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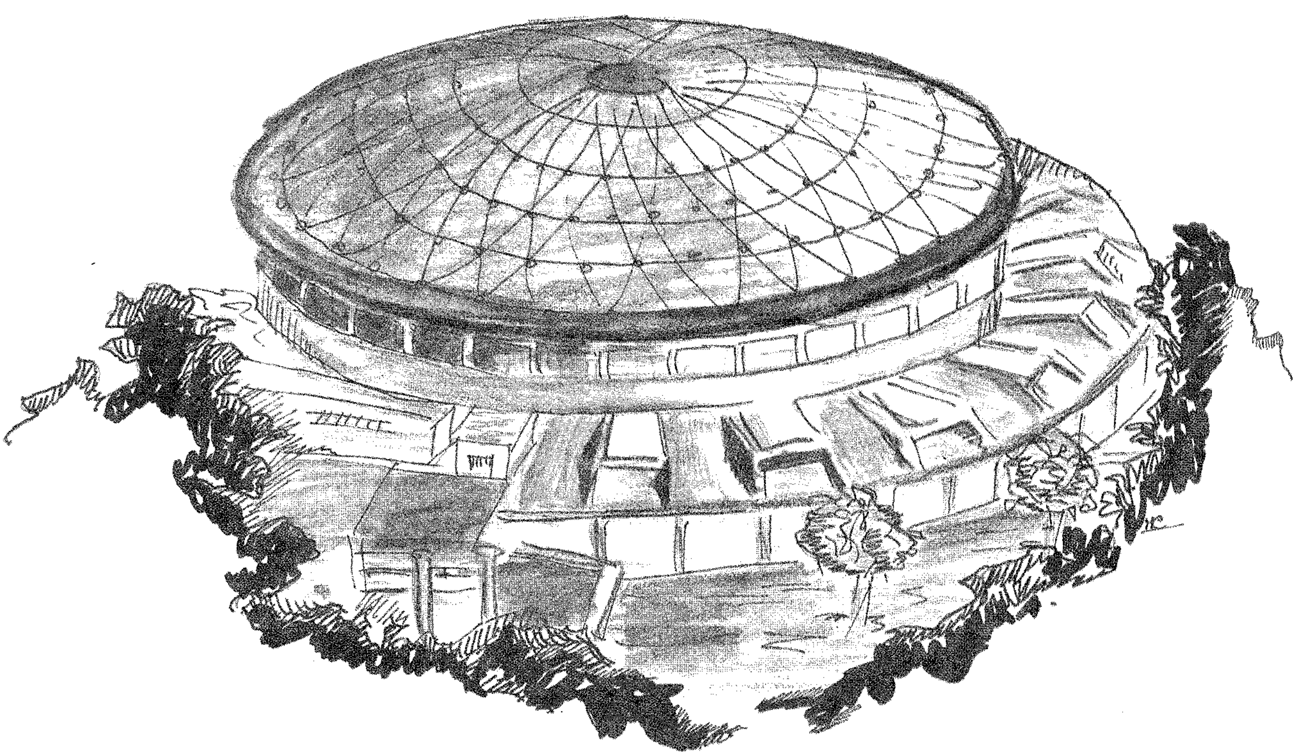

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HIGHER ORDER CORRECTIONS TO QCD JETS: GLUON - GLUON PROCESSES

F. Aversa, M. Greco,
INFN - Laboratori Nazionali di Frascati, C. P. 13, 00044 Frascati, Italy

P. Chiappetta, J.Ph. Guillet[(+)]
Centre de Physique Théorique, CNRS, Luminy, Marseille and Institut de Physique Nucléaire, Université de Lausanne

ABSTRACT

We evaluate the full $o(\alpha_s^3)$ cross sections for the processes $g + g \to g + X$ and $g + g \to q + X$, relevant for jet production at large p_T at very high energies. Some phenomenological applications of our results at $Sp\bar{p}S$ and Tevatron energies are also presented.

In a recent letter we have presented[1] the first results of a general calculation of QCD one-loop corrections to the production of jets in hadronic collisions. Indeed, motivated by recent calculations [2] of matrix elements for $o(\alpha_s^3)$ parton-parton scattering processes, we have given in ref.[1] the results for parton subprocesses involving only quarks of different flavour. In view of the well known dominance of gluon-gluon interactions at present hadron colliders we have considered as a next step the $o(\alpha_s)$ radiative corrections to gluon-gluon scattering.

(+) Institut de Physique Nucléaire, Université de Lausanne

In the present letter we present the evaluation of the full QCD $o(\alpha_s^3)$ cross section for the real and virtual processes

$$\begin{array}{lll} \text{I} & g + g \to g + g + (g) & \\ & & (1) \\ \text{II} & g + g \to q + \bar{q} + (g) & \end{array}$$

More precisely we evaluate the cross sections to order α_s^3 for the reactions $g + g \to g + X$ and $g+g\to q + X$. Some phenomenological applications of our results at the Sp$\bar{\text{p}}$S and Tevatron energies are also presented.

The method followed is described in [1,3,4]. Starting from the expressions of matrix elements in n dimensions we perform the phase space integration of the real processes, then we cancel the $(1/\varepsilon^2)$ divergences [$\varepsilon = n - 4$] by adding the virtual contributions. The left over $(1/\varepsilon)$ terms, corresponding to mass singularities, are then absorbed into the structure and fragmentation functions beyond the leading order. The algebraic manipulations are done in parallel with two independent programmes, one using REDUCE and the other one MACSYMA.

The inclusive cross section for the one-hadron inclusive production at large transverse momentum

$$H_1(K_1) + H_2(K_2) \to H_3(K_3) + X \qquad (2)$$

is given[1] by

$$E_3 \frac{d\sigma}{d^3k_3} = \sum_{i,j,l} \frac{1}{\pi S} \int_{1-V+VW}^{1} \frac{dx_3}{x_3^2} \int_{\frac{VW}{x_3}}^{1-\frac{(1-V)}{x_3}} \frac{dv}{1-v} \int_{\frac{VW}{x_3 v}}^{1} \frac{dw}{w}$$

$$D_{p_l}^{H_3}(x_3, M_f^2)\, F_{p_i}^{H_1}(x_1, M^2)\, F_{p_j}^{H_2}(x_2, M^2) \qquad (3)$$

$$\left\{ \frac{1}{v} \frac{d\sigma^o}{dv}_{p_i p_j \to p_l}(s, v)\, \delta(1-w) + \frac{\alpha_s(\mu^2)}{2\pi} K_{p_i p_j \to p_l}(s, v, w) \right\}$$

where the hadronic variables V and W are related to the Mandelstam variables as $V = 1+T/S$, $W=-U/(S+T)$, and similarly for the partonic variables v, w and s, t, u. Furthermore $x_1=VW/x_3vw$, $x_2 = (1-V)/x_3(1-v)$ and $s=Sx_1x_2$. $F_{pi}^H(x, M^2)$ are the structure functions of a parton p_i inside the hadron H at the factorization mass scale M^2, and $D_{p1}{}^{H}(x_3, M_f{}^2)$, similarly, are the fragmentation functions of a parton p_l into the hadron H. For the processes (1) under consideration, we restrict $p_i = p_j = g$ and $p_l = g, q$ respectively. Furthermore the factorization mass scales M^2 and $M_f{}^2$ have been, in principle, kept distinct. The Born cross sections $ds^o{}_{gg \to pl}(s,v)$ for the two subprocesses (1) are [5]

$$\frac{d\sigma^o}{dv_{gg \to gg}} = \pi \frac{4N^2}{N^2-1} \frac{\alpha_s^2(\mu^2)}{s} \left[3 - v(1-v) + \frac{v}{(1-v)^2} + \frac{1-v}{v^2}\right]$$

$$\frac{d\sigma^o}{dv_{gg \to q\bar{q}}} = \pi \frac{1}{2N(N^2-1)} \frac{\alpha_s^2(\mu^2)}{s} \left[\frac{N^2-1}{v(1-v)} - 2N^2\right] \left[v^2 + (1-v)^2\right] \tag{4}$$

where averaging over spin and colour has been performed. Finally $K_{p_ip_j \to p_l}(s,v,w)$ are the partonic K-factors and $\alpha_s(\mu^2)$ is the running coupling constant evaluated to the renormalization scale μ^2:

$$\alpha_s(\mu^2) = \frac{2\pi}{\beta_o \ln(\mu^2/\Lambda^2)} \left[1 - \frac{\beta_1 \ln(\ln(\mu^2/\Lambda^2))}{\beta_o^2 \ln(\mu^2/\Lambda^2)}\right] \tag{5}$$

with

$$\beta_o = \frac{11N}{6} - \frac{N_F}{3}, \quad \beta_1 = \frac{17N^2}{6} - \frac{5NN_F}{6} - \frac{C_F N_F}{2}$$

and $N = 3$ (N_F) being the number of colours (flavours) with $C_F = (N^2-1)/2N$.

So far eq.(3) describes the one-hadron inclusive production, with the appropriate definition of the fragmentation function. In case of inclusive jet production at large transverse momentum one can also use eq.(3) with the following simple phenomenological prescription. As stated above, in perturbation theory the distribution functions $F^o{}_{pi}(x)$ and $D^o{}_{pl}(x)$ are transformed, after cancellation of virtual and real singularities, into scale - breaking distributions $F_{pi}(x, M^2)$ and $D_{pl}(x, M^2{}_f)$ beyond the leading order, which absorb the collinear left over singularities. Correspondingly the partonic K-factors $K_{pipj \to pl}(s,v,w)$ have a residual dependence on the factorization scales M^2 and $M^2{}_f$, in addition to the renormalization scale μ^2, which has not been explicitly shown in our short-hand notation for $K_{pipj \to pl}(s,v,w)$. It is clear then that the contribution to jet production from the parton processes $p_i + p_j \to p_l + X$ can be simply calculated by replacing $D_{pl}(x_3, M_f{}^2)$ with $\delta(1 - x_3)$, the sensitivity to the phenomenological jet algorithm being related to the variation with M_f^2 of the partonic K-factors.

Alternatively one can use, as in ref. [4], a description of jets à la Sterman and Weinberg[6], namely the jet is defined as an arbitrary set of hadrons contained within a cone of given opening angle δ. It is clear that the above fragmentation scale is of order $M_f \sim E_{jet} \sin\delta$.

We have studied the production of jets also in this framework. For the time being we only give here explicit results for gluon-gluon subprocesses within the first algorithm. A more complete phenomenological analysis of jet production, including all parton subprocesses, will be given elsewhere.

The inclusive production of a gluon in a g – g collision takes contributions from the processes

$$\begin{aligned} &g+g \to g+(g+g) \\ &g+g \to g+(q+\bar{q}), \end{aligned} \qquad (6)$$

where one integrates over the unobserved final partons. Furthermore, following ref. [1,3,4], after the cancellation of the $(1/\varepsilon^2)$ divergences from real and virtual gluon emission, the collinear singularities are absorbed into $F_g(x, M^2)$ and $D_g(x, M_f^2)$ by adding the bremsstrahlung contributions from the initial and final parton legs as follows:

$$E_3 \frac{d\sigma^{gg \to (g \to H_3) + X}}{d^3 k_3} = \frac{I}{\pi S} \frac{dx_3}{x_3^2} \frac{dv}{1-v} \frac{dw}{w} F_g^o(x_1) F_g^o(x_2) D_g^o(x_3)$$

$$\left\{ \frac{1}{v} \frac{d\sigma^o}{dv_{gg \to gg}}(s,v)\,\delta(1-w) + \frac{\alpha_s}{2\pi}\,\theta(1-w) \left[k_{gg \to g}(s,v,w) + \right.\right.$$

$$+ \frac{1}{v} H_{gg}(w) \frac{d\sigma^o}{dv_{gg \to gg}}(ws, v) + \frac{1}{1-vw} H_{gg}\left(\frac{1-v}{1-vw}\right)$$

$$\frac{d\sigma^o}{dv_{gg \to gg}}\left(\frac{1-v}{1-vw} s, vw\right) + \frac{1}{1-v+vw} \tilde{H}_{gg}(1-v+vw)$$

$$\frac{d\sigma^o}{dv_{gg \to gg}}\left(s, \frac{vw}{1-v+vw}\right) + \frac{2N_F}{v} H_{qg}(w) \frac{d\sigma^o}{dv_{qg \to qg}}(ws, 1-v)$$

$$+ \frac{2N_F}{1-vw} H_{qg}\left(\frac{1-v}{1-vw}\right) \frac{d\sigma^o}{dv_{qg \to qg}}\left(\frac{1-v}{1-vw} s, vw\right)$$

$$+ \frac{2N_F}{1-v+vw} \tilde{H}_{gq} (1-v+vw) \frac{d\sigma^o}{dv_{gg \to q\bar{q}}} (s, \frac{vw}{1-v+vw}) \Bigg] \Bigg\} \quad (7)$$

where [7]

$$H_{pi\,pj}(x) = \ln\left(\frac{M^2}{\mu^2}\right) P_{pi\,pj}(x) + f_{pi\,pj}(x)$$

$$\tilde{H}_{pi\,pj}(x) = \ln\left(\frac{M_f^2}{\mu^2}\right) P_{pi\,pj}(x) + d_{pi\,pj}(x) \quad (8)$$

and $P_{pipj}(x)$, $f_{pipj}(x)$ and $d_{pipj}(x)$, are, respectively, the Altarelli - Parisi kernels and the finite $o(\alpha_s)$ corrections to the structure and fragmentation functions.

So far only $f_{qq}(x)$ and $d_{qq}(x)$ have been calculated explicitly [8]. As well known, they contain next-to-leading terms which become particularly large near the boundary of the phase space. Furthermore this kind of corrections can be simply taken into account by the appropriate use of the correct kinematical limits in the various processes and in particular by incorporating an explicit dependence of the running coupling constant on the kinematic variables in the Altarelli - Parisi evolution equations [9]. For gluon initiated processes, we can similarly incorporate in the f's and d's the relevant kinematical factors, as done for example in ref. [10], by "multiplying" $P_{pipj}(x)$ by ln $[(1-x)/x]$ for $f_{pipj}(x)$ and by $\ln[(1-x)\cdot x^2]$ for $d_{pipj}(x)$. Furthermore imposing energy-momentum sum rules we obtain the following expressions for $f_{pipj}(x)$ and $d_{pipj}(x)$:

$$f_{gg}(x) = 2N \left\{ x \left[\frac{\ln(1-x)}{1-x} \right]_+ - \frac{x \ln x}{1-x} + \left(\frac{5N_F}{24\,N} - \frac{\pi^2}{6} - \frac{1}{2} \right) \delta(1-x) \right\}$$

$$d_{gg}(x) = 2N \left\{ x \left[\frac{\ln(1-x)}{1-x} \right]_+ + \frac{2 \ln x}{1-x} + \left(\frac{7N_F}{16\,N} + \frac{\pi^2}{3} - \frac{17}{4} \right) \delta(1-x) \right\}$$

$$f_{qg}(x) = \frac{1}{2} \left[x^2 + (1-x)^2 \right] \ln\left(\frac{1-x}{x} \right) \quad (9)$$

$$d_{qg}(x) = \frac{1}{2}\left[x^2 + (1-x)^2\right] \ln\left[x^2(1-x)\right]$$

$$f_{gq}(x) = C_F \left\{\frac{1+(1-x)^2}{x} \ln\left(\frac{1-x}{x}\right) - \frac{4}{3}\right\}$$

$$d_{gq}(x) = C_F \left\{\frac{1+(1-x)^2}{x} \ln\left[(1-x)\,x^2\right] - 2\right\}$$

$$f_{qq}(x) = C_F \left\{ (1+x^2)\left[\frac{\ln(1-x)}{1-x}\right]_+ - \frac{3}{2}\,\frac{1}{(1-x)_+} - \frac{1+x^2}{1-x}\ln x + 3 + 2x - \left(\frac{9}{2} + \frac{\pi^2}{3}\right)\delta(1-x)\right\}$$

$$d_{qq}(x) = C_F \left\{ (1+x^2)\left[\frac{\ln(1-x)}{1-x}\right]_+ + 2\,\frac{1+x^2}{(1-x)}\ln x - \frac{3}{2}\,\frac{1}{(1-x)_+} + \frac{3}{2}(1-x) + \left(\frac{2}{3}\pi^2 - \frac{9}{2}\right)\delta(1-x)\right\} \tag{9}$$

where, for convenience, we have also reported the complete expressions[8] for the quark-quark case(*). We think that the above definition of f'_{ij}s and d'_{ij}s are more physical ones, instead of the naive choice $f_{ij} = d_{ij} = 0$ (i,j = g), because it reduces the large correction terms of kinematical origin, common to all processes involving gluons [10, 11], and should be absorbed into the structure and fragmentation functions.

From eqs. (7-8-9) we finally obtain an explicit expression for the (gg→g) K-factor. Due to the very long and cumbersome expression of $K_{gg\to g}$ (s,v,w), we only report in Table I the coefficients of the distributions in w, with the same notation of ref. [1]

$$K_{gg\to g}(s,v,w) = \frac{1}{v}\,\frac{d\overset{o}{\sigma}}{dv_{gg\to gg}}(s,v)\left\{\left[C_1 + \tilde{C}_1 \ln\left(\frac{s}{M^2}\right)\right.\right.$$

(*) (We disagree with ref.[10] for the coefficient of the δ(1 - x) term in f_{gg})

$$+ \tilde{\tilde{C}}_1 \ln (\frac{s}{M_f^2}) + \hat{C}_1 \ln (\frac{s}{\mu^2}) \Bigg] \delta(1-w) +$$

$$+ \left[C_2 + \tilde{C}_2 \ln (\frac{s}{M^2}) + \tilde{\tilde{C}}_2 \ln (\frac{s}{M_f^2}) \right] \frac{1}{(1-w)_+} +$$

$$+ \quad C_3 \left[\frac{\ln (1-w)}{(1-w)} \right]_+ \Bigg\} + K'_{gg \to g}(s,v,w) \tag{10}$$

where $K'_{gg\to g}(s,v,w)$ is finite for $w\to 1$. Notice the explicit dependence on M_f^2, M^2 and μ^2.

Comparing the gluon-gluon results of Table I with the analogous ones of ref [1], for the quark-quark case, one simply finds $C_3^{(gg)}/C_3^{(qq)} = N/C_F$, in agreement with the expectations from the Sudakov form factor. Indeed, as shown in [1], C_3 is the coefficient of the leading $(\ln k_T^2/k_T^2)$ term of the relative transverse momentum distributions. Notice that the result $C_3^{(gg)} = 4N$ in eq. (10) depends critically upon the definition of f_{gg} and d_{gg} made in eq. (9). The naive choice $f_{gg} = d_{gg} = o$ would in fact lead to $C_3^{(gg)} = 10\,N$, similarly to $C_3^{(qq)} = 10\,C_F$, that could have been found in [1] for $f_{qq} = d_{qq} = 0$.

We present now, in analogy to eq. (7), our result for the case $g + g \to q + X$:

$$E_3 \frac{d\sigma^{gg \to (q \to H_3) + X}}{d^3 k_3} = \frac{1}{\pi S} \int \frac{dx_3}{x_3^2} \int \frac{dv}{1-v} \int \frac{dw}{w} F_g^o(x_1) F_g^o(x_2) D_q^o(x_3)$$

$$\Bigg\{ \frac{1}{v} \frac{d\sigma^o}{dv_{gg \to q\bar{q}}}(s,v)\,\delta(1-w) + \frac{\alpha_s}{2\pi}\,\theta(1-w) \Big[K_{gg \to q}(s,v,w) +$$

$$\frac{1}{v} H_{gg}(w) \frac{d\sigma^o}{dv_{gg \to q\bar{q}}}(ws,v) + \frac{1}{1-vw} H_{gg}(\frac{1-v}{1-vw}) \frac{d\sigma^o}{dv_{gg \to q\bar{q}}}(\frac{1-v}{1-vw} s, vw)$$

$$+ \frac{1}{v} H_{qg}(w) \frac{d\sigma^o}{dv_{qg \to qg}}(ws,v) + \frac{1}{1-vw} H_{qg}(\frac{1-v}{1-vw}) \frac{d\sigma^o}{dv_{qg \to qg}}(\frac{1-v}{1-vw} s, 1-vw)$$

$$+ \frac{1}{1-v+vw} \tilde{H}_{qq}(1-v+vw) \frac{d\sigma^o}{dv_{gg \to q\bar{q}}}(s, \frac{vw}{1-v+vw}) + \frac{1}{1-v+vw} \tilde{H}_{qg}(1-v+vw) \cdot$$

$$\cdot \frac{d\overset{o}{\sigma}}{dv_{gg\to gg}} \left(s, \frac{vw}{1-v+vw}\right)\Bigg]\Bigg\} \qquad (11)$$

The same result holds for the reaction $g + g \to q + X$. With the help of eqs.(8) and (9) we finally get an expression for $K_{gg \to q}(s,v,w)$, similar to eq.(10). The corresponding results for the coefficients $C_{(i)}$ are shown in Table II.

Following ref.[1], the relative transverse momentum distributions of two jets produced in a gluon-gluon collision, can be simply obtained from eq. (10) for $gg \to g + X$ and the analogous one for $gg \to q + X$, using the general formula given in [1], with the appropriate substitution of the Born cross sections and the coefficients C_i in the Sudakov form factor and in the regular terms.

We present now some phenomenological numerical consequences of our results. We define R to be the ratio of the cross sections (3) to order $(\alpha_s{}^2 + \alpha_s{}^3)$ to the cross section including $o(\alpha_s{}^2)$ only, for $gg \to g + X$ and $gg \to q + X$. Thus $R = 1$ would imply the exact validity of the leading-order formula. To be consistent, the numerator of R has been calculated with $\alpha_s(\mu^2)$ to two lops, while the Born cross section in the denominator has been evaluated at one loop order only. As discussed above, we have taken $D\,(x_3, M_f{}^2) = \delta(1 - x_3)$ in eq. (3), for the jet algorithm.

In Figs. (1a, 1b) we show for $p\bar{p}$ collisions at $\sqrt{S} = 0.63$ TeV and with the set of structure functions of Duke-Owens [12], the ratio R as a function of $\eta \equiv 2p_T/\sqrt{S}$, p_T being the transverse momentum of the jet. The results have been obtained for $\theta_{jet} = 90^o$, but do not change significantly at $\theta_{jet} = 60^o$. The full curves (respectively dashed curves) correspond to f_{gg} and d_{gg} given by eq. (9) (resp. $f_{gg} = d_{gg} = 0$). The upper (lower) curves correspond to $\mu^2 = M^2 = M_f{}^2 = 4p_T{}^2$ ($M^2 = M^2 = M_f{}^2 = p_T{}^2/4$). As clear from Figs. 1, the increasing behaviour of R is much weakened for a "physical" definition of gluon structure functions, when the main kinematical corrections are taken into account in the structure and fragmentation functions. Furthermore the corrections are stable and reasonably small for $\mu^2 = M^2$ in the range $p_T{}^2/4 \div p_T{}^2$. The energy dependence of R for $gg \to gX$ is shown in Fig. 2, where the results of Fig. 1a at $\sqrt{S} = 0.63$ TeV (full lines) are compared with the analogous one at 1.8 TeV (dashed lines). In Fig.3 we show the dependence of R on the fragmentation mass M_f, for fixed $M = \mu = p_T/2$.

Finally in Fig. 4, we present our results for a jet algorithm à la Sterman-Weinberg, namely when the jet produced at large p_T is defined by a cone of semiaperture δ. Full and dashed-dotted lines refer respectively to $\delta = 0.1$ and $\delta = 0.05$, and the limiting case $\delta = 1$ is presented with a dashed line. Of course, with this definition of jets one accounts for all possible partons (g, q, $\bar{q}$) produced inclusively. Notice that the corresponding distribution functions do not enter in the calculation because the related $(1/\varepsilon)$ terms appearing in the real contributions to the cross section are automatically subtracted by the analogous contribution from two partons in the jet. A more detailed analysis will be presented elsewhere.

TABLE I

$$\hat{C1} = \frac{2\,nf}{3} - \frac{11\,n}{3}$$

$$\tilde{C1} = \frac{n\,(6\log(v) - 6\log(1-v) + 11)}{3} - \frac{2\,nf}{3}$$

$$\approx\!\!C1 = \frac{n\,(12\log(v) + 11)}{6} - \frac{nf}{3}$$

$$\begin{aligned} C1 = {} & n\,(90\,(6\,v^6 - 21\,v^5 + 45\,v^4 - 58\,v^3 + 54\,v^2 - 30\,v + 10)\log^2(v) \\ & - 90\log(1-v)\,(4\,v^6 - 16\,v^5 + 37\,v^4 - 50\,v^3 + 47\,v^2 - 26\,v + 8)\log(v) \\ & - 30\,(v^2 - v + 1)\,(11\,v^4 - 15\,v^3 + 4\,v^2 + 22\,v - 11)\log(v) \\ & + 90\log^2(1-v)\,(2\,v^6 - 7\,v^5 + 14\,v^4 - 16\,v^3 + 14v^2 - 7\,v + 2) + 120\,\pi^2 v^6 \\ & + 914\,v^6 - 360\,\pi^2 v^5 - 2742\,v^5 + 765\,\pi^2 v^4 + 5349\,v^4 - 930\,\pi^2 v^3 - 6128\,v^3 \\ & + 30\log(1-v)\,v\,(v^2 - v + 1)\,(7\,v^2 + 8\,v + 7) + 855\,\pi^2 v^2 + 5349\,v^2 \\ & - 450\,\pi^2 v - 2742\,v + 120\,\pi^2 + 914)\,/\,(180\,(v^2 - v + 1)^3) \\ & + (-9\,nf\,(v-1)\,v^2\,(v^2 + v - 1)\log^2(v) \\ & + 6\,nf\,(v^2 - v + 1)\,(4\,v^4 - 3\,v^3 - v^2 + 8\,v - 4)\log(v) \\ & + 18\,nf\log(1-v)\,(v-1)\,v\,(2\,v^2 - 2\,v + 1)\log(v) \\ & - nf\,(83\,v^6 - 249\,v^5 + 18\,\pi^2 v^4 + 444\,v^4 - 36\,\pi^2 v^3 - 473\,v^3 + 27\,\pi^2 v^2 \\ & + 444\,v^2 - 9\,\pi^2 v - 249\,v + 83) - 6\,nf\log(1-v)\,v\,(v^2 - v + 1) \\ & (5\,v^2 - 2\,v + 5) + 9\,nf\log^2(1-v)\,(v-1)^2\,v\,(v^2 - 3\,v + 1))\,/\,(72\,(v^2 - v + 1)^3) \end{aligned}$$

$$\tilde{C2} = 4\,n$$

$$\approx\!\!C2 = 2\,n$$

$$C2 = \frac{n\,(12\,(2\,v^2 - 3\,v + 3)\log(v) - 11\,(v^2 - v + 1) + 12\log(1-v)\,v)}{6\,(v^2 - v + 1)} + \frac{nf}{3}$$

$$C3 = 4\,n$$

TABLE II

$$\hat{C1} = \frac{n\,nf\,(2\,v^2 - 2\,v + 1)}{3(n\,v^2 - n\,v + cf)} - \frac{nf}{3\,n\,(n\,v^2 - n\,v + cf)} + \frac{11}{6\,(n\,v^2 - n\,v + cf)}$$

$$\tilde{C1} = \frac{6\log(v) - 6\log(1-v) + 11}{6\,(n\,v^2 - n\,v + cf)} + \frac{n\,nf\,(2\,v^2 - 2\,v + 1)}{3\,(n\,v^2 - n\,v + cf)} - \frac{nf}{3\,n\,(n\,v^2 - n\,v + cf)}$$

$$\approx\!\!\!\!\!C1 = \frac{4\,(v^2 - v + 1)\log(v) + 3\,(v^2 - v + 1)}{4\,(n\,v^2 - n\,v + cf)} + \frac{-4\log(v) - 3}{8\,n^2\,(n\,v^2 - n\,v + cf)}$$

$$\begin{aligned} C1 = {} & (3\,(v^3 - 16\,v^2 + 17\,v - 8)\log^2(v) + 12\log(1-v)\,(2\,v^2 - 2\,v + 1)\log(v) \\ & + 3\,(v - 1)\,(v^2 - 9\,v + 6)\log(v) - 3\log(1-v)\,v\,(v^2 + 7\,v - 2) \\ & - 3\log^2(1 - v)\,(v-1)\,(v^2 - 2\,v + 2) - 8\,\pi^2 v^2 - 39\,v^2 + 8\,\pi^2 v + 39\,v \\ & - 4\,\pi^2 - 18)\,/\,(12\,(2\,v^2 - 2\,v + 1)\,(n\,v^2 - n\,v + cf)) \\ & + ((v^2 + 1)\log^2(v) + (v-3)\,(v-1)\log(v) + \log^2(1-v)\,(v^2 - 2\,v + 2) \\ & + 2\,v^2 + \log(1 - v)\,v\,(v + 2) - 2\,v + 1)\,/\,(4\,n^2\,(2\,v^2 - 2\,v + 1)\,(n\,v^2 - n\,v + cf)) \\ & + \frac{5\,nf}{12\,n\,(n\,v^2 - n\,v + cf)} \end{aligned}$$

$$\tilde{C2} = \frac{2}{n\,v^2 - n\,v + cf} - \frac{2\,n^2\,(2\,v^2 - 2\,v + 1)}{n\,v^2 - n\,v + cf}$$

$$\approx\!\!\!\!\!C2 = \frac{v^2 - v + 1}{n\,v^2 - n\,v + cf} - \frac{1}{2\,n^2\,(n\,v^2 - n\,v + cf)}$$

$$C2 = \frac{n^2\,((4\,v^2 - 6\,v + 3)\log(v) + \log(1 - v)\,(2\,v - 1))}{n\,v^2 - n\,v + cf} + \frac{-3\log(v) - \log(1 - v)}{n\,v^2 - n\,v + cf}$$

$$C3 = \frac{2\,n^2\,(2\,v^2 - 2\,v + 1)}{n\,v^2 - n\,v + cf} - \frac{2}{n\,v^2 - n\,v + cf}$$

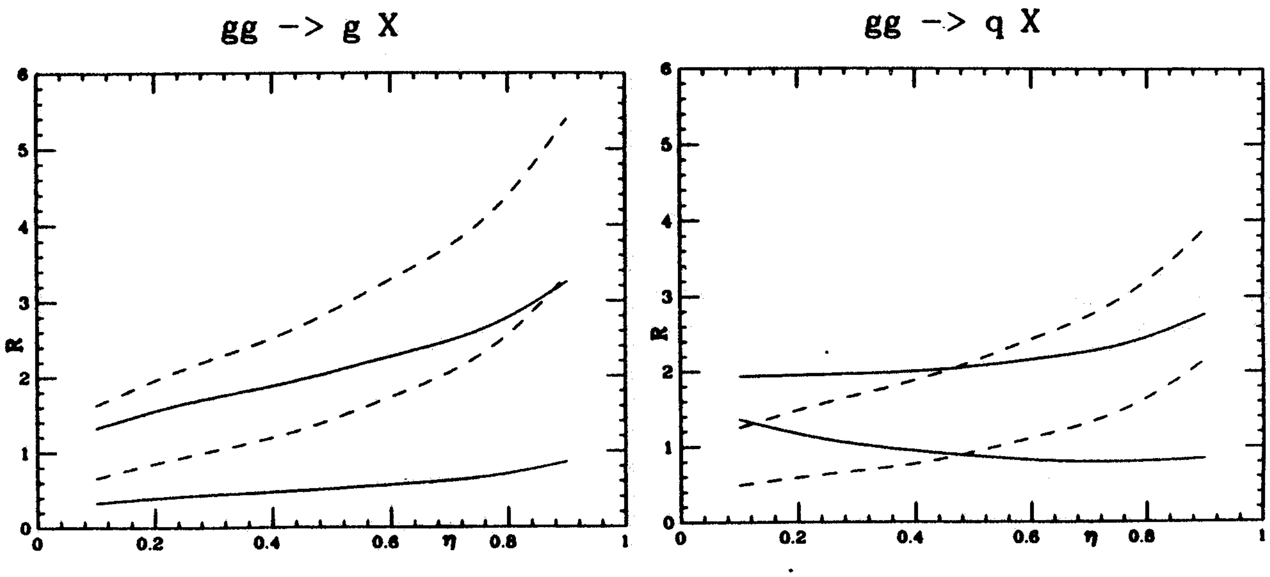

FIG. 1 - The ratio R of $o(\alpha_s^2 + \alpha_s^2)$ cross section to the cross section of $o(\alpha_s^2)$ only as a function of $\eta \equiv 2p_T/\sqrt{S}$, for $gg \to gX$ (Fig. 1a) and $gg \to qX$ (Fig. 1b). Full (dashed) curves correspond to f_{gg} and d_{gg} given by eq. (9) (resp. $f_{gg} = d_{gg} = 0$). The upper (lower) curves correspond to $\mu^2 = M^2 = M_f^2 = 4\, p_T^2$ ($\mu^2 = M^2 = M_f^2 = p_T^2/4$). The energy is $\sqrt{S} = 0.63$ TeV.

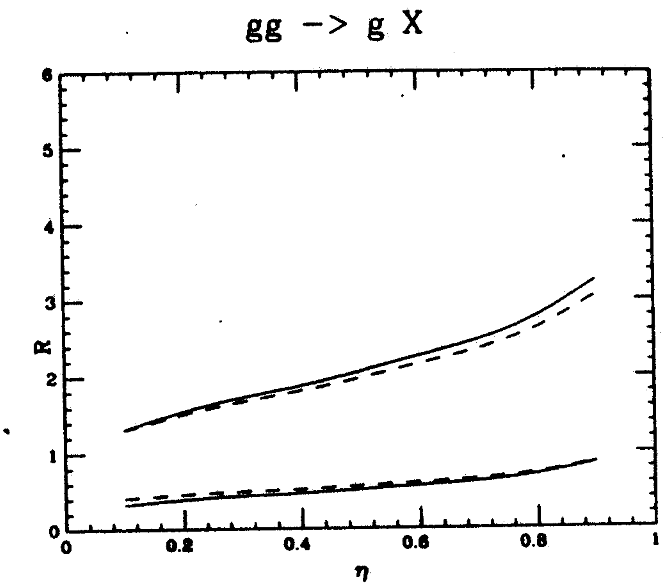

FIG. 2 - Same as in Fig. 1a for the full lines. Dashed lines correspond to the analogous case for $\sqrt{S} = 1.8$ TeV.

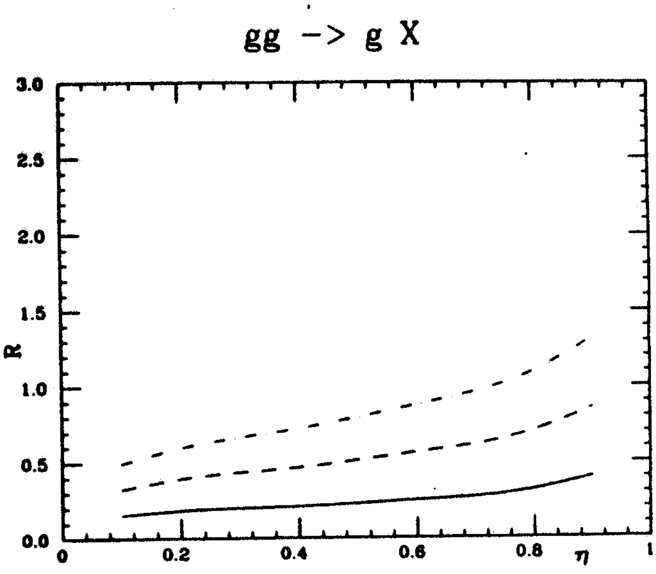

FIG. 3 -Dependence of R upon the factorization mass M_f. Full, dashed, dotted-dashed lines correspond to $M_f = p_T/4$, $p_T/2$ and p_T respectively, at $\sqrt{S} = 0.63$ TeV.

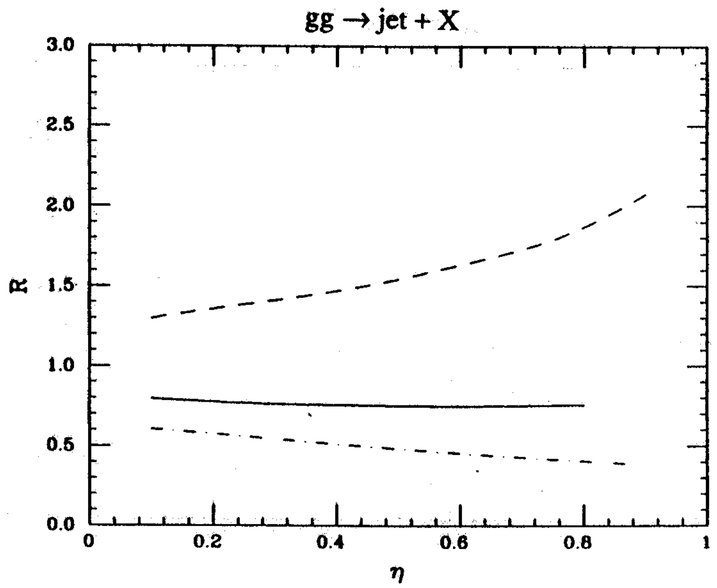

FIG. 4 - The ratio R for jets à la Sterman and Weinberg. Full, dashed-dotted and dashed lines refer to $\delta = 0.1$, 0.05 and $\delta = 1$, respectively.

In conclusion we have presented the $o(\alpha_s^3)$ corrections to gluon-gluon processes. The corrections are reasonably small and stable after taking into account in the definition of structure and fragmentation functions the main logarithmic corrections of kinematical origin.

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