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QCD AT FINITE BARYON DENSITY

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QCD AT FINITE BARYON DENSITY

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ABSTRACT

Direct attempts to measure the chiral condensate in QCD with dynamical fermions and non-zero chemical potential fail to predict the chiral symmetry restoration transition point. We propose an alternative method and present some early results.

1. - INTRODUCTION

The work reported here is being in done in collaboration with Ian Barbour of Glasgow University and Clive Baillie of Caltech. The computers used are an IBM in Glasgow and the Crays XMP in Rutherford and S.Diego. We have studied the chiral symmetry restoration transition at (almost) zero temperature and non-zero chemical potential in SU(3) with dynamical fermions. We have been constrained to very small lattices and very low statistics which, we hope, will reveal the basic mechanism characterising the transition, although they will not give physically reliable answers. The chemical potential μ (in lattice units) is included by multiplying lattice links in the positive (negative) direction by a factor e^{μ} ($e^{-\mu}$) [1,2].

The objective is to evaluate the chiral condensate as an order parameter for the restoration of chiral symmetry with incresing μ. This transition is expected to occur at a critical value $\mu_C=(m_p/N_C)$ (m_p:= the proton mass, N_C:= number of colours) [2] but quenched approximation results suggest a value $\mu_C=m_\pi/2$ (m_π:= the pion mass) [3]. The two values coincide for SU(2) ($N_C=2$, $m_p \equiv m_\pi$) but for SU(3) this is a discrepancy, which has been attributed to the quenched approximation [4].

2. - MC SIMULATIONS WITH DYNAMICAL FERMIONS

In order to resolve the above discrepancy, we include Kogut Susskind fermions in the theory. One typical MC (multi)sweep consists of visiting half the lattice hypercubes, lapping around each of them 4 times, hitting each of its links 10 times. While we update, we use the (exact) Lanczos algorithm [5] to calculate the ratio of fermionic determinants and the difference of the traces of the inverse of the fermion matrix M. To this end, we need M^{-1} and M^{-2} for the hypercube in question. Details can be found in [6]. The Wilson action S_g is updated in standard Metropolis fashion. For SU(3), such a multisweep takes 15mins. on the Cray.

The most serious problem is that for SU(3), detM is complex when $\mu \neq 0$. Since the Metropolis algorithm relies on configurations being generated via a real positive weight W, we use $W=|\det M| \exp(-S_g)$. With this weight, the condensate becomes a ratio of two observables

$$<\bar{\varphi}\,\varphi> = \frac{<\mathrm{Tr}(M^{-1})\ \exp(i\phi)>_W}{<\exp(i\phi)>_W} \tag{1}$$

where ϕ is the phase of detM. Note that the imaginary parts of both the numerator and denominator of the above ratio vanish identically [7]. We performed MC measurements of the numerator and denominator of eqn.(1) at bare quark mass $m_q=0.1$ and inverse coupling $\beta=1.5$ on a 4^4 lattice. We work in strong coupling so as to be able to compare with similar results [8] and also in order to decrease correlations. We measure after updating each hypercube (i.e. 32 measurements/sweep) for 5 sweeps and for 5 values of $\mu=0.3$, 0.4, 0.5, 0.9, 1.0 . Because our measurements (which were found to be uncorrelated) were wildly fluctuating, we present results through the normalised distribution function ρ of the numerator and denominator. In terms of $\rho(x)$, the average of any observable x can be written as

$$<x> := \frac{1}{N}\sum_{i=1}^{N} x_i \approx \int_{x_{min}}^{x_{max}} x\ \rho(x)\ dx \tag{2}$$

for a large number of measurements N.

In Fig. 1 we show $\rho(\cos\phi)$ for μ=0.3. Its assymetry resulted to our being able to obtain a condensate of about 200 (in lattice units), which agrees with naive expectations. In the language of refs.[3,4] this is the case where the mass is outside the strip of eigenvalues of the massless fermion matrix. In Fig. 2 we plot the results for the denominator and numerator at μ=0.5. Since the distributions are compatible with being symmetric, the condensate is compatible with the indeterminate 0/0 answer. The same behaviour occurs at μ=0.4 and 0.9. In this region, the mass is inside the strip of eigenvalues of $M(m_q=0)$ [3,4], and MC appears to be failing. This behaviour, a result of a rapidly fluctuating phase of detM, is in disagreement with the analysis of [7]. But at μ=1.0, Fig. 3 suggests that the numerator has a symmetric distibution (i.e. it vanishes) but the denominator does not. Thus the condensate is either very small or compatible with 0. In this case the mass sits inside the cavity formed by the eigenvalues [3,4].

The above results suggest that in the region $\mu \in (0.3, 1.0)$, the condensate cannot be measured by MC; this behaviour may signal that we are near the critical region. At any rate, our results differ from the quenched result μ_C=0.3. This is mildly encouraging.

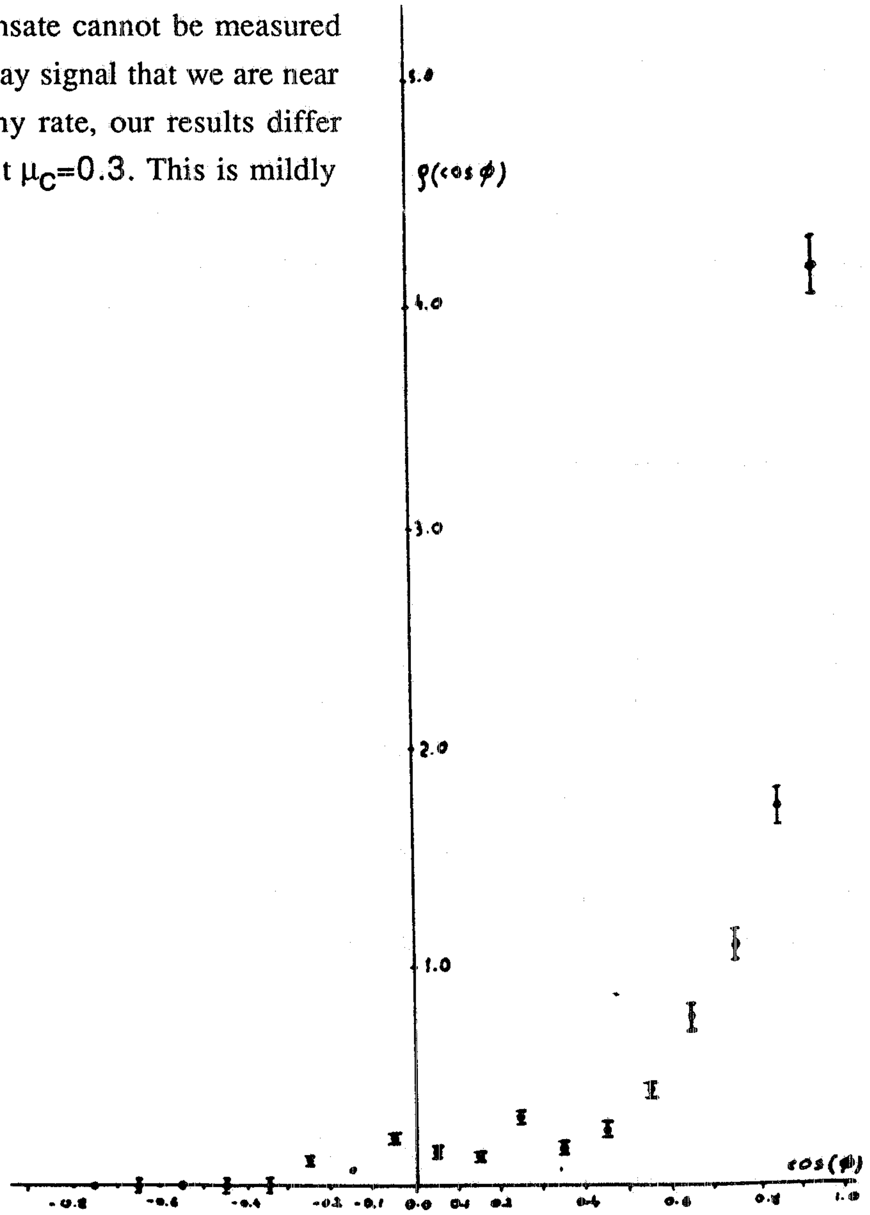

FIG. 1 - The normalised distribution of measurements for the denominator of the condensate at μ=0.3

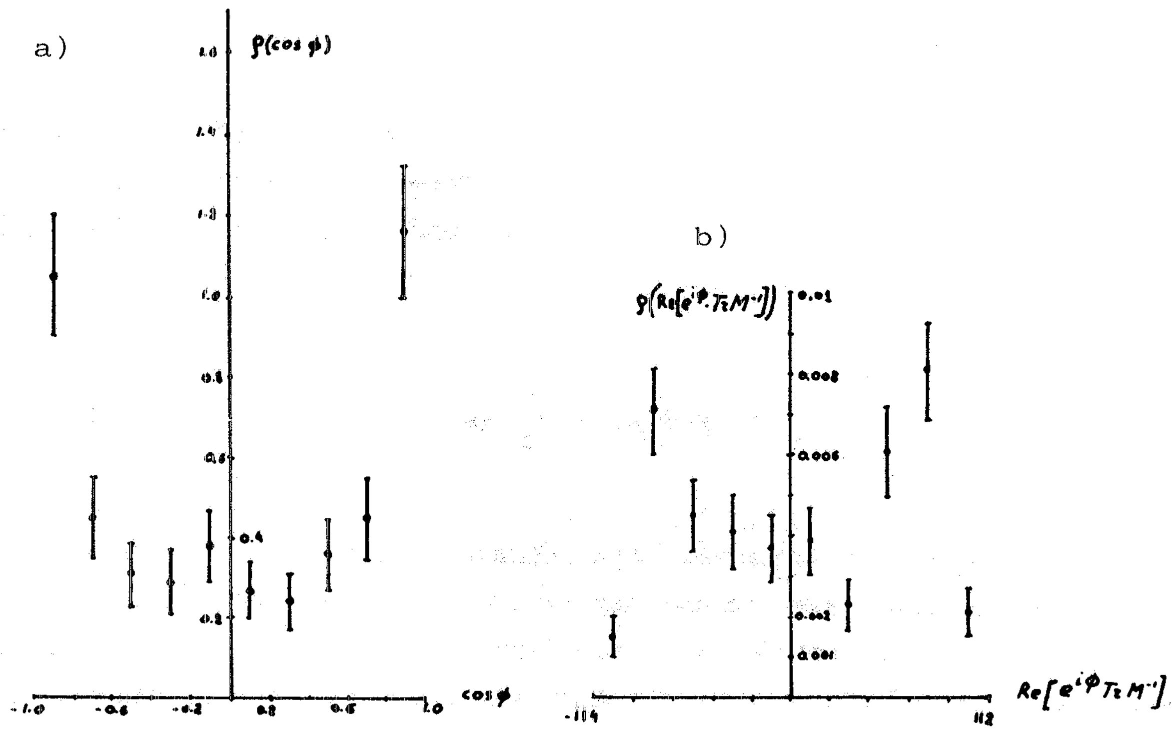

FIG. 2 - (a) Same as in Fig.1 for μ=0.5, (b) Same as in Fig.2a for the numerator of the condensate.

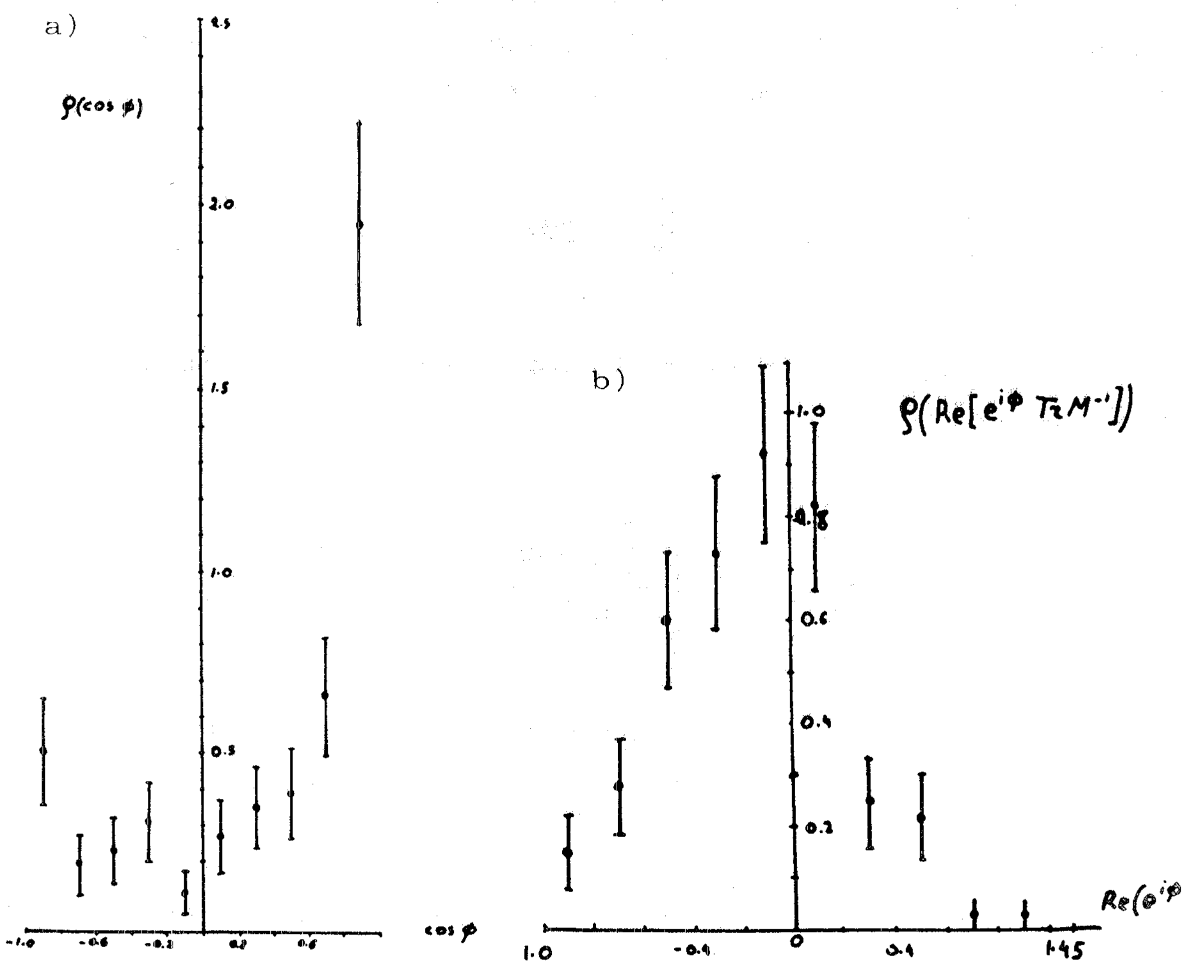

FIG. 3 - a) Same as in Fig.2a for μ=1.0; b)Same as in Fig.2b for μ=1.0

3. - THE DETERMINANT EXPANSION

In order to deal with the above problems, we use the fact that detM can be expanded in terms of Wilson loops and Polyakov lines. We group these expansion terms according to the number of time-like Polyakov lines they contain. Each such Polyakov line contributes a factor $\exp(N_t\mu)$ or $\exp(-N_t\mu)$ and thus we have

$$\det M = c_0 + \sum_{m=1}^{M} [\, c_m \exp(mN_t\mu) + c_m^* \exp(-mN_t\mu) \,] \qquad (3)$$

where $M := N_S{}^3 N_C$ in obvious notation. The coefficients contain contributions from space-like loops and lines, as well as from time-like loops which have as many links in the +t as in the -t direction. This explains the complex conjugate relationship of the coefficients of the $+\mu$ and $-\mu$ exponentials in eqn.(3). The last terms' coefficient is 1, because it consists of the determinant of the product of all the time-like Polyakov lines. Each of the terms in the expansion can be interpreted as representing a contribution from quarks (anti-quarks) winding around in the forward (backward) time direction m times.

Now recall that under Z_3 transformations of the t-like gauge links, we obtain a valid SU(3) configuration which leaves the path integral measure and the pure gauge action invariant but changes detM. We may thus write the partition function as follows

$$Z = \frac{1}{3}\int [DU] \, \exp(-S_g) \sum_{i=1}^{3} \det(M_i) \qquad (4)$$

where the 3 determinants differ by such a Z_3 trnsformation. Thus, the problematic denominator of eqn.(1) can now be obtained as

$$\int [DU] \, \exp(-S_g) \, |\det M_1| \, \frac{\sum_{i=1}^{3} \det M_i}{|\det M_1|} = \left\langle \frac{\sum_{i=1}^{3} \det M_i}{|\det M_1|} \right\rangle_W \qquad (5)$$

with the same MC weight as before. The important fact is that the sum of 3 determinants which appears in eqns.(4) and (5), when evaluated using eqn.(3), becomes:

$$\sum_{i=1}^{3} \det M_i = 3c_0 + \sum_{m=1}^{M} [\, c_{3m} \exp(3mN_t\mu) + c_{3m}^* \exp(-3mN_t\mu) \,] \qquad (6)$$

i.e only loops that wind around the lattice 3,6,9 etc times contribute. The cancelling terms of eqn.(3) are, however, "noise" which is present in eqn.(1) and will require enormous statistics to cancel and may be hidding our singal when evaluating eqn.(2). If, however, we evaluate $<\cos\phi>$ using eqn.(6), the noise is cancelled explicitly.

To assess how relevant these considerations are in practice, we examine the relative strength of each contribution from the positive exponential terms in the expansion (6). In Table I we show such contributions from equilibrium configurations at different μ values, for a very small 2^3X4 lattice. Note that for small μ the zeroth order term c_0 dominates. Thus, for this small lattice we were able to measure the condensate at μ=0.4 and found it to agree with its μ=0 value. As μ increases, the strength of the powers of $\exp(\mu)$ takes over and more terms come into play. These terms correspond to the lattice being gradually filled up by quarks that wind around 3,6,9 etc times in the standard picture of ref.[2], until at large enough μ, the whole lattice is completely filled up (i.e. the last term with $\exp(N_tM\mu)$ dominates). Unfortunately, we see from Table I that the intermediate values $\mu\sim0.6$ are characterised by cancelling dominant terms, which means that obtaining reliable results is again difficult. We encountered similar difficulties even at the smaller μ=0.4 value when dealing with a 4^4 lattice. It may thus be necessary to clear up the signal from further noise by using the same trick of Z_3 group rotations in all four directions. Then, instead of dealing with the sum of 3 determinants, our $<\cos\phi>$ is given by the sum of 3^4determinants. Early trial runs have shown that a few results obtained in this way at μ=0.4 on a 4^4 lattice are certainly very different from those obtained in Section 2 and possibly in accordance with the theoretical expectations of [2]. Work which is currently in progress along these lines will hopefully clarify the situation.

TABLE I - The real part of the terms $\{c_{3m}\ \exp(3mN_t\mu)\}$ of the determinant expansion (eqn.(6)), for typical configurations at different values of μ. The lattice size is 2^3x4.

m	μ=0.2	μ=0.4	μ=0.5	μ=0.65
0	5.25	1.28	0.08	0.75E-4
1	0.74E-1	0.81E-1	0.30	-0.40E-2
2	0.32E-2	0.37E-1	0.81	-0.14E-1
3	-0.48E-4	0.12E-1	-0.97	0.88
4	-0.11E-6	0.20E-3	-0.17	-0.86
5	-0.32E-9	0.11E-5	-0.27E-2	-0.67
6	-0.16E-12	0.55E-7	-0.49E-4	-0.20E-1
7	0.42E-15	0.34E-9	0.75E-6	0.59E-2
8	0.14E-19	0.25E-12	0.11E-7	0.53E-3

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