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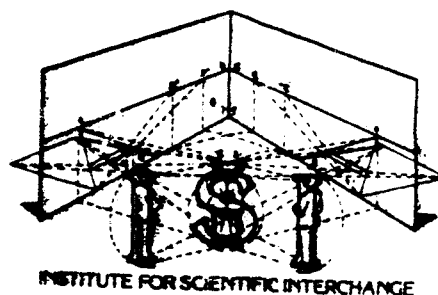
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# SUPERCONDUCTIVE PARTICLE DETECTORS

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## NEUTRINO MASS DETERMINATION BY $\beta$ DECAY SPECTRUM: PRESENT AND FUTURE SITUATION

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We report and discuss the results of the recent experiments on neutrino  $\nu_e(\bar{\nu}_e)^*$  mass, which used the electron spectrum from tritium  $\beta$  decay.

Up to now all the experiments (not only by  $\beta$  decay) performed on the determination of  $m(\nu_e)$  gave no solid proof of  $m(\nu_e) \neq 0$ .

We analyze the energy resolution we need to determinate the eventual mass of  $\nu_e$  in tritium  $\beta$  decay by the measurement of the end-point in the Kurie plot.

The present experimental apparatuses based on Si-Li detectors have a resolution of the order of 200 eV and in the near future people hope to reach resolutions of about 10 eV by improvements in the present techniques. But it needs to reach resolutions less than 10 eV.

Actually the only way to reach resolutions of the order of eV is by using superconducting junction in the tunnel regime with materials having enough low value of the energy gap (0.2 + 2 meV).

\* This writing is only to remember the possibility of Dirac or Majorana neutrinos.

## 1. INTRODUCTION

First of all we have to start saying that up to now there is no definitive evidence of massive neutrinos. We will take in account two ideas generally admitted and invoked as "common sense":

A - An a priori hierarchy among neutrino masses, in this scheme  $m(\nu_e)$ ,  $m(\nu_\mu)$ ,  $m(\nu_\tau)$  are in the ratio of  $m(e)$ ,  $m(\mu)$ ,  $m(\tau)$ . This means that  $\nu_\tau$  is at least 20 times heavier than  $\nu_\mu$ , itself 200 times heavier than  $\nu_e$ .

B - A bound on the sum of neutrino masses that the astrophysicists gave us which corresponds to  $\sim 50 + 100$  eV.

With these two inputs it is easy to derive sensible guesses for the masses:  $m(\nu_\tau)$  of order 10 eV,  $m(\nu_\mu)$  of order 1 eV,  $m(\nu_e)$  of order  $10^{-2}$  eV or less.

We will not speak about the mixing as a possible direct consequence of the non zero neutrino mass but we will consider the reverse situation: to use the mixing or rather the neutrino oscillation to have limits on neutrino mass.

With these a priori expectations we see that the laboratory experiments are at present hopeless.

Now, infact, the experiments have an energy resolution of about 200 eV and in the near future should reach the level of  $\sim 10$  eV accuracy.

But to improve on this it will be extremely difficult with the present techniques. So we hope to reach in the future the 1 eV level by the new superconducting tunnel junction.

The present results (1986-87) from tritium  $\beta$  decay give us:

$$\text{Moscow ITEP} \quad 20 \text{ eV} < \quad m(\nu_e) < 40 \quad \text{eV}^{(1)}$$

$$\text{Zürich} \quad \quad \quad \quad \quad \quad \quad m(\nu_e) < 18 \quad \text{eV}^{(2)}$$

$$\text{Los Alamos} \quad \quad \quad \quad \quad \quad \quad m(\nu_e) < 29 \quad \text{eV}$$

$$\text{Tokyo INS} \quad \quad \quad \quad \quad \quad \quad m(\nu_e) < 32 \quad \text{eV}^{(3)}$$

Anyway people prefer to remember that all these results are compatible with 0 mass.

From the double  $\beta$  decay experiments with  $\text{Ge}^{76}$  we have:

$$m(\nu_e) < 1+2 \text{ eV} \quad (4), (5)$$

This seems a very stringent limit, on order of magnitude better than the tritium case. From the recent experiments on neutrino oscillations<sup>(6)</sup> we have that they exclude masses down to  $\sim 0.1 \text{ eV}$ .

Obviously the experiments on neutrino oscillations give us only the  $m_2^2 - m_1^2$ , but admitting the hierarchy, as we said before, we can set limit on neutrino mass too. The technique of the time of flight used in the case of the burst (s) from SN1987A<sup>(7)</sup> gives us a limit on  $m(\nu_e)$  of order 10 eV, at the level of the tritium measurements. At the end we would like to recall the results about  $\nu_\mu$  and  $\nu_\tau$  masses.

$$m(\nu_\mu) < 270 \text{ keV}^{(8)}$$

$$m(\nu_\tau) < 70 \text{ MeV}^{(9)}$$

## 2. THEORETICAL BACKGROUND

The electron energy spectrum in nuclear  $\beta$  decay is affected by finite  $\nu_e$  mass as shown in Fig. 1 where we report the classical Kurie plot showing deviation from straight line for non zero neutrino mass.

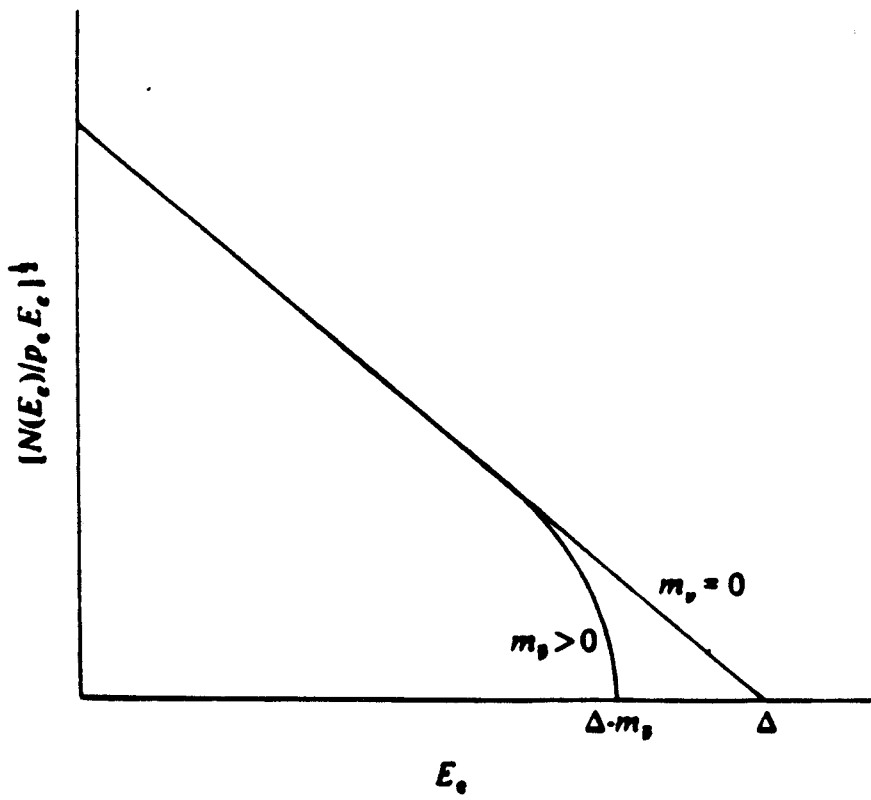


FIGURE 1

The differential transition probability for allowed  $\beta$  decay, averaged over nuclear spin and integrated over  $e^-$  and  $\nu_e$  directions, is:

$$dW = (4G_F^2 \cos^2\theta_1)/2\pi^3 \xi F(Z,E) p_e^2 dp_e p_\nu^2 dp_\nu \delta(\Delta - E - E_\nu) \quad (1)$$

Including the possibility of finite neutrino mass

$$m_\nu = (E_\nu^2 - p_\nu^2)^{1/2} \quad (2)$$

Writing  $p_\nu^2 dp_\nu = p_\nu E_\nu dE_\nu$ , and integrating over  $E_\nu$ , we obtain the electron energy spectrum:

$$* \quad N(E) = G_F^2/2\pi^2 \cos^2\theta_1 \xi F(Z,E) p_e E (\Delta - E) \sqrt{(\Delta - E)^2 - m_\nu^2} \quad (3)$$

However, the final atom may be in an excited state  $i$ , since each such state  $i$  is associated with a different energy  $\Delta_i$ , (3) must be replaced by

$$N(E) = G_F^2/2\pi^2 \cos^2\theta_1 \xi F(Z,E) p_e E \sum_i W_i (\Delta_i - E) \sqrt{(\Delta_i - E)^2 - m_\nu^2} \quad (4)$$

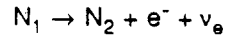
where  $W_i = |\langle \psi_{\text{atom}}^f | \psi_{\text{atom}}^i \rangle|^2$ .

If the sum dominated by a single term and  $m_\nu = 0$ , a plot of  $\sqrt{N(E)}/pE$  versus  $E$  (kurie plot) is a straight line with negative slope, which intercepts the  $E$  axis at  $\Delta$  (see FIG. 1). A finite mass causes the line curve downward near the endpoint and to intersect the axis with infinite negative slope as shown in FIG. 1.

The existence of other final atomic states usually cause the line to curve the other way, obscuring the effect of finite mass.

\* Note:  $\theta_1 = \theta_{\text{Cabibbo}}$  according the convention of the Kobayashi-Maskawa matrix.

In the three-body  $\beta$  decay we talked about we have the situation:



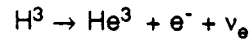
for massive mixed neutrinos such a decay will consist of  $n$  components with maximal electron energies (end point energies)

$$\Delta_{\max}^i = [M_1^2 + m_e^2 - (M_2 + m_{\nu_i})^2] / 2M_1 \quad i = 1, n \quad (5)$$

where  $M_1$  is the initial mass of the nucleus and  $M_2$  is the final mass. The corresponding Kurie plot  $\sqrt{N}/E_e p_e$ , neglecting for a moment the Coulomb correction factor, will have, as said before, a kink at each energy  $E_{\max}^i$ , but up to now due to poor energy resolution of the present detectors, as we will discuss later, it was not possible to see this effect.

The method of neutrino mass determination based on three-body decays is more sensitive to small neutrino masses, since there is a linear mass dependence of the end point, the accuracy of the neutrino mass determination is essentially equal to the energy resolution of the electron spectrometer.

The best studied case among the three-body  $\beta$  decay is the tritium one.



A finite antineutrino mass  $m_{\bar{\nu}}$  will cause deviations from the straight Kurie plot at distances  $\sim m_{\bar{\nu}}$  from the energies determined



by (5).

In particular we recall that, at least theoretically, the Kurie plot will have an infinite slope at highest end point but in practice, due to the finite resolution of the electron spectrometer, the slope of the Kurie plot is decreased near the end point. So, with a finite neutrino mass the Kurie plot near the end point will be somewhat steeper than expected from the performance of the measuring device assuming  $m_\nu = 0$  see FIG. 2 remembering that  $\Delta =$  end point energy. The full curves are the expected behavior for  $m_\nu = 0$  and  $m_\nu = 1$  unit of energy, for the indicated effective resolution width  $W$ , the dashed lines represent straight line extrapolation

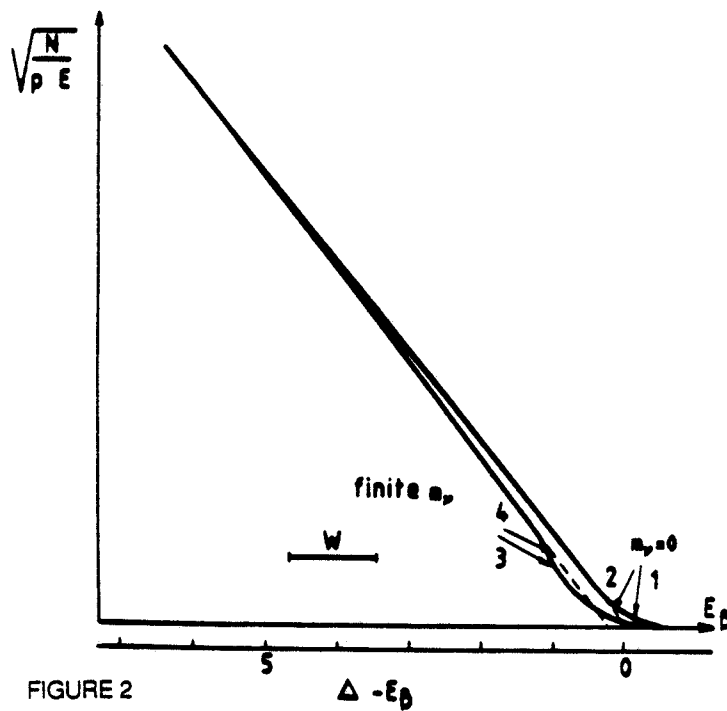


FIGURE 2

Tritium is particularly suited for measurements of  $\beta$  decay due

to its low end point energy  $\Delta = 18.6$  keV, low  $Z$  value, and convenient half life  $T_{1/2} = 12.3$  year. But the energy resolution of present detectors is poor as we said and so at this level of accuracy an additional difficulty arose.

The initial state for a free tritium atom has one bound electron with binding energy  $E_B = 13.6$  eV during the beta decay nuclear charge suddenly changes by a factor of two. The initial bound spectator (atomic) electron can be shaken up to the  $2S_{1/2}$  or to some other excited state of the final  $\text{He}^3$  atom.

For a free atom the probability of the final  $1S_{1/2}$  state is 70%, of the  $2S_{1/2}$  state 25%, and the remaining 5% of the electrons go to higher excited states or to the continuum<sup>(10)</sup>.

This estimate can obviously be effected in a complicated way by the chemical and crystalline structure of the tritium source.

From the point of view of the neutrino mass searches the atomic effects play a role resembling the finite momentum resolution of the electron spectrometer. In fact one is dealing with a decay having several end point energies and effectively leading to a modification of the line shape, essentially a broadening to  $\sim 70$  eV, see FIG. 3 which illustrates the effects of finite experimental resolution 55 eV and distribution of endpoint energies due to atomic effects in tritium<sup>(11)</sup>.

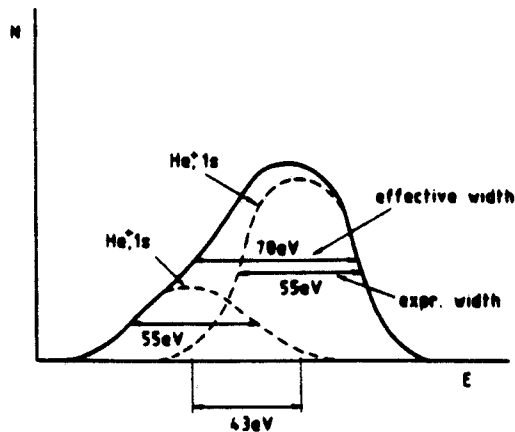


FIGURE 3

So it is possible to perform an experiment with a relatively strong tritium source combined with a good energy resolution but the atomic final state effects represent a serious obstacle to a further improvement of accuracy.

Summarizing it is opportune to have a  $\beta$  source with a low  $\Delta$  and an half-life reasonably short in order to have an high  $\beta$  emission rate as we said for the tritium  $\Delta = 18.6$  keV and  $T_{1/2} = 12.3$  years and in the sensible zone of 100 eV the  $\beta$  particles fraction is  $\sim 10^{-6}$  whereas in the sensible region of 10 eV is  $\sim 10^{-9}$ .

In fact remembering:

$$N_{\beta} \propto F(E,Z) p_0 E (\Delta-E) \sqrt{(\Delta-E)^2 - m_{\nu}^2} \quad (6)$$

we can see that the stronger change occurs in an interval  $\Delta E$  below  $\Delta$  which is at least few times the neutrino mass and the fraction of the number of  $\beta$  particles having  $E > \Delta - \Delta E$  (for  $m_{\nu} = 0$ ) is proportional to  $(\Delta E/\Delta)^3$  with proportional constant  $\cong 7$  assuming that  $F(E,Z) =$  constant, as said before the finite resolution of the detector has the effect to deform the "tail" of  $\beta$  spectrum (Kurie plot) in the opposite sense with respect to the effect of neutrino mass.

The resolution, as FWHM, was ever greater than neutrino mass, almost in the experiment performed up to now, so we need to know very well the resolution function to calculate the neutrino mass.

The theoretical  $\beta$  spectrum and Kurie plot have to be calculated, at the experimental energy points, by convoluting the resolution function with the theoretical  $\beta$  spectrum:

$$N(E) = C F(E,Z) p_0 E (\Delta-E) \sqrt{(\Delta-E)^2 - m_{\nu}^2} \quad (7)$$

The constant  $C$  could be determined by an opportune fit of the data for particular values of  $\Delta$  over the  $\beta$  kinetic energy range from 10 to 17 keV; also it is possible to use a modified non relativistic coulomb correction factor  $F(E,Z)$  according<sup>(12)</sup> we have to take into

account that the main effect of the resolution function  $R$  is to "force" the data in interval of width  $R \cong \text{FWHM}$ .

So in a region of energy  $R$  ( $R > m_\nu$ ) below the extrapolated end point  $\Delta$  there will be a counting number

$$A(m_\nu) \propto \int N_\beta(E) dE \propto (R^2 - m_\nu^2)^{3/2} \quad (8)$$

assuming that the factor  $F(E,Z) p_e E$  is constant in the integration region.

However if  $R$  is over estimated of a quantity  $\epsilon$  then  $A(m_\nu)$  can be calculated assuming a neutrino mass  $M_\nu > m_\nu$ , where  $M_\nu$  is a fictitious mass and  $m_\nu$  the true mass so:

$$M_\nu^2 - m_\nu^2 \cong 2 \epsilon R \quad (9)$$

If e.g.  $m_\nu = 0$  strictly we will have a fictitious mass:

$$M_\nu \cong \sqrt{2\epsilon R} \quad (10)$$

And this shows that if  $R > m_\nu$  the accuracy increases not only improving the resolution but also increasing the knowledge of the resolution function which depends from each different detector.

At the end of this section we would like to recall another possible way to determine the neutrino mass in a three-body  $\beta$  decay using the internal bremsstrahlung electron capture and measuring the photon spectrum shape<sup>(13)</sup>. For several nuclei e.g.  $\text{Pt}^{193}$  the  $\Delta$  value is so low that K (or L) capture is forbidden. This leads to an enhancement of the photon rate near the end point.

The atomic final state effects discussed above are also essentially absent; nevertheless, the practical utilization of this

method is very difficult on account of low efficiency of the resolution (few eV) photon spectrometer required.

### 3. RECENT EXPERIMENT ON $H^3$ BETA DECAY

Starting from the conclusions of the previous section we would like to recall the main requirements on the apparatus: good energy resolution, good luminosity, since only a very small fraction  $\sim 10^{-6}$  of the spectrum is of interest, and low background, furthermore at the level of accuracy achieved in recent experiments 20 + 50 eV resolution, some additional difficulties arise.

People have to take account that the  $He^3$  atom may be left in an excited state with a probability around 30 - 40%, depending on the chemical composition of the source; the excitation energies are typically 13-20 eV and the calculation of the spectrum of final states is straightforward for free atoms, but more complicated for molecules. As we said the shape of the energy response function giving the spread of a monoenergetic beam through the spectrometer must be known with great accuracy: overestimating the width can mimic a neutrino mass; underestimating it can hide a mass.

An important contribution to the response function comes from the energy losses in the source which must be known with good accuracy.

The difficulties stemming from final-state effects and energy losses were solved in an elegant way by Simpson<sup>(12),(14), (15)</sup>.

He implanted  $H^3$  atoms in a Si(Li) detector which was functioning as an active source. Despite of Monte Carlo treatment the data were consistent with a zero mass, and Simpson's result was  $m_{\nu_e} < 65$  eV (95% CL).

Unfortunately this method is limited by the resolution of the Si(Li) counter: 300 eV FWHM at 18 KeV, which cannot be significantly improved.

The best instrument to study  $H^3$  beta decay seems to be the

magnetic spectrometer, here to minimise energy very thin sources are used.

Bergkvist<sup>(11)</sup> performed a very careful experiment using a  $\pi\sqrt{2}$  spectrometer with central circle radius of 50 cm.

The energy resolution was 40 eV FWHM, and the source consisted of a thin foil of Al with tritium ions implanted near the surface.

Bergkvist found no evidence for a finite non zero mass and reported the limit  $m_{\nu_e} < 55$  eV (90% CL).

At ITEP in Moscow a toroidal spectrometer with total deflection angle  $720^\circ$  was built and a source of  $H^3$  enriched Valine  $C_5 H_{11} NO_2$  was used in all the measurements<sup>(16), (17), (18)</sup>.

In the last upgraded version of the spectrometer the scans are performed electrostatically at a constant magnetic field so that all the electrons hitting the detector have the same energy, reducing systematic uncertainties. The electrons emerging from the source are accelerated to an energy above the end-point energy; the threshold of the detector is high enough so that only these accelerated electrons are seen.

This drastically reduces the background due to  $H^3$  contamination of the entire apparatus.

They used an extended source giving a high activity mounted on a resistive plate with an electric field gradient in order to make it look like a point source. A multiwire proportional chamber was used as detector. The last analysis, incorporating additional data and the most recent calculation of the final-state spectrum of Valine<sup>(19)</sup>, presented by Boris et al.<sup>(20)</sup> gives a good agreement between the experimental points and the fitted curve for  $m_{\nu_e} = 34.8 \pm 1.9$  eV. Just to show that the sensitivity to the final states is not too bad let us mention that taking the distribution for free  $H^3$  atoms one obtains

$$m_{\nu_e} = 30.9 \text{ eV.}$$

Taking, quite unrealistically, the distribution for for  $H^3$  nuclei one obtains  $m_{\nu_e} = 13.8$  eV.

Trying to take into account the uncertainty from the final states, Boris et al. conclude that a realist estimate is

$$20 < m_{\nu_e} < 40 \text{ eV}$$

The most advanced of the competing experiments is being performed by a group at the University of Zurich<sup>(21)</sup>, a toroidal spectrometer, similar to the ITEP design, has been built. Here, however, the electrons are decelerated after the source which allows them to achieve a good energy resolution with a comparatively large luminosity; the source consists of tritium implanted in carbon.

The response function was determined by using X-ray source and an electron gun, the background was low and they refer a resolution of 25 eV but the data are again consistent with a vanishing mass. Taking into account statistical errors only the limit  $m_{\nu_e} < 10 \text{ eV}$  (95% CL) is derived, but an analysis making allowance for possible systematic errors, including the uncertainty on the final states, gives the firm upper limit

$$m_{\nu_e} < 18 \text{ eV}$$

in disagreement with the ITEP result.

Another interesting programme is being pursued at Los Alamos [22] also with a toroidal spectrometer, but the most salient feature is the source, which will consist of free tritium atoms. In this case the final-state spectrum is particularly simple.

#### 4. CONCLUSION AND OUTLOOK

Looking back at all the preceding sections one concludes that the theoretical situation is clear enough but in spite of a vast experimental effort, there is presently no strong evidence for  $\nu_e$  mass. The result obtained from the study of tritium  $\beta$  decay by Moscow group  $20 < m_{\nu_e} < 40 \text{ eV}$  is contradicted by the Zurich

limit  $m_{\nu_e} < 18$  eV.

Other competing experiments are in progress, but at present level of accuracy the experimental problems are quite formidable, and the results are coming out slowly. Using electron Spectrometer with tunnel junction in superconducting regime made by material with low energy gap (0.2 + 20 meV) we will expect for the energy resolution(23):

$$\Delta E/E = 2.35 \sqrt{FW/E} \quad (11)$$

where  $F$  is the Fano factor ( $0 < F \leq 1$ ) and  $W$  is twice the gap energy of the material. We shall consider the case of the tin having  $E_{\text{gap}} \cong 0.5$  meV and the worse situation with  $F = 1$  so we have:

$$\Delta E/E = 7.4 \cdot 10^{-2} E^{-1/2} \quad (12)$$

In FIG. 4 we plot  $\Delta E$  versus  $E$  to show the expected behaviour of this energy resolution in the 0 + 20 KeV region (where one should have to do the measurements for  $\beta$  spectrum from  $H^3$ ).

Expected  $\Delta E(E)$  for tunnel junction

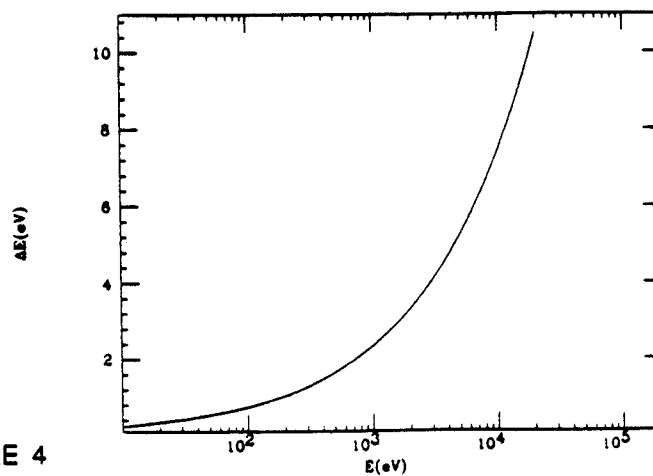


FIGURE 4



In the same previous approximation we have for the energy resolution function:

$$R(E) \propto E^{1/2} \quad (13)$$

while for the tritium end point  $\Delta \cong 18$  KeV it will have an energy resolution  $R(\Delta) \cong 10$  eV about one order of magnitude below the present apparatuses.

Doing the convolution between this resolution function (13) and the theoretical spectrum given by (7)

$N(E) = C F(E,Z) p_0 E (\Delta-E) [(\Delta-E)^2 - m_\nu^2]^{1/2}$ , we calculate the expected experimental spectrum  $N_s$ :

$$N_s = \int N(E') R(E'-E) dE' \quad (14)$$

So in our case we have, assuming  $F(E,Z) p_0 E$  constant:

$$N_s \propto \int_E^\Delta (\Delta-E') [(\Delta-E')^2 - m_\nu^2]^{1/2} (E'-E)^{1/2} dE' \quad (15)$$

where  $0 < E < \Delta$ .

In the simplified case assuming  $m_\nu = 0$  we have

$$N_s \propto \int_E^\Delta (\Delta-E')^2 (E'-E)^{1/2} dE' \quad (16)$$

$$N_s \propto (\Delta-E)^{7/2} \quad (17)$$

So for the Kurie plot we expect for the variable  $K$  a dependence from the energy  $E$  of the kind:

$$K \propto (\Delta - E)^{7/4} \quad (18)$$

as shown in Fig. 5.

Actually this is the more reasonable prediction we can do about what we expect to see of a tritium  $\beta$  spectrum using this kind of devices.

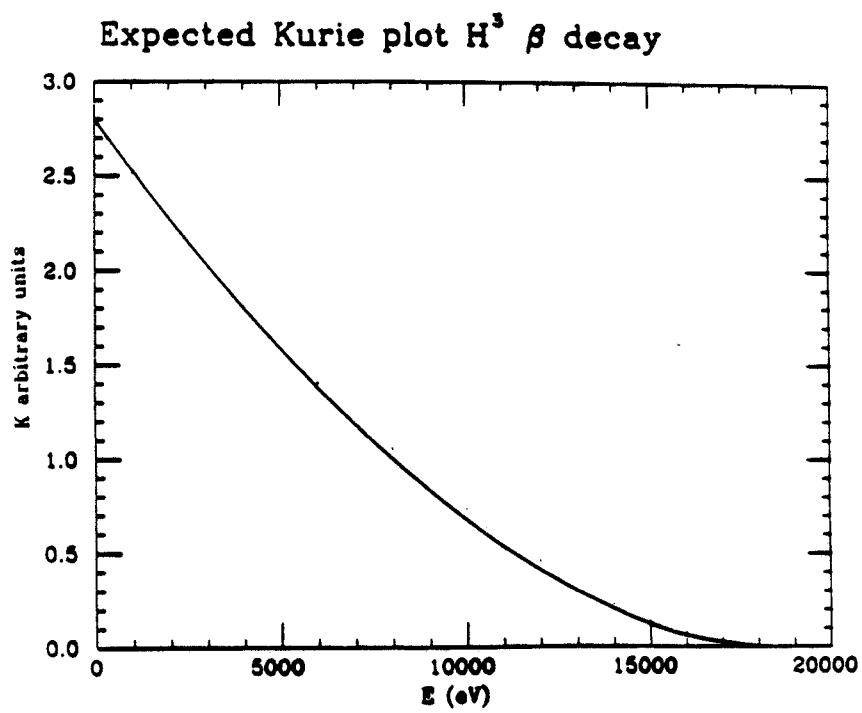


FIGURE 5

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