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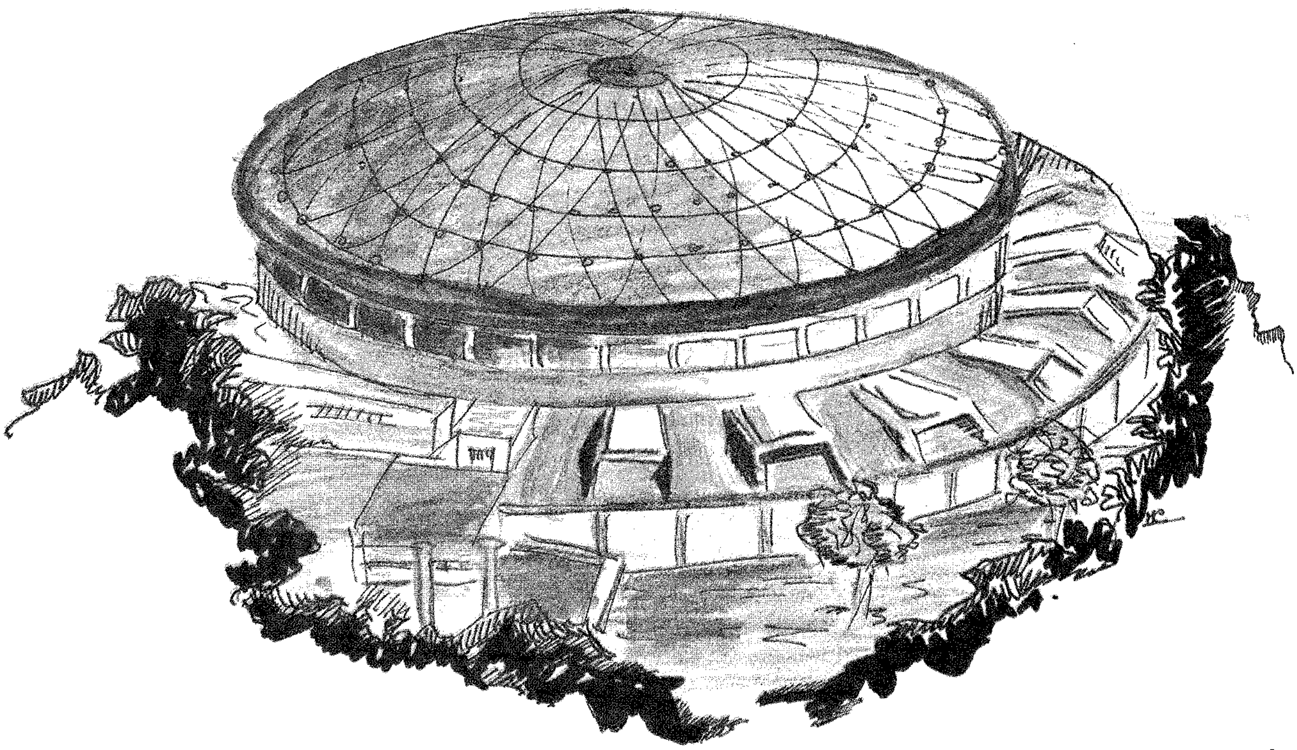

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QUANTUM STABLE VORTICES IN THE LATTICE U(1)-HIGGS MODEL

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Abstract

In the framework of the U(1) Gauge - Higgs model in four dimensions we show that vortices are present and are stable under quantum fluctuations, and so they can be considered as true non-trivial vacua of the quantum theory.

In recent times the lattice approach to quantum field theories has been extensively applied to the study of Gauge-Higgs models. The understanding of the dynamical mass generation mechanism in the standard model is one of the motivations for a such study.

On the other hand, another exciting non-perturbative phenomenon which can be analysed using the lattice approach is the relevance of classical, topologically non trivial field configurations in the quantum theory. Many phenomenological models rely on the relevance of this kind of non trivial solutions of the classical equations. One example is the vortex solution in the U(1)-Higgs model.

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A vortex in three spatial dimensions is a string with finite energy per unit length, and is caracterized by a topologically quantized charge which reflects the fact that the field configurations cannot be deformed continuously to the trivial vacuum configuration.

The existence of these topologically non trivial solutions in 4-dimensional Gauge-Higgs theories was found the first time by Nielsen and Olesen in 1973[1]; on the other hand, the relevance of the vortex solution to the superconductivity phenomenon in solid-state physics is known since long ago[2].

In this paper we report results of numerical simulations of the standard U(1) lattice Gauge-Higgs model which give a quite unambiguous evidence for the stability of these objects under quantum fluctuations.

The euclidean continuum action for this model is

$$S_c=\int d^4x\{ (D_\mu\phi_c^*)(D_\mu\phi_c) + m_c^2\phi_c^*\phi_c + \lambda_c(\phi_c^*\phi_c)^2 + \frac{1}{4}F_{\mu\nu}F_{\mu\nu} \} \tag{1}$$

where $D_\mu = \partial_\mu + igA_\mu$ is the U(1) covariant derivative and $F_{\mu\nu}$ the electromagnetic tensor. The lattice version of the action (1) is

$$\begin{aligned} S = - \beta \sum_{plaq}(\mathrm{Re}\, U_{plaq}) + \sum_n(\rho_n{}^2 - \ln \rho_n + \lambda(\rho_n{}^2-1)^2) - \\ - \frac{1}{2}\kappa \sum_{n\mu}\rho_n\rho_{n+\mu}(\zeta_n U_{n\mu}\zeta^+_{n+\mu} + \zeta^+_n U^+_{n\mu}\zeta_{n+\mu}) \end{aligned} \tag{2}$$

The first term in (2) is the standard Wilson action for the pure gauge theory ($\beta=\frac{1}{g^2}$) and the scalar field at site n is given by

$$\phi_n = \rho_n \zeta_n$$

ζ_n being an element of U(1) and ρ_n a real number running from 0 to infinity. The integration measure for the action (2) is

$$\prod_n \rho_n d\rho_n\, d\zeta_n \prod_\mu dU_{\mu n}$$

with the Haar measure for the the gauge group.We have included the ρ_n term from the measure in S. The relations between lattice and continuum variables are given by

$$\phi_c(x) = \frac{\kappa^{1/2}}{a}\phi_n \;,\quad \lambda_c = \frac{\lambda}{\kappa^2} \;,\qquad m_c^2 = \frac{1-2\lambda-8\kappa}{\kappa a^2}$$

In our numerical simulations we have always worked at fixed ρ and carried simulations on 6^4, 8^4 and 10^4 lattices using as dynamical variables the gauge invariant combinations $W_{n\mu} = \zeta_n^+ U_n \zeta_{n+\mu}$. The U(1)-gauge group was in some runs discretized as Z(12) and in the remaining we used the full U(1) group. No significant differences were found in the explored region.

In order to characterize quantum vortices, we look for several signals:
i) From the continuum analysis, a string-like structure with finite energy per unit length can be characterized by a (localized) increase of the link energy density around the vortex singularity line. Therefore, if we define the mean link energy :

$$<E_{l\mu}> = <\sum_n (1\text{-Re } W_{n\mu}) > \tag{3}$$

we expect that:

$$E_{l\text{-vortex}} = <\sum_\mu E_{l\mu} >_{\text{vortex}} \;>\; E_{l\text{-no vortex}} \,. \tag{4}$$

Additionally, when a vortex moves in (euclidean) time it becomes a surface in a 4-dimensional space and then

$$< E_l >_{\text{vortex}} - < E_l >_{\text{no vortex}} \tag{5}$$

will be proportional to the area spanned by the vortex. These features will survive in a quantum simulation only if the vortex is quantum-mechanically stable.

ii) The vortex field configurations break (lattice) rotational invariance. Therefore, if vortex configurations are stable under quantum fluctuations we expect to see a signal of rotational invariance breaking, because of the high energy cost for changing the vortex orientation.

iii) In the continuum, a classical vortex string is caracterized by a quantized magnetic flux. Using the Stokes theorem, the magnetic flux through a plane perpendicular to the vortex

$$\Phi = \int_c A_\mu dx_\mu = \frac{2\pi}{g} n \tag{6}$$

is quantized and proportional to the winding number n.

In the compact lattice version of this model, there are several possible definitions for the magnetic flux and topological charge. Later, we will give several definitions and we will show the corresponding numerical results.

In the numerical simulation we have made several runs starting from different initial conditions. In Fig. 1a we plot the quantity $E_{1,1} - E_{1,3}$ at β=1.15 and $\kappa = 0.44$, normalized to the lattice volume (8^4) as a function of the Monte Carlo time, averaged in groups of six configurations for a hot start.

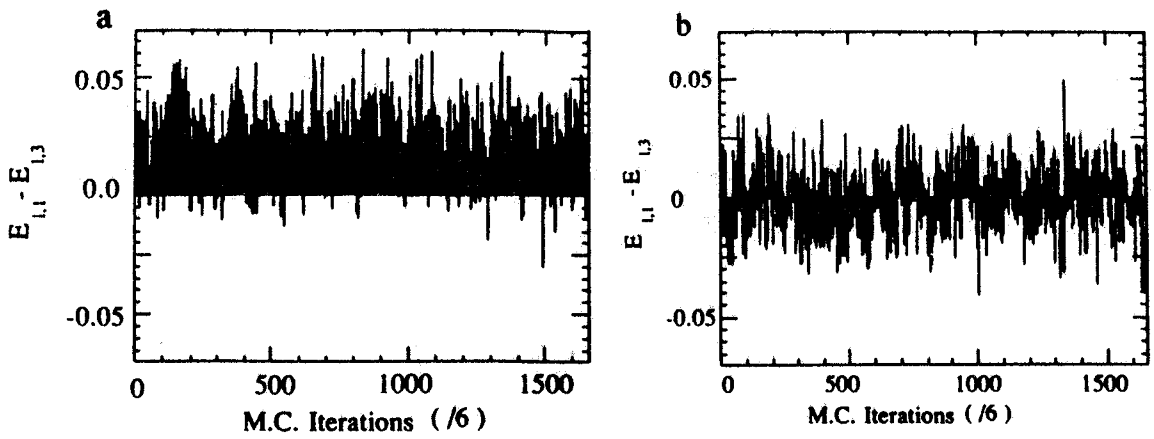

FIG. 1 - $E_{11}+E_{12}-E_{13}-E_{14}$ as a function of M.C. time at β=1.15 , κ=0.44 , 10000 M.C. iterations averaged in groups of 6, 8^4 lattice, Z(12) : a) hot start , b) cold start.

Since this quantity is in the average different from zero, we interpret it as a clear evidence for rotational invariance breaking; moreover these results were stable even after 20000 M.C. iterations. The corresponding results at the same κ–β values obtained from a run with cold start (Fig 1.b) were compatible with rotational invariance within statistical errors. Also lower values for the mean energy per plaquette

$$< \sum_{plaq} (1 - \mathrm{Re}\, W_{plaq}) >$$

and the mean link energy were systematically found. This fact can be interpreted as a signal of the existence of a 2-dimensional (quantum) stable structure in the equilibrium configuration reached from a hot start. The high energy cost prevents the formation of such structures from ordered initial conditions in at least 50000 M.C. iterations.

We define the mean link energy in the μ–ν plane as

$$E_{\mu\nu}(i,j) = < \sum_{n'\sigma} (1\text{-Re } W_{n'\sigma}) > \qquad (7)$$

where n' stands for the sum over the sites for which the (μ,ν) components take values (i,j). This quantity is reported in Figs. 2 in a 10^4 lattice, for 4000 M.C. sweeps, after 2600 thermalization iterations. A strong peak (Fig. 2.a) in this quantity is observed in planes 2-3 for random start, in contrast with a completely flat distribution obtained for a cold start case. In Fig. 2.b an 2.c we show this mean link energy distribution in the 1-2 and 1-4 planes (distributions similar to fig 2.b were found for the 2-4,1-3 and 3-4 planes). On the other hand we have observed that the peak in the mean link energy distribution is very stable in the Monte Carlo and real time evolution and we needed 2000 - 3000 MC iterations to move it of a lattice spacing (in an 8^4 lattice).

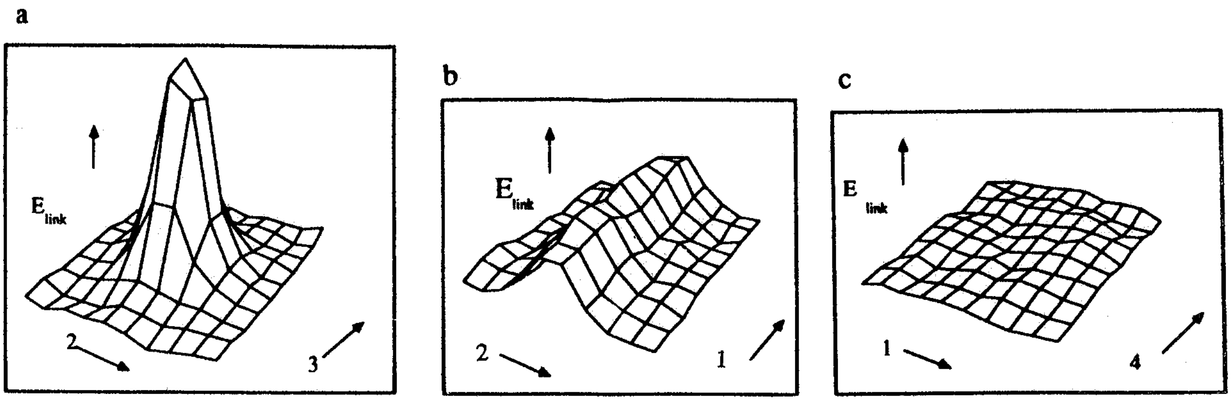

FIG. 2 - Average link energy distribution at β=1.15 , κ= 0.44 , 4000 M.C. iterations, 10^4 lattice, U(1): a) planes 2-3, b) planes 1-2 , c) planes 1-4.

All these results tell us that a bidimensional structure was always present in these runs. This is particulary remarkable in a lattice simulation where the trivial topology of discretized space-time does not support topological stability.

In order to get additional information about the physical properties of this anomalous "vacuum" we have measured the distribution of magnetic flux summed as in (7). In Fig. 3 we present the magnetic flux distribution for the hot start run in the 8^4 lattice. The magnetic flux per plaquette is taken as the sum of the link phases around the plaquette.

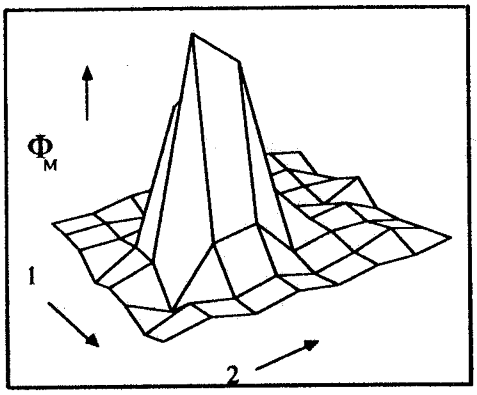

FIG. 3 - Magnetic flux distribution in planes 1-2 at β=1.15 , κ=0.44 , 8^4 lattice, Z(12).

Periodic boundary conditions fix the total magnetic flux in each plane to be zero. We found nonetheless a clear evidence for a magnetic flux tube orthogonal to the 1-2 plane in quite good agreement with the position of the mean link energy distribution.

Another way to characterize a vortex is to look at the topological charge, which in the continuum is proportional to the magnetic flux; in the lattice however this may not be strictly true and one can give at least two definitions of quantized topological charge[3]:

i) Isolating "large" from "small" fluctuations of the plaquette phase through the definition

$$\Phi = \Phi_p + 2\pi n \qquad (8)$$

where Φ_p is the sum of the the phases of the link variables around a plaquette. Φ_p takes values between -4π and 4π and F is constrained to take values between $-\pi$ and $+\pi$ [3]; the topological charge is then $Q=n$.

This definition is not additive in the sense that if we apply it to loop formed by two plaquettes the result will be different from the sum of the topological charges of the two plaquettes. A possible solution in order to avoid this problem is to apply the previous procedure to the phase of each link in the plaquette (ii).

A further motivation for this choice is that this definition is more sensitive to local properties that may be hidden in a more global quantity as the plaquette phase.

Using the first defintion, the topological charge distribution in the 1-2 plane (in units of 2π, with a definition similar to (7)) has a sharp peak for runs in which a vortex is present (Fig.4-a). Furthemore the sum over all 1-2 planes gives the (normalized) value 0.905 ± 0.002 against a near zero value (-0.040 ± 0.002) in the cold start. Note that the definition (i) is not constrained to be zero by the boundary conditions, if applied to a whole plane.

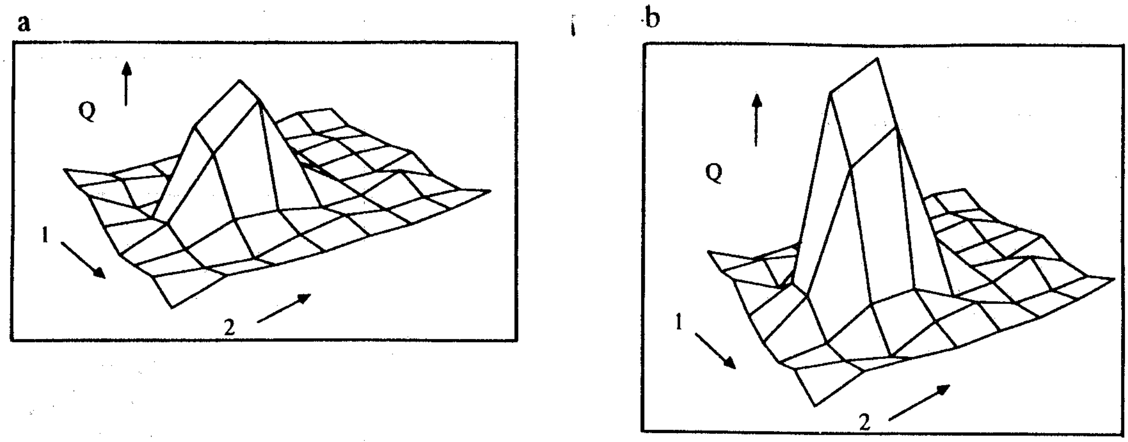

FIG. 4 - Topological charge distribution in planes 1-2 at β=1.15, κ=0.44, 8^4 lattice, Z(12): a) definition (i) , b) definition (ii).

The distribution of topological charge with definition (ii) is plotted in Fig.4-b. Again a sharp peak is found in the 1-2 plane in correspondence with the magnetic flux and mean link energy distributions peaks. In contrast to the previous case, now periodic boundary conditions fix this topological charge to be zero in each plane. If we sum the values on a loop of side 3 lattice units around the peak, we get instead the value -0.95±0.05. All these results were obtained at β=1.15, κ=0.44, well in the Higgs phase.

As we pointed out before, a four dimensional vortex is a bidimensional structure and therefore, if we define ΔS as the difference of the expectation values of the total action density in the "vortex" vacuum and in the standard vacuum respectively, we expect a number proportional to the surface spanned by the vortex, normalized to the total volume, i.e. proportional to L^{-2} where L is the lattice size, at least if the vortex is extended to the whole lattice (we have checked this looking at the energy distribution at each temporal plane). In Fig. 5 we plot the ΔS values we have obtained for several lattice sizes. The continous line is a fit of the form

$$\Delta S = \alpha L^{-2} \qquad (9)$$

with α=1.8. The linear behaviour of ΔS with L^{-2} is very well reproduced in the numerical simulation. We stress that in our runs we have never found vortices that are "closed" in the sense that extend to a limited volume of the lattice; we found only "open" vortices that are closed only by the periodic boundary conditions. This happens probably because a closed vortex can shrink to a point and then disappear, while this is not possible for an open vortex.

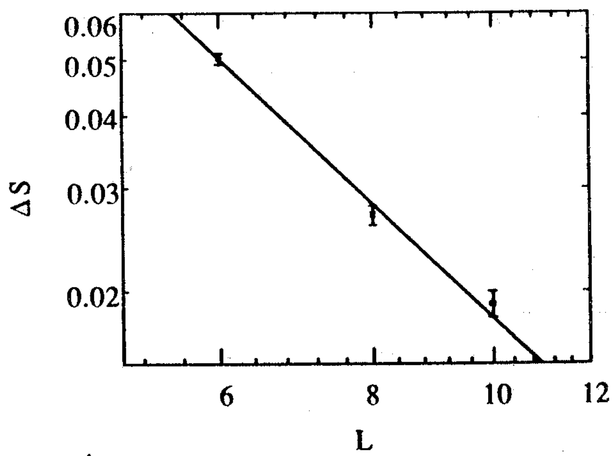

FIG. 5 - ΔS as a function of lattice size at β=1.15, κ=0.44, 8^4 lattice, U(1) ; the line is a fit of the form $\Delta S = \alpha L^{-2}$.

The vortex "vacuum" configurations are not reached for every random start. Only if the initial hot configurations is, in same sense, near to a vortex configuration, the system fluctuates around the vortex vacuum. We have verified this by checking that for some random starts the system goes to the same equilibrium configurations as in the cold start case. Hovewer, perturbing the system with an external magnetic pulse acting at t=0, the system goes to the vortex vacuum and stays there even after we take off the external field.

We expect that the vortex stable solutions will disappear or will change drastically when we go to the Coulomb phase of the model since the photon is massless there. To check this point we have made several runs in a 8^4 lattice, starting from the equilibrium vortex configuration at κ = 0.44 (β = 1.15) and decreasing κ in several steps by changing κ ---> $\kappa+\delta$ with δ=-0.005. At each point we have run 4000 MC iterations.

In Fig. 6 we plot topological charge distribution (definition ii) for the final point (κ =0.335) in the 1-2 plane. No peak can be observed in this Figure and the same result is obtained if we plot the mean link energy distribution.

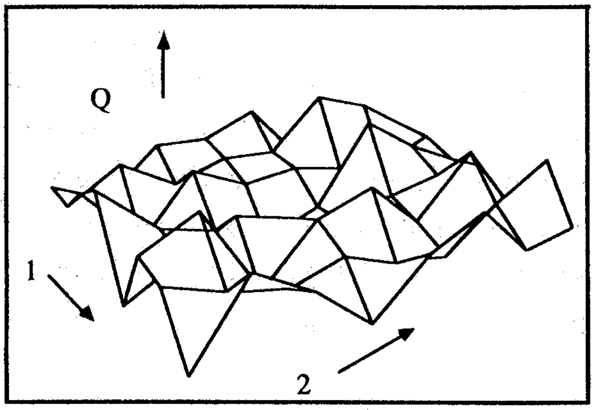

FIG. 6 - Topologial charge distribution in planes 1-2 (definition ii) at β=1.15, κ=0.335, 8^4 lattice, Z(12).

Hovewer the numerical value for the total topological charge in the 1-2 plane (definiton i) is 0.891± 0.004 to be compared with the corresponding result for the cold start case (-0.050±0.002). These results indicate that we are not in the standard vacuum, as confirmed by the results on the mean energy per plaquette which are slightly greater than in the cold start case. The absence of a structure similar to Fig. 2-a can be explained by the fact that, beeing the photon massless, the magnetic flux has not to be confined in a narrow tube and so the vortex can occupy to the whole plane.

These results are also applicable to Type II superconductors in an approach similar to Ref.4. Our results say us that once the magnetic vortex is created in the superconductor, this flux tube is stable and remain even if the external field is removed.

The main comclusion one can get from this analysis is that, in the context of the U(1)-Higgs model, a non perturbative calculation based on the lattice formulation confirms the possible relevance at the quantum level of the vortex configurations, beyond the semiclassical approximation in which they are introduced. One advantage of the present formulation as compared to the semiclassical approximation is that the same analysis can be extended in a simple way to theories with fermion fields.

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