



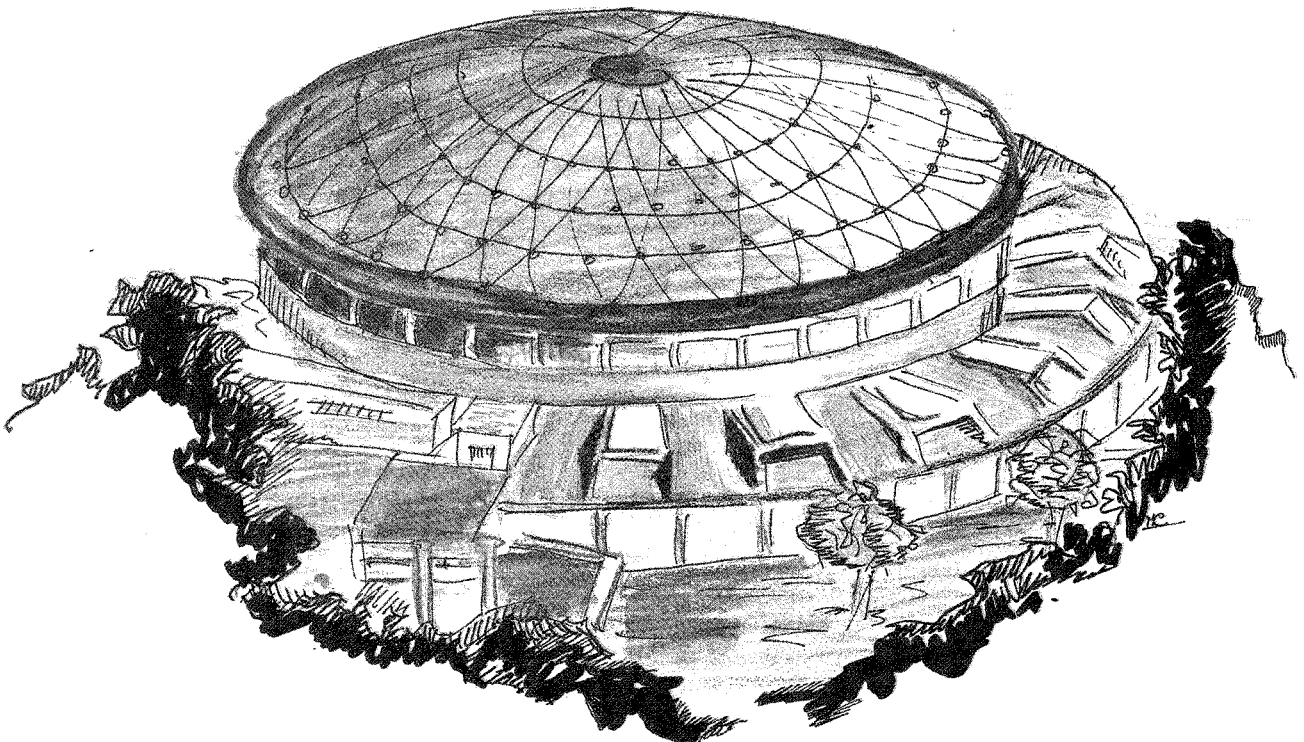
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## A NEW MODEL FOR ELASTIC DEUTERON-DEUTERON SCATTERING

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### ABSTRACT

Straightforward application of the Glauber multiple scattering theory is dramatically challenged by data on elastic deuteron-deuteron (d-d) scattering. The challenge has been argued to be met by an improved representation of the ground state wave function of the deuteron as an admixture of S- and D-waves. In the light of the failure of the Glauber and geometrical picture models in general, to explain proton-proton (pp) and proton-antiproton ( $p\bar{p}$ ) scattering data up to and including collider energies and for all momentum transfers, this argument becomes less and less compelling and more and more unconvincing. A model inspired by unitarity and which produces substantial elastic scattering through a unitarity sum over a specific class of intermediate states is presented. The model fits not only d-d, but also pp,  $p\bar{p}$  and  $\alpha N \rightarrow \alpha N$  ( $N = \alpha, d, {}^3\text{He}$ ) data for all energies and momentum transfers. No detailed knowledge of ground state wave functions is required.

### 1. - INTRODUCTION

In the light of recent experimental results on proton-proton (pp) and proton-antiproton ( $p\bar{p}$ ) collisions at collider energies<sup>(1)</sup>, it seems that a profound revision of the geometrical picture of diffraction scattering<sup>(2,3)</sup> is becoming necessary. By geometrical picture<sup>(4)</sup> we mean, generally, that

model of hadron-hadron, hadron-nucleus and nucleus-nucleus scattering patterned after diffraction phenomena in optics. It comprises, therefore, the eikonal approximation and, more specifically the Glauber<sup>(2)</sup> and Chou-Yang models<sup>(3)</sup>. The characteristic of this picture is that a high energy projectile passing through a composite target (e.g. a nucleon made up of quarks or a nucleus made up of nucleons) suffers very little deviation of its path and is outside of the target long before the changes it induces in the target take place. Consequently when the projectile collides with any one of the constituents of the target, the others can, for all practical purposes, be considered to be inactive spectators. Thus arises the spectator model. Because the projectile suffers very little deviation of its trajectory, it loses, on average, a very small fraction of its energy per collision with a target constituent. Unless the target is unusually dense, multiple collisions with constituents are rare and contribute small calculable corrections to the dominant single one particle collisions. These multiple scattering corrections appear in the form of a cluster-type expansion<sup>(2)</sup> with successive multiple scattering terms contributing with opposite signs. Interferences between the multiple scattering terms are, therefore, expected to give rise to dips and re-enforcements in the differential cross-section. The first such interference dip has been observed in many reactions but the successive ones have been disappointingly absent, particularly, in hadron-hadron cross-sections<sup>(1,5)</sup>. In hadron-nucleus<sup>(5)</sup> and nucleus-nucleus<sup>(7)</sup> collisions there are additional structures in the differential cross sections which have been interpreted as evidence of these multiple scattering dips. These additional structures are not present in deuteron-deuteron (d-d) differential cross-sections<sup>(8)</sup> indicating a limitation of the above interpretation.

In the case of d-d scattering the absence of multiple dips has been explained, but not completely satisfactorily, by the assumption that the deuteron ground state wave function is not pure S-wave but a superposition of S- and D-waves<sup>(9)</sup>. In hadron-hadron scattering the absence of multiple dips has, at first, been extremely baffling.

Attempts to explain it point to the obvious fact that the scattering amplitude cannot be purely imaginary, as simple geometrical models assume. Ad hoc complexifications of the amplitude have therefore been attempted. They are no more successful than the simple, purely imaginary amplitude models: the multiple dips persist for high momentum transfers. There is a further complication: these complexifications can no longer be carried out in an ad hoc manner. Experiments at collider energies have been able to measure the ratio of real to imaginary parts of the amplitude<sup>(10)</sup>. Many theoretical models cannot accommodate the results of these measurements: the ratio of real to imaginary parts seems "unusually" large.

One conclusion that is obliged to be drawn from this analysis is that high energy hadron-hadron as well as hadron-nucleus and nucleus-nucleus collisions may not, after all, be as similar to optical diffraction as has hitherto been assumed. The optical analogy derives basically from the picture of the target as a geometrical obstacle in the path of incoming high frequency waves to which one applies Fraunhofer diffraction theory. Fraunhofer diffraction theory explains how light "bends round corners" and leads to a structured penumbra characterized by alternating destructive and constructive

interference pattern. There is certainly much more in high energy elastic and quasi-elastic hadron-hadron scattering than the geometry of the target as an obstacle along an optical path: there is unitarity. The opening up of a large number of multi-particle channels at high energies substantially reduces the probability for direct two-particle to two-particle scattering. These two-to-two-particle scatterings, including elastic scattering, occur therefore mainly through the intermediate state mediation of these huge number of multi-particle states. There is thus, as a consequence of unitarity, substantial elastic scattering where very little would have been expected from direct two-to-two-particle scattering. In the optical analogy, this is similar to the existence of the penumbra where there should have been shadow from strict application of linear (geometrical) optics. The analogy however stops there. Elastic final states in hadron-hadron scattering is produced more by the characteristic interaction responsible for the multi-particle intermediate states than by the geometry of the target as an assembly of scattering centres, or equivalently as an obstacle along an optical path. Multiple dips in the cross-section, corresponding to a structured penumbra in the optical analogy, may not be produced in the case of elastic hadron scattering which takes place as above described. In other words, unitarity contributions of multi-particle intermediate states to elastic scattering do not correspond to and are not to be compared with the multiple scattering corrections of the geometrical picture. Consequently from the point of view of the overall experimental situation on high energy scattering, the apparent success of the latter theory in explaining the structures in some hadron-nucleus and nucleus-nucleus cross-sections seems now more of an accident than a consistent approximation to the underlying physical phenomena. Unfortunately there is as yet no model of high energy diffraction scattering based directly on the unitarity contributions of multi-particle intermediate states. In support of the need for such a model we quote the work of Landshoff and Donnachie<sup>(11)</sup> based on S-matrix theory and a dynamical model for the Pomeron. These authors obtain quite satisfactory fits for  $pp$  and  $p\bar{p}$  data at collider energies. Their model is not of the geometrical type. Effectively it produces elastic scattering through a particular class of intermediate states represented by the exchange of the Pomeron.

The purpose of this paper is to present a simple model of elastic scattering based upon summation over the unitarity contributions of a specific class of intermediate states. The model is a hybrid in two respects:

- (i) it combines the multiple scattering approximation of geometrical models, restricted however to a so-called "hard" part of the amplitude. Multi-particle unitarity contributions are associated with the "soft" part of the amplitude;
- (ii) the complexification of the amplitude is not carried out in an analytic manner but results from separate approximations for the real and imaginary parts. The one, that is the real part, is assumed to be overwhelmingly dominant at high momentum transfers and the other, the imaginary part, at small momentum transfers.

To stress the relevance of the multi-particle contributions we apply this model to deuteron-deuteron scattering where straightforward application of geometrical models runs into difficulty. The success of the model in this particular application underlines the fact that there is an alternative explanation to this "recalcitrant" case which does not require any detailed knowledge of the ground state wave function of the deuteron. The S-wave Gaussian wave function is found to be amply sufficient. There is more to it than this: the same model has been applied with equal level of success to  $pp^{(12,13)}$ ,  $p\bar{p}^{(12,13)}$  and  $\alpha N \rightarrow \alpha N$  ( $N = \alpha, d, {}^3\text{He}$ )<sup>(14)</sup> collisions at different CM energies and all available momentum transfers. The conclusion is then that, unlike geometrical multiple scattering models, this hybrid scheme provides a better approximation to the dynamics of all of these processes.

A description of the model is given in the next section. Comparison with data on d-d scattering together with comments and conclusions, is given in sect. 3.

## 2. - DESCRIPTION OF THE MODEL

In view of the intermediate states approach which we intend here to develop, consider the S-matrix picture of the scattering process  $|i\rangle \rightarrow |f\rangle$  for any given pair of initial and final states  $|i\rangle$ ,  $|f\rangle$ , respectively.

Let us express the S-matrix in the usual form

$$S := 1 + iT \tag{1}$$

with T, the scattering operator, given in the form

$$T = e^{iA} T_0 e^{-iA} \tag{2}$$

where A is Hermitian and  $T_0$  is some basic scattering operator. Eq. (2) is quite general and follows only from the unitarity of S. It may therefore be specialised for particular uses. We do so with the assumption that  $T_0$  describes the "hard" scattering part of T while the operator A allows to take into account the "soft" parts. This assumption is inspired by the so-called Bremsstrahlung model<sup>(15)</sup> or equivalently by the theory of infrared radiative corrections<sup>(16)</sup>. It entails that the dominant part of the scattering, described by T, is contained in the operator  $T_0$  while the remaining part can be calculated as corrections through the operator A.

By assumption, A is a "soft" operator in the sense that it "creates" and "annihilates" particles with negligibly small energy and momentum, so small in fact that an infinite number of such particles leads only to a very small change in the total energy-momentum in a given process. Consequently a given state  $|a\rangle$  and  $|a'\rangle = e^{iA}|a\rangle$  are almost degenerate in energy-momentum.

In this sense these states may be said to be "close", "closeness" being defined, in this case, with respect to the 4-momentum operator. The idea of "closeness" of states may be defined with respect to other operators.

There is a general intuitive understanding that the phenomenon of diffraction involves transitions between states which are "close" in the above sense. This intuitive understanding forms the Good-Walker<sup>(17)</sup> approach. There is a need, therefore, for some formal definition of this concept, at least as far as diffraction is concerned. To this end, one starts from the Good-Walker model<sup>(17)</sup> of diffraction and assumes that the operator  $T_0$  in Eq. (2) is diagonal in the basis of physical states  $|i\rangle$ ,  $|j\rangle$ ,  $|k\rangle$ , ..., that is<sup>(12)</sup>

$$T_0 |i\rangle = \eta_i |i\rangle \quad (3)$$

with eigenvalues  $\eta_i$ .

Taking matrix elements of (2) between the states  $|i\rangle$ ,  $|f\rangle$  one gets

$$\langle f | T | i \rangle := T_{fi} = \sum_k \eta_k \langle f | e^{iA} | k \rangle \langle k | e^{-iA} | i \rangle \quad (4)$$

where, if  $T_{fi}$  is diffractive, then by assumption, not only the states  $|i\rangle$ ,  $|f\rangle$  are "close" but are also "close" to a subset of the set of intermediate states  $|k\rangle$  ( $k = 1, 2, \dots$ ) which dominate in the sum in Eq. (4). The problem has, thus, been reduced to specifying the action of the operator  $A$  between any pair of "close" states. Alternatively, this means defining the concept of "closeness" in the Hilbert space of physical states  $|i\rangle$ ,  $|j\rangle$ ,  $|k\rangle$ , ... with respect to the operator  $A$ .

To do this we introduce the operator

$$D = \frac{1}{2} (1 - e^{iA}) \quad (5)$$

and note that it satisfies the unitarity relation

$$DD^\dagger = D^\dagger D = \frac{1}{2} (D + D^\dagger) \quad (6)$$

where

$$D^\dagger = \frac{1}{2} (1 - e^{-iA}) \quad (7)$$

is the Hermitian adjoint of  $D$ .

Two states  $|i\rangle$  and  $|j\rangle$  will be said to be "close", or equivalent, with respect to A ( $|i\rangle \equiv |j\rangle \pmod{A}$ )(\*) if there exists a state  $|\lambda\rangle$  and non-zero complex functions  $\varphi_{i\lambda}$  and  $\varphi_{j\lambda}$  such that

$$\frac{D|i\rangle}{\varphi_{i\lambda}} = \frac{D|j\rangle}{\varphi_{j\lambda}} = D|\lambda\rangle \quad (8)$$

This is an equivalence relation. By iterating it, it is easy to establish that the state  $|\lambda\rangle$  is an eigenstate of D, that is

$$D|\lambda\rangle = \lambda|\lambda\rangle \quad (9)$$

where  $\lambda$  is the corresponding eigenvalue. Making use of (8) in (4) one gets

$$T_{fi} = \eta_i \delta_{fi} + 2 \varphi_{f\lambda}^* \varphi_{i\lambda} [i(\Delta\eta_{i\lambda} - \Delta\eta_{f\lambda}) \text{Im}(\lambda) + (\Delta\eta_{i\lambda} + \Delta\eta_{f\lambda}) \text{Re}(\lambda)] \quad (10)$$

where

$$\Delta\eta_{i\lambda} := \eta_\lambda - \eta_i \quad (11)$$

and

$$\eta_\lambda = \sum_k \eta_k |\varphi_{k\lambda}|^2 = \langle \lambda | T_o | \lambda \rangle = \langle \lambda | T | \lambda \rangle \quad (12)$$

Note, how with Eq. (8), one is able to carry out the summation in Eq. (4) over the set of "close" intermediate states, giving rise to Eq. (10).

Note too, that, on account of (6), the eigenvalue  $\lambda$  satisfies

$$|\lambda|^2 = \text{Re}(\lambda) \quad (13)$$

For elastic scattering, Eq. (10) therefore becomes

$$T_{ii} = \eta_i + 2\lambda |\varphi_{i\lambda}|^2 \Delta\eta_{i\lambda} \quad (14)$$

This formula is simple to apply. Consider deuteron-deuteron (d-d) scattering for definiteness. The first term in Eq. (14), corresponds, in the geometrical picture, to scattering by a black disk; that is (with  $|i\rangle \equiv |dd\rangle$ )

$$\eta_{dd}(s,b) = \frac{\sigma_{dd}(s)}{4\pi a_d(s)} e^{-b^2/2 a_d(s)}; \quad (15)$$

---

(\*) Or equivalently  $|i\rangle \equiv |j\rangle \pmod{D}$  since D is a function of A.

in the impact parameter representation  $\sigma_{dd}(s)$  is the total d-d cross-section at CM energy  $\sqrt{s}$ ;  $a_d(s)$  is the corresponding slope.

The states equivalent to the initial deuteron-deuteron state  $|dd\rangle$  are assumed to be  $|dd, nq\rangle$ , where  $nq$  stands for  $n$  quanta ( $n = 0, 1, 2, 3, \dots$ ). The quanta may be quarks, pions etc. For the wave functions of these states we take<sup>(18)</sup>

$$|\varphi_{i\lambda}|^2 := |\varphi_{ddn}(b_1 \dots b_n)|^2 = P_n \prod_{j=1}^n |\psi(b_j)|^2 \quad (16.a)$$

$$\psi(b) = \frac{e^{-b^2/4R_0}}{\sqrt{2\pi R_0}} \quad (16.b)$$

$$P_n = e^{-\bar{n}} \frac{(\bar{n})^n}{n!} \quad (16.c)$$

$R_0$  is a free parameter. The wave function of the incoming d-d state is taken to be a plane wave in Eq. (16.a).

The  $n$  quanta are assumed to be Poisson distributed according to Eq. (16.c). For the elastic amplitudes  $\eta_{ddn}(s, b, b_1, \dots, b_n)$  we use the Glauber model<sup>(2,12-14)</sup> ..., i.e.

$$\eta_{ddn}(s, b, b_1 \dots b_n) = 1 - (1 - \eta_{dd}(s, b)) \prod_{j=1}^n (1 - \eta_{dq}(s, b, b_j)) \quad (17)$$

where, similar to  $\eta_{dd}(s, b)$  in Eq. (15), we take

$$\eta_{dq}(s, b) = \frac{\sigma_{dq}(s)}{4\pi a_q(s)} e^{-b^2/2a_q(s)} \quad (18)$$

$\sigma_{dq}(s)$  is thus the total deuteron-quantum cross-section at energy  $\sqrt{s}$  and corresponding slope  $a_q(s)$ . Substituting from eqs. (15) - (18) into (14) and taking Fourier transform, one finds, for the elastic differential cross-section

$$\frac{d\sigma_{dd}(s, t)}{d|t|} = \pi |T_G(s, t)|^2 \quad (19.a)^{(*)}$$

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(\*)  $T_G(s, t)$  stands for the amplitude given by the Glauber model ( $G \equiv$  Glauber)



$$T_G(s, t) = i \int_0^{\infty} db b J_0(b\sqrt{t}) T_G(s, b) \quad (19.b)$$

$$T_G(s, b) = \eta_{dd}(s, b) + |2\lambda|^2 (1 - \eta_{dd}(s, b)) (1 - e^{-h(s, b)}) \quad (19.c)$$

where

$$h(s, b) = \frac{\bar{n} \sigma_{dq}(s)}{4\pi R(s)} e^{-b^2/2 R(s)} \quad (20.a)$$

$$R(s) = R_0(s) + a_q(s) \quad (20.b)$$

Note from eqs. (15) - (20) that the scattering amplitude  $T_G(s, t)$  is purely imaginary.

This amplitude alone does not fit the data for large momentum transfers  $t^{(12)}$ . It leads to multiple dips which are not observed experimentally.

The combination<sup>(13,14)</sup>

$$T(s, b) = T_G(s, b) - i T_{CY}(s, b) \quad (21)$$

where

$$T_{CY}(s, b) = 1 - e^{-h_{CY}(s, b)} \quad (22.a)$$

$$h_{CY}(s, b) = \frac{\bar{n} \sigma_{dq}(s)}{4\pi} \frac{\mu^2}{48} (\mu b)^3 K_3(\mu b) \quad (22.b)$$

complexifies the scattering amplitude but not in an analytic fashion.  $T_{CY}(s, b)$  is the Chou-Yang model amplitude<sup>(3)</sup> with the form factor of the deuteron taken to be of the dipole form  $F_d(t) = (1 + |t|/\mu^2)^{-2}$ , just as for the proton<sup>(13,14)</sup>. The mass scale parameter  $\mu$  is to be fitted to experiment. The Chou-Yang amplitude thus contributes a purely real part to the total amplitude  $T(s, t)$  defined, from eqs. (19) and (21) by

$$T(s, t) = i \int_0^{\infty} db b J_0(b\sqrt{t}) T(s, b) \quad (23)$$

The rationale behind the Ansatz in Eq. (21) is the following: separately the Glauber and Chou-Yang models fit the data in the near forward direction and both quickly lead to multiple diffraction

dips<sup>(\*)</sup> as  $t$  increases. A purely imaginary or purely real amplitude severely ruins the unitarity of the amplitude and leads to the unacceptable multiple dip structure in the differential cross-section.

Because  $T(s, b)$ , as given by Eq. (21), is not an analytic function<sup>(\*\*)</sup> it does not give rise to a unitarity S-matrix. It does, however, give rise to a possibility to eliminate the dips in the differential cross-section by superposing incoherently the two amplitudes  $T_G(s, t)$  and  $T_{CY}(s, t)$  whose dip structures do not match. The dip structures depend on the parameters of the models. Their matching would be a remarkable accident.

The choice of fit parameters in the amplitudes  $T_G(s, t)$  and  $T_{CY}(s, t)$  is made in such a way that the one ( $T_G(s, t)$ ) dominates for  $t \rightarrow 0$  while the other ( $T_{CY}(s, t)$ ) for  $|t| \rightarrow \infty$ . This choice of parameters is non-standard: the cross-section  $\sigma_{dq}(s)$  in eqs. (20.a) and (22.b) is very small and does not correspond to the values which obtain in the usual fits of the Glauber or Chou-Yang models to data. The consequence of this is to push (but does not eliminate) the oscillations in  $|T_G(s, t)|^2$  and  $|T_{CY}(s, t)|^2$  out to  $|t| \rightarrow \infty$ . But by then  $|T_G(s, t)|^2$  is so small with respect to  $|T_{CY}(s, t)|^2$  to be completely negligible. Vice versa  $|T_{CY}(s, t)|^2$  is so small with respect to  $|T_G(s, t)|^2$  for  $t \rightarrow 0$  as not to influence the first dip structure in the latter. In the intermediate  $t$ -region the two amplitudes combine incoherently to enhance each other's contributions with no possibility for dips. The success of the approach depends therefore largely on the fact that the structures of these two model amplitudes are well known. It is a hybrid model. Comparison with data is carried out in the next section.

### 3. - COMPARISON WITH DATA

The model described in sect. 2 is compared with data on elastic d-d scattering in Figs. (1) - (3) for CM energies<sup>(8)</sup>  $\sqrt{s} = 4, 6.12$  and  $63$  GeV, respectively. The fits have only one dip in the differential cross-section, in agreement with experiments.

Important too is the fact that the agreement is not restricted to the near forward direction (i.e.  $t \rightarrow 0$ ) only but extends to all the available values of momentum transfer, with  $t$  going up to  $2$  (GeV)<sup>2</sup>. To appreciate the importance of this result, two observations must be made:

- (i) the same model fits alpha-nucleus ( $\alpha N \rightarrow \alpha N$ ,  $N = \alpha, d, {}^3\text{He}$ ) elastic differential cross-sections, for various energies and all available momentum transfers<sup>(14)</sup> as well as  $pp$  and  $p\bar{p}$  data up to collider energies<sup>(12,13)</sup>;

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(\*) The Glauber and Chou-Yang models are two different realisations of the geometrical picture of diffraction scattering.

(\*\*) The real and imaginary parts of  $T(s, t)$  come from completely different models (or dynamics) and cannot therefore satisfy the Cauchy integral (i.e. dispersion) relation linking the real to the imaginary parts of an analytic function.

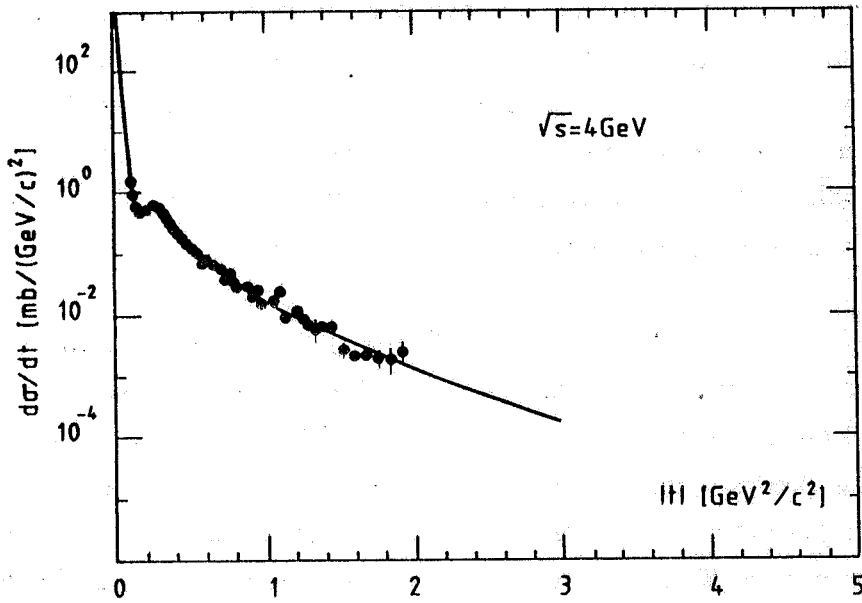


FIG. 1 - Plot of d-d elastic differential cross section  $d\sigma(s,t)/dt$  against  $t$  for CM energy  $\sqrt{s}=4 \text{ GeV}$ . The data points are from Ref. (8a).

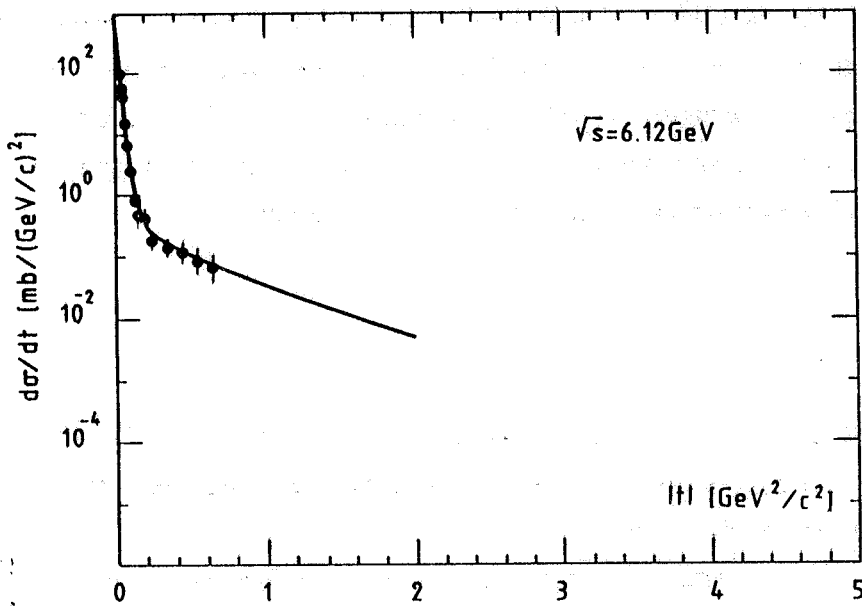


FIG. 2 - Same as in Fig. 1, for  $\sqrt{s}=6.12 \text{ GeV}$  and data points from Ref. (8b).

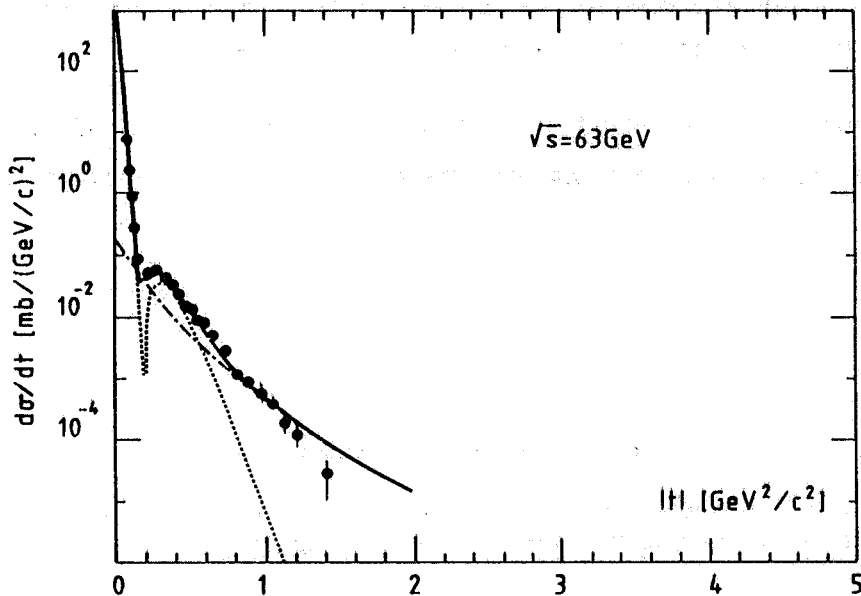


FIG. 3 - Same as in Fig. 1, for  $\sqrt{s}=6.3 \text{ GeV}$  and data points from Ref. (8c).

- (ii) a detailed knowledge of the ground state wave function of the nucleus or nucleon in these comparisons is not required. In fact an S-wave ground state described by a Gaussian is used in all these cases and is found to be sufficient. On the contrary, d-d scattering was always found to provide a challenge to the Glauber model and that the challenge could be met by taking the ground state of the deuteron to be an admixture of S- and D-waves<sup>(9)(\*)</sup>. Our model, therefore, provides an alternative explanation for the deuteron data which is, besides, applicable to all other processes.

A few comments on the fit parameters are in place. These parameters are given in Table (I) for the three energies  $\sqrt{s} = 4, 6.12$  and  $63$  GeV. In place of  $\sigma_{dq}$  (s) the table gives values for the product  $\sigma_q$  (s) =  $\bar{n} \sigma_{dq}$  (s). Compared to the d-d total cross-section  $\sigma_{dd}$  (s),  $\sigma_q$  (s) is small, as pointed out in sect. (2). Secondly the mass scale parameter  $\mu$  in the dipole form factor is not the same as for the nucleon form factor ( $\mu = 0.85$  GeV) although not very much different from it. The parameter  $|\lambda|^2$  is large and of the same order as found for  $pp^{(12,13)}$   $p\bar{p}^{(12,13)}$  and in  $\alpha N^{(14)}$  scattering.

TABLE I - Values of the parameters used in the fits.

$\sqrt{s}$ (GeV)	$ \lambda ^2$	$a_{dd}$ (GeV/c) <sup>-2</sup>	$\sigma_{dd}$ (mb)	$R$ (GeV/c) <sup>-2</sup>	$\sigma_q$ (mb)	$ \lambda ^2 \sigma_q$ (mb)	$m$ (GeV)
4	113	128	559	44	2.2	249	1.24
6.12	84	1451	618	99	1.3	112	1.56
63	90	104	450	39	0.96	86	0.938

There are very many parameters in the model: five in all. This number is not more than in the usual versions of the geometrical model. They arise mostly from the simplification of using Gaussians for the wave functions  $\psi_{i\lambda}$  (cf. Eq. (16)) and for the input scattering amplitudes  $\eta_{dd}$  (s, b) and  $\eta_{dq}$  (s, b).

Less understandable is the dependence of these parameters on the CM energy  $\sqrt{s}$ . Again, this is a feature shared by geometrical models. The specific values of these parameters should perhaps not be taken too seriously. The quality of the fits remains over a fairly wide range of values of the parameters. Most important for the success of the approach is the change in the point of view: that is, from a model of multiple scattering from a geometrical distribution of scattering centres to a unitarity inspired model for multi-particle scattering amplitudes and their contribution to a unitarity sum. The sum over intermediate states has been carried out with the help of a special assumption which restricts the sum to a dominant class of "close" states. The resulting amplitude is not analytic not because of the summation but because of an extrapolation procedure which allows to cover the entire range of momentum transfer.

(\*) The deuteron has spin one and therefore possesses a quadrupole moment

Various other simplifying assumptions have been made along the way<sup>(\*)</sup>. Important is that, despite all this, the model compares very well with experiment as if to say that the departure from the geometrical picture is what is really crucial. We draw this conclusion. Further support for it comes from the good fits of Landshoff and Donnachie<sup>(11)</sup> to pp and  $p\bar{p}$  data based on S-matrix theory and a dynamical model for the Pomeron.

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<sup>(\*)</sup> e.g. the use of Gaussian wave functions and Gaussian input amplitudes  $\eta_{dd}(s,b)$  and  $\eta_{dq}(s,b)$

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