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THE ENERGY OF TWO COLOR CHARGES IN THE $g\to\infty$ LIMIT OF QCD IN THE POLAR REPRESENTATION

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The energy of two color charges is studied in the $g\to\infty$ limit of QCD in the polar representation. It is found that there is no confining potential, and confinement depends on whether the Gauss constraint is imposed in classical or in operator form. Only in the latter case quarks cannot be separated.

In this letter we study the energy of two quarks in the $g\to\infty$ limit of QCD in the polar representation [1,2].

In this limit there are no spatial derivatives in the gauge field hamiltonian, so that the discretization which is understood in the hamiltonian formalism can be performed straightforwardly. The price to be paid for such a simplification is that, in the absence of spatial derivatives, the hamiltonian cannot describe moving particles but only static energies.

Due to the peculiarity of such a situation it is not obvious how general our results are, and which modifications might be necessary to handle the full theory. In the present framework, however, we find that there is no confining potential, and confinement depends on whether the Gauss constraint is imposed in classical or in operator form. Only in the latter case quarks cannot be separated.

The polar representation has been worked out only for the SU(2) color group, so that we have to restrict ourselves to this case. The gauge field reads [2]

$$A_{ia}=\sum_{n=1}^{3}\left(f_{in}(\theta)\lambda_n h_{na}(\varphi)-\frac{1}{2g}\sum_{b,c=1}^{3}\varepsilon_{abc}h_{nb}(\varphi)\partial_i h_{nc}(\varphi)\right), \tag{1}$$

where θ_i and φ_a are two sets of three angles which parametrize the orthogonal matrices f and h, and ε is the antisymmetric tensor.

The Gauss constraint takes the form

$$-(1/g)\mathrm{D}_{kab}E_{kb}+j_{0a}=L_a(\varphi)+j_{0a}=0\,. \tag{2}$$

In the above equation D_k are the covariant derivatives, E_{ka} the electric strength components, j_{0a} the matter field color charge densities and L_a the cartesian components of the orbital angular momentum expressed in terms of the Euler angles φ_1, φ_2, φ_3. The variables θ_i and λ_n are therefore gauge-invariant.

In the large-g limit the hamiltonian is [2]

$$H=\frac{1}{2}\int \mathrm{d}^3x\Bigg(\sum_n -\frac{\delta^2}{\delta\lambda_n^2}-2\delta(0)\sum_{n>m}\frac{1}{\lambda_n^2-\lambda_m^2}(\lambda_n\delta/\delta\lambda_n-\lambda_m\delta/\delta\lambda_m)$$

$$+\frac{1}{2}\sum_{m,n}\varepsilon_{mnk}^2\frac{1}{(\lambda_n^2-\lambda_m^2)^2}[(\lambda_n^2+\lambda_m^2)(L_k'^2+l_k^2)+4\lambda_m\lambda_n l_k L_k']+g^2(\lambda_1^2\lambda_2^2+\lambda_1^2\lambda_3^2+\lambda_2^2\lambda_3^2)\Bigg). \tag{3}$$

In the above equation L_k' and l_k are the "proper frame" components of angular momentum

$$L'_k = h_{km}(\varphi) L_m(\varphi), \quad l_k = f_{km}(\theta) L_m(\theta) . \tag{4}$$

Let us perform the space discretization which is understood in the hamiltonian formalism and which is straightforward here due to the absence of spatial derivatives

$$\lambda_n(\boldsymbol{x}) \to \lambda_{n\mu}, \quad \delta/\delta\lambda_n(\boldsymbol{x}) \to l^{-3}\partial/\partial\lambda_{n\mu}, \quad L_k(\boldsymbol{x}) = l^{-3} L_{k\mu}, \quad j_{0a}(\boldsymbol{x}) = l^{-3} j_{0a\mu},$$

$$\int \mathrm{d}^3x \to l^3 \sum_{\mu}, \quad \delta(\boldsymbol{x}-\boldsymbol{y}) \to l^{-3}\delta_{\mu,\nu} , \tag{5}$$

where μ represents the lattice site and l the lattice spacing.

It is also convenient to introduce dimensionless variables

$$\xi_{n\mu} = l\lambda_{n\mu} , \tag{6}$$

so that the hamiltonian and the Gauss constraint become

$$H = \frac{1}{2l}\sum_{\mu}\Bigg(\sum_{n} - \frac{\partial^2}{\partial\xi_{n\mu}^2} - 2 \sum_{n>m} \frac{1}{\xi_{n\mu}^2 - \xi_{m\mu}^2} (\xi_{n\mu}\partial/\partial\xi_{n\mu} - \xi_{m\mu}\partial/\partial\xi_{m\mu})$$

$$+ \frac{1}{2}\sum_{m,n} \varepsilon_{mnk}^2 \frac{1}{(\xi_{n\mu}^2 - \xi_{m\mu}^2)^2} [(\xi_{n\mu}^2 + \xi_{m\mu}^2)(\tilde{L}'^2_{k\mu} + \tilde{l}^2_{k\mu}) + 4\xi_{m\mu}\xi_{n\mu}\tilde{l}_{k\mu}\tilde{L}'_{k\mu}] + g^2(\xi_{1\mu}^2\xi_{2\mu}^2 + \xi_{1\mu}^2\xi_{3\mu}^2 + \xi_{2\mu}^2\xi_{3\mu}^2)\Bigg) , \tag{7}$$

$$\tilde{L}_{a\mu} + \tilde{j}_{0a\mu} = 0 . \tag{8}$$

In the above equations there are no terms connecting the sites, so that the state functionals can be written as products of ordinary wave-functions at each site.

The fact that the Gauss constraint is expressed by a local equation in spite of the presence of spatial derivatives on the LHS of eq. (2), is a general result. Eq. (2) is independent of g, and its validity does not rely on the polar representation [3].

The absence of spatial derivatives in the hamiltonian is instead related to the use of the polar representation and the large-g limit. This is the drastic simplification and restriction of the present study.

According to the Dirac theory of constrained systems [4] [#1]. Eq. (8) can be imposed in two ways. In its operator form the Gauss law is imposed on physical state functionals

$$(\tilde{L}_a + \tilde{j}_{0a})\Psi = 0 . \tag{9}$$

In its classical form the Gauss law is considered along with the classical form of the hamiltonian, and it is imposed as an equation among field variables and supplemented by a gauge-fixing condition. After solution of the Gauss and gauge-fixing constraints and replacement of the Poisson brackets by Dirac brackets the quantization is performed in the standard way.

In the first case, since L_3 has integral eigenvalues, only integral eigenvalues of j_{03} are admissible, and therefore we must have at each point in space either two quarks or one quark and one antiquark (assumed to be in the fundamental representation): Quarks cannot be separated.

Such a property is lost by adopting the classical form of the Gauss constraint [#2]. This is due to the fact that L_a is replaced by j_{0a} at the classical level, where both take continuous values. After the quantization is performed, states are admissible with any eigenvalue of j_{03}, including half-integer ones.

Let us consider the gauge field in the presence of two color charges. If we adopt the operator form of the

[#1] See ref. [5] for a review.

[#2] The fact that the classical and operator form of the Gauss constraint are not equivalent to each other was already noticed in the case of abelian gauge theories [6].

Gauss constraint we can assign arbitrarily their common position, let us say ν, but not their color state which is determined by the Gauss constraint itself.

The wave-functions in the presence of the charges can differ from the vacuum wave-function only at the site ν. The following equations will therefore refer to this site only.

The wave-function in the presence of the charges can be written

$$\Psi_{JM}=\Phi_J \sum_{M'} D^J_{M'M}\xi^J_{-M'}\langle JM'\, J-M'\,|\,0\,0\rangle\,, \tag{10}$$

where $\langle JM'\, J-M'\,|\,0\,0\rangle$ are Clebsch–Gordan coefficients, $D^J_{M'M}$ are eigenfunctions of $\tilde{L}^2$, $\tilde{L}_3$ and $\tilde{L}'_3$ with eigenvalues $J(J+1)$, M' and M respectively, χ^J_M are eigenfunctions of $\tilde{j}^2_0$, $\tilde{j}_{03}$ with eigenvalues $J(J+1)$ and M respectively, and $\phi_J=\phi_J(\lambda_n, \theta_i)$.

The vacuum wave-function can be written as

$$\Psi_{\mathrm{V}}=\Phi_{\mathrm{V}} D^0_{00}\,. \tag{11}$$

It is obvious that $\Psi_{00}=\Psi_{\mathrm{V}}\chi_0$, so that the color charges have zero energy in such a state. To evaluate the energy of the states Ψ_{1M} we perform a variational calculation. When $J\neq 0$ there is a term which diverges like $(\xi_{n\nu}-\xi_{m\nu})^{-2}$ in the hamiltonian. We therefore assume as variational wave-function

$$\Phi_1=\Phi_{\mathrm{V}} F_\nu \tag{12}$$

with

$$F_\nu=\prod_{n>m}\{1-\exp[-(\xi_{n\nu}-\xi_{m\nu})^2]\}\,. \tag{13}$$

It is obvious that the above factor yields only a finite contribution to the energy.

If we adopt the classical form of the Gauss constraint, on the other hand, we must fix the gauge. It is convenient to do it by the equations

$$h_{na}=\delta_{na}\,, \tag{14}$$

so that $L'_a=L_a$ and using the Gauss constraint we can replace L_a by j_{0a}. It is now obvious that we can put two color charges at different sites because the eigenvalues of j_{0a} are no longer constrained to be integers. The color state of charge can also be assigned arbitrarily.

A variational calculation can again be performed by introducing a factor $F_{\nu_1}F_{\nu_2}$ if the charges are at the sites ν_1, ν_2. It turns out that their energy is finite and independent of $|\nu_1-\nu_2|$ for $\nu_1\neq\nu_2$. Since a variational calculation provides an upper bound to the energy, we conclude that the quark–quark potential cannot increase with distance in the limit of large g.

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