 + 13\,v^4w^3 - 40\,v^3w^3$
$\quad + 12\,v^4w^2 - 35\,v^3w^2 + 67\,v^2w^2 - 9\,v^3w + 21\,v^2w - 36\,vw + 2\,v^2 + 2)/(v-1)^2 w(vw-1)^3$

$$\quad - \frac{Nv(2\,v^2w^2 + v^2 - 8\,v + 9)}{(v-1)^2(vw-1)}$$

$c_9 = C_F(4\,v^3w^3 + 19\,v^3w^2 - 24\,v^2w^2 - vw^2 - 5\,v^3w - 3\,v^2w - 17\,vw + w - 3\,v^2 - 3)/(v-1)^2 w(vw-1)$

$$\quad - \frac{Nv(2\,v^2w^2 + 10\,v^2w - 10\,vw - 3\,v^2 - 2\,v - 9)}{(v-1)^2(vw-1)}$$

$c_{10} = -C_F(2\,v^4w^4 - 2\,v^4w^3 + 2\,v^3w^3 + v^4w^2 - 8\,v^3w^2 + 11\,v^2w^2 - 10\,v^2w + 2\,vw + v^2 + 1)/(v-1)^2 w(vw-1)$

$$\quad - \frac{Nv(3\,v^3w^3 - 7\,v^2w^2 + 2\,v^2w - vw + 3)}{(v-1)(vw-1)^3}$$

$\tilde{c}_{11} = -C_F(4\,v^7w^7 - 8\,v^7w^6 - 4\,v^6w^6 + 6\,v^7w^5 + 14\,v^6w^5 - v^5w^5 + v^4w^5 - 2\,v^7w^4 - 16\,v^6w^4 - 5\,v^5w^4$
$\quad - 11\,v^4w^4 - 2\,v^3w^4 + 7\,v^6w^3 + 14\,v^5w^3 + 8\,v^4w^3 + 15\,v^3w^3 - 8\,v^5w^2 - 10\,v^4w^2 - 11\,v^3w^2 + 3\,v^2w^2 + 2\,vw^2$
$\quad + 7\,v^4w - 4\,v^3w + 14\,v^2w - 12\,vw - w - 2\,v^3 + 2\,v^2 - 2\,v + 2)/(v-1)^2 w(vw-1)^3(vw-v+1)$

$$\quad - \frac{Nv(v^2w^2+1)(2\,v^2w^2 - 2\,v^2w - 2\,vw + v^2 + 1)}{(v-1)^2(vw-1)^3}$$

$c_{11} = C_F(2\,v^6w^6 - 4\,v^6w^5 - 2\,v^5w^5 + v^6w^4 - 6\,v^5w^4 + 7\,v^4w^4 - 14v^3w^4 + v^6w^3$
$\quad + 20\,v^5w^3 - 21\,v^4w^3 + 18\,v^3w^3 + 14\,v^2w^3 - 19\,v^5w^2 + 58\,v^4w^2 - 73\,v^3w^2 + 6\,v^2w^2 + 14\,vw^2$
$- 13\,v^4w + 44\,v^3w - 51\,v^2w + 32\,vw - 14\,w - v^3 + 5\,v^2 - 5\,v + 1)/2\,(v-1)^2 w(vw-1)^2(vw-v+1)$
$\quad + N(v^5w^4 - 2\,v^4w^4 - 3\,v^5w^3 + 4\,v^3w^3 + 3\,v^5w^2 - 4\,v^3w^2 - 4\,v^4w + 9\,v^3w$
$\quad - 4\,vw - 3\,v^3 + 6\,v^2 - 5\,v + 2)/(v-1)(vw-1)^3(vw-v+1)$

$$c_{12} = -\frac{N(v+1)}{v-1} - \frac{8\,C_F}{(v-1)^2}$$

$$c_{13} = \frac{2\,N(5\,v^2+3)}{(v-1)^2} - \frac{2\,C_F(9\,v^2+5)}{(v-1)^2}$$

$$c_{14} = \frac{8\,C_F(v+1)}{v-1} - \frac{N(7\,v^2+1)}{(v-1)^2}$$

$$f_g = \frac{Nv(v^2w^2+1)f_{gq}[(v-1)/(vw-1)]}{C_F(v-1)(vw-1)^2} + \frac{(v^2w^2+1)f_{gq}[(v-1)/(vw-1)]}{(v-1)w}$$

Table 3
For process III.

$\hat{c}_1=4\,T_r/3-11\,N/3$

$\tilde{c}_1=C_F[8\log v-4\log(1-v)+9]/2$

$c_1=C_F[192\,v^2\log^2 v-168\,v\log^2 v+84\log^2 v-192\log(1-v)\,v^2\log v+18\,v^2\log v+192\log(1-v)\,v\log v$
$-42\,v\log v-96\log(1-v)\log v+33\log v-18\log(1-v)\,v^2+20\,\pi^2 v^2+12\,v^2+24\log^2(1-v)\,v-6\log(1-v)\,v$
$-20\,\pi^2 v-12\,v-12\log^2(1-v)-9\log(1-v)+10\,\pi^2+6]/6(2\,v^2-2\,v+1)$
$-N[144\,v^2\log^2 v-126\,v\log^2 v+63\log^2 v-216\log(1-v)\,v^2\log v+216\log(1-v)\,v\log v-18\,v\log v$
$-108\log(1-v)\log v+18\log v+36\,\pi^2 v^2-340\,v^2+36\log^2(1-v)\,v-36\log(1-v)\,v$
$-36\,\pi^2 v+340\,v-18\log^2(1-v)+18\,\pi^2-170]/18\,(2\,v^2-2\,v+1)-20\,T_r/9$

$\tilde{c}_2=6\,C_F$

$c_2=3\,C_F[6\log v-6\log(1-v)+1]-4\,N[\log v-2\log(1-v)]$

$c_3=2^2 C_F$

$c_5=-C_F(2\,v^4w^4-4\,v^4w^3+2\,v^3w^3+6\,v^4w^2+14\,v^3w^2-18\,v^2w^2-vw^2-40\,v^4w+108\,v^3w$
$-106\,v^2w+39\,vw-w+4\,v^4-12\,v^3+14\,v^2-8\,v+2)/(v-1)w(vw-v+1)$
$-Nv(2\,v^4w^4-4\,v^4w^3-4\,v^3w^3+12\,v^4w^2-8\,v^3w^2-v^2w^2-20\,v^4w$
$+48\,v^3w-40\,v^2w+12\,vw+10\,v^4-36\,v^3+51\,v^2-34\,v+9)/(vw-v+1)^3$

$$c_6=\frac{2\,C_F v(2\,v^2w^2+2\,v^2w-4\,vw+4\,v^2-6\,v+3)}{v-1}-Nv$$

$c_7=Nv(3\,v^3w^3-8\,v^4w^2+15\,v^3w^2-9\,v^2w^2+16\,v^4w-47\,v^3w+46\,v^2w-15\,vw-8\,v^4+29\,v^3-41\,v^2+27\,v-7)/(vw-v+1)^3$

$$+\frac{4\,C_F v(2\,vw-2\,v+1)}{vw-v+1}$$

$c_8=-C_F v(6\,v^4w^4-4\,v^4w^3-8\,v^3w^3+6\,v^4w^2-14\,v^3w^2+17\,v^2w^2+24\,v^4w$
$-66\,v^3w+70\,v^2w-31\,vw-24\,v^3+61\,v^2-55\,v+18)/(v-1)(vw-1)(vw-v+1)$

$$-\frac{Nv(2\,v^2w^2+4\,vw-14\,v^2+22\,v-11)}{vw-v+1}$$

$c_9=C_F(4\,v^3w^3-6\,v^3w^2-14\,v^2w^2-vw^2+40\,v^3w-54\,v^2w+26\,vw-w-6\,v^3+12\,v^2-9\,v+3)/w(vw-v+1)$

$$-\frac{Nv(2\,v^2w^2-5\,vw+10\,v^2-13\,v+6)}{vw-v+1}$$

$c_{10}=Nv(3\,v^3w^3+v^3w^2-v^2w^2-7\,v^3w+12\,v^2w-5\,vw+3\,v^3-9\,v^2+9\,v-3)/(vw-v+1)^3$
$-C_F(2\,v^4w^4-2\,v^3w^3+4\,v^4w^2+2\,v^3w^2-5\,v^2w^2-8\,v^4w+30\,v^3w-36\,v^2w+14\,vw+2\,v^4-6\,v^3+7\,v^2-4\,v+1)/(v-1)w(vw-v+1)$

$\tilde{c}_{11}=-C_F(4\,v^7w^7-12\,v^7w^6+4\,v^6w^6+20\,v^7w^5-15\,v^6w^5+2\,v^5w^5-v^4w^5-36\,v^7w^4+67\,v^6w^4-50\,v^5w^4+19\,v^4w^4-2\,v^3w^4$
$+44\,v^7w^3-119\,v^6w^3+128\,v^5w^3-68\,v^4w^3+15\,v^3w^3-24\,v^7w^2+63\,v^6w^2-44\,v^5w^2-16\,v^4w^2+34\,v^3w^2-15\,v^2w^2+2\,vw^2+4\,v^7w$
$+4\,v^6w$
$-64\,v^5w+144\,v^4w-149\,v^3w+79\,v^2w-19\,vw+w-4\,v^6+20\,v^5-42\,v^4+48\,v^3-32\,v^2+12\,v-2)/(v-1)w(vw-1)(vw-v+1)^3$

$$-\frac{Nv(v^2w^2+v^2-2\,v+1)(2\,v^2w^2-4\,v^2w+2\,vw+2\,v^2-2\,v+1)}{(vw-v+1)^3}$$

$c_{11}=C_F(6\,v^7w^7-16\,v^7w^6+4\,v^6w^6-2\,v^7w^5+38\,v^6w^5-27\,v^5w^5-2\,v^4w^5+36\,v^7w^4-130\,v^6w^4+122\,v^5w^4-25\,v^4w^4-4\,v^3w^4-26\,v^7w^3$
$+92\,v^6w^3-134\,v^5w^3+116\,v^4w^3-48\,v^3w^3-4\,v^7w^2+28\,v^6w^2-44\,v^5w^2+2\,v^4w^2+40\,v^3w^2-26\,v^2w^2+4\,vw^2+6\,v^7w-26\,v^6w$
$+45\,v^5w-50\,v^4w+48\,v^3w-30\,v^2w+5\,vw+2\,w-6\,v^6+30\,v^5-61\,v^4+64\,v^3-36\,v^2+10\,v-1)/2\,(v-1)w(vw-1)(vw-v+1)^3$
$+N(2\,v^6w^5-4\,v^6w^4-v^5w^4-v^4w^4+4\,v^6w^3-v^5w^3+5\,v^4w^3-2\,v^3w^3-4\,v^6w^2+5\,v^5w^2+6\,v^4w^2-11\,v^3w^2$
$+2\,v^6w-v^5w-13\,v^4w+19\,v^3w-9\,v^2w+2\,vw-2\,v^5+7\,v^4-8\,v^3+4\,v^2-2\,v+1)/(vw-1)(vw-v+1)^3$

$c_{12}=-N(8\,v^2-14\,v+7)-8\,C_F(2\,v-1)$

$c_{13}=2\,C_F(18\,v^2-14\,v+7)-N(8\,v^2-6\,v+3)$

$c_{14}=2\,N(2\,v-1)-8\,C_F v^2$

$$f_g=\frac{2\,C_F(v^2w^2+v^2-2\,v+1)d_{qg}(vw-v+1)}{(v-1)w(vw-v+1)}+\frac{2\,Nv(v^2w^2+v^2-2\,v+1)d_{qg}(vw-v+1)}{(vw-v+1)^3}$$

choice $\mu^2 = t$ is made. It is interesting to note that, indicating by Q^2 the arbitrary momentum squared which sets the scale of the virtual radiative corrections (see, for example, eq. (2.9) of ref. [4]), which is in principle distinct from μ^2 and M^2, the dependence on Q^2 drops out in the final expressions for the three processes under consideration.

In the case of inclusive jet production at large p_T the expressions (3), (7) have to be used, with the fragmentation function $D(x_3, M^2)$ replaced by $\delta(1-x_3)$, i.e. $p_1 = K_3$ ($p_T \equiv K_{3T}$). Of course this substitution has to be made only after cancelling all $(1/\varepsilon)$ mass singularities.

We would like to discuss now the jet–jet relative transverse momentum distributions. Besides to pure QCD tests, these distributions are of interest for background estimates to other rare standard model processes (i.e. $Q\bar{Q}$ production, $t \to W + X$, ...) or unconventional production of jets from new physics mechanisms. The $o(\alpha_s^2)$ results from ($2 \to 3$) parton subprocesses are known [5], as well as the leading terms corresponding to soft multigluon emission [8]. In the following we give our results, following closely the analysis of ref. [9], for the p_T distributions of vector bosons and Drell–Yan pairs.

In the parton subprocess $p_i(k_1) + p_j(k_2) \to p_l(k_3) + p_m(k_4) + g(k)$ we define $k_\perp^2 \equiv (k_4 + k)^2 = sv(1-w)$. Indeed the elastic process $p_i + p_j \to p_l + p_m$ corresponds to $\delta(1-w) = sv\,\delta(k_\perp^2)$, as is clear from eq, (3). Using $D(x_3, M^2) = \delta(1-x_3)$ and transforming the variables (x_1, x_2) or (v, w) to $(v, k_\perp^2)$ one gets from eq. (3)

$$E_3 \frac{d\sigma}{d^3k_3} = \sum_{i,j} \frac{1}{\pi S} \int_{VW}^{V} \frac{dv}{1-v} \int_0^{A_t^2} dk_\perp^2 F_{p_i}^{H_1}(x_1, M^2) F_{p_j}^{H_2}(x_2, M^2)$$

$$\times \left(\frac{1}{v} \frac{d}{dv} \sigma^0_{p_i p_j \to p_l}(s, v) \delta(k_\perp^2) + \frac{\alpha_s(\mu^2)}{2\pi v s} K_{p_i p_j \to p_l}(s, v, w) \right), \tag{9}$$

where

$$x_2 = \frac{1-V}{1-v}, \quad x_1 = \frac{VW}{vw}, \quad w = \frac{SVW(1-V)}{k_\perp^2(1-v) + SVW(1-V)}, \tag{10}$$

and

$$A_t^2 = S(1-V)(v - VW)/(1-v).$$

From the structure of $K(s, v, w)$, explicitly shown in eq. (7), it follows that the integrand of eq. (9) is of the general form $A\delta(k_\perp^2) + B[1/k_\perp^2]_+ + C_3[\ln(k_\perp^2)/k_\perp^2]_+ + Y$, with Y finite as $k_\perp \to 0$ and C_3 given in eq. (7). Furthermore, as in the Drell–Yan case [9], the distribution functions $F_i(x, M^2)$ evoluted at the scale $M^2 \sim A_t^2$, possess the convenient structure to reproduce, in the b impact parameter space, the expected $1/b^2$ dependence.

Without entering into details, which will be given elsewhere, after performing the Fourier transform of the singular part (as $k_\perp \to 0$) into impact parameter space, and exponentiating the leading logarithmic terms one finally gets, for the reaction $p_i + p_j \to p_l + \ldots$ a structure of the type

$$E_3 \frac{d\sigma}{d^3k_3\, dk_\perp^2} = \frac{1}{\pi S} \sum_{i,j} \int_{VW}^{V} \frac{dv}{1-v} \left(\frac{1}{v} \frac{d\sigma^0(s_0, v)}{dv} \int \frac{d^2\boldsymbol{b}}{4\pi} \exp(-i\boldsymbol{b}\cdot\boldsymbol{k}_\perp) F_i(x_1^0, 1/b^2) F_j(x_2^0, 1/b^2) \right.$$

$$\left. \times \exp[S(b^2, A_t^2, s_0, v)][1 + o(\alpha_s)] + Y(k_T^2, v, s) \right), \tag{11}$$

where x_1^0, x_2^0 and s_0 are calculated at $k_\perp^2 = 0$, and the Sudakov form factor is of the form

$$S(b^2, A_t^2, s_0, v) = \int_0^{A_t^2} \frac{dk^2}{k^2} \frac{\alpha_s(k^2)}{2\pi} [J_0(bk) - 1][+C_3 \ln(k^2/s_0 v) + B(s_0, v)], \tag{12}$$

with $B(s_0, v) = C_2 + \tilde{C}_2 \ln(s_0/M^2)$.

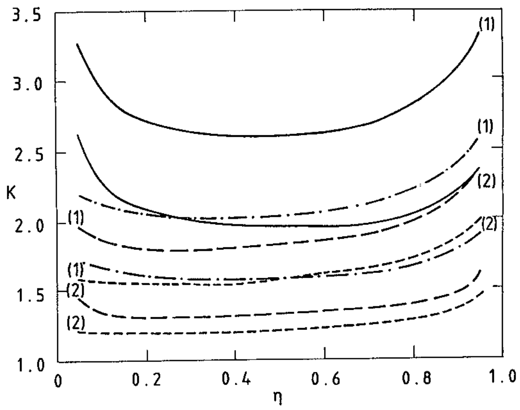

Fig. 1. The ratio K of $o(\alpha_s^2)+o(\alpha_s^3)$ jet cross section to the $o(\alpha_s^2)$ jet cross section as a function of η for process I. Curves labelled by (1) correspond to the choice $M^2=\mu^2=s$ and curves labelled by (2) to the choice $M^2=\mu^2=p_T^2$. Solid curves (respectively dot-dashed curves): K with lowest order evaluated with α_s at two-loop order for $\sqrt{S}=62$ GeV (respectively $\sqrt{S}=1.8$ TeV). Dashed curves (respectively small dashed curves): K with lowest order evaluated with α_s at one-loop order for $\sqrt{S}=62$ GeV (respectively $\sqrt{S}=1.8$ TeV).

This concludes the discussion of the jet–jet transverse momentum properties.

As a preliminary indication of the phenomenological consequences of our results, we show in the following the effect of the one-loop corrections to the three processes (1). We consider pp collisions at $\sqrt{S}=62$ GeV and 1.8 TeV. Of course one expects gluon–gluon interactions, which are not included in this analysis to be dominant at the latter energy. We use the set of structure functions of Duke–Owens [10] for $\Lambda=200$ MeV. We give the ratio K of the $o(\alpha_s^2)+o(\alpha_s^3)$ jet cross section to the $o(\alpha_s^2)$ jet cross section at $\theta=90°$ as a function of $\eta=2p_T/\sqrt{s}$, p_T being the transverse momentum of the jet. Our results are shown in figs. 1–3 for $M^2=\mu^2=p_T^2$ and $M^2=\mu^2=s$ with two choices for the lowest order jet cross section: α_s evaluated at two-loop order and α_s evaluated at one-loop order. The corrections to processes (1) are found to be large in the first case. They decrease by roughly 40% in the second case, indicating a sensitivity to the lowest-order prediction. The largest value is obtained for $M^2=\mu^2=s$. The size of the QCD corrections decreases when S increases. The general trend of fig. 1, which of course reflects the results already obtained in ref. [1], is maintained in the other two processes under consideration.

A more complete phenomenological analysis, including also gluon–gluon interactions, will be given elsewhere.

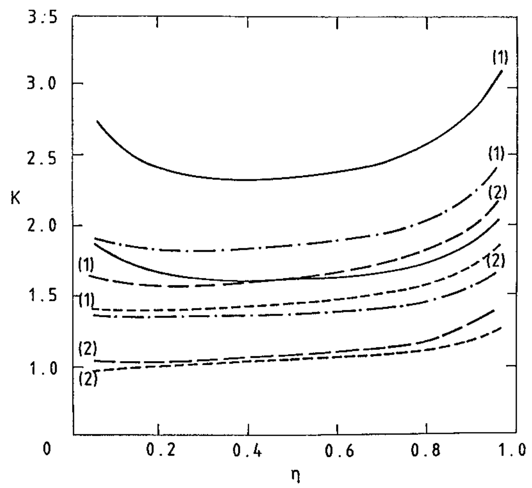

Fig.2. Same caption as in fig. 1 for process II.

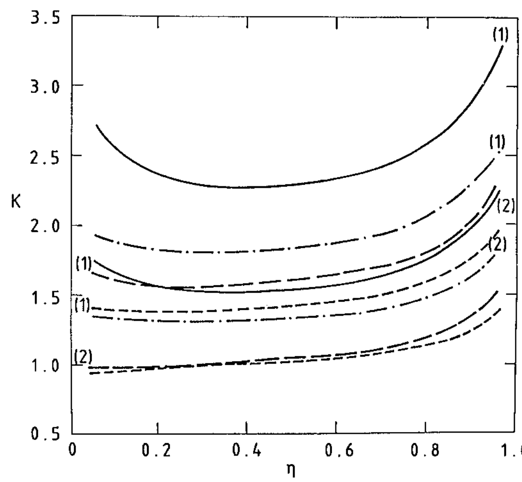

Fig. 3. Same caption as in fig. 1 for process III.

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