Laboratori Nazionali di Frascati

M.J. Lavelle, M. Schaden and A. Vladikas:

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PROPAGATORS AND RELAXATION FOR STOCHASTICALLY QUANTISED U(1) IN THE TEMPORAL GAUGE

M.J. Lavelle[*] and M. Schaden
Institut für Theoretische Physik, Universität Regensburg, 8400 Regensburg, W. Germany

A. Vladikas
INFN - Laboratori Nazionali di Frascati, CP 13, 00044 Frascati, Italy

ABSTRACT

We show that stochastic quantisation can be used to provide a simple derivation of the propagator for a maximal axially fixed gauge field. The relaxation and correlation properties of systems both unfixed and axially fixed are examined and compared. A comparison of the two methods of gauge fixing used in stochastic quantisation is made.

1. - INTRODUCTION

The maximal axial gauge is a method of gauge fixing which has given rise to some controversy concerning its Feynman rules ([1-5] and references therein). The correct longitudinal gauge field propagator in this gauge has been obtained recently by a variety of methods [1-4]. In the present paper, we provide a new derivation using Stochastic Quantisation (S.Q.) [6]. The main advantage of this approach is its conceptual simplicity; a carefully derived result in refs [1-5], now becomes a straightforward result of S.Q.

For simplicity we will confine ourselves to the Abelian case; this could be generalised to the non-Abelian case straightforwardly. In Section 2 we first review known results on fixing the axial gauge in SQ. We then present the stochastically quantised, completely fixed gauge theory, and derive its propagator, which for $\tau \to \infty$ equilibrises to the known [1-4] propagator.

In Section 3 we compare the relaxation properties of observables with and without gauge fixing by examining the spectrum of the Fokker-Planck Hamiltonian. It is shown that in the method due to Lim ([7] and below) not all modes relax equally quickly, a fact which might lead to difficulties in calculations using this approach [8]. The correlation and relaxation properties of observables are also shown to coincide.

2 - PROPAGATOR FOR AXIAL GAUGES

We first consider the unfixed Langevin equation in Euclidean momentum space for a free electromagnetic field

$$\frac{\partial A_\mu(k,\tau)}{\partial \tau} = -(k^2 \delta_{\mu\nu} - k_\mu k_\nu) A_\nu(k,\tau) + \eta_\mu(k,\tau) \tag{1}$$

where $\eta_\mu(k,\tau)$ is a Gaussian noise obeying

$$\langle \eta_\mu(k,\tau) \rangle = 0 \quad ; \quad \langle \eta_\mu(k,\tau) \eta_\nu(k',\tau') \rangle = 2 \delta_{\mu\nu} \delta(k+k') \delta(\tau-\tau') \tag{2}$$

The 2-point correlation function then is[6]

$$\langle A_\mu(k,\tau) A_\nu(k',\tau') \rangle = \delta^4(k+k') [\frac{1}{k^2} (\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}) \left(\exp(-k^2|\tau-\tau'|) - \exp(-k^2(\tau+\tau')) \right)$$

$$+ \frac{2k_\mu k_\nu}{k^2} \min(\tau,\tau')] \tag{3}$$

Lim [7] fixes the axial gauge in this theory by inserting a δ-functional in the generating functional, $Z[J]$. For the temporal gauge ($A_0=0$) this leads to the Langevin equation [7]

$$\frac{\partial A_i(k,\tau)}{\partial \tau} = -(k^2 \delta_{ij} - k_i k_j) A_j(k,\tau) + \eta_i(k,\tau) \tag{4}$$

($i=1,2,3$) where the η_i have the same variances as in eqn. (2). With the boundary conditions $A_\mu(k,\tau=0) = 0$, the two-point function is [7]

$$\langle A_i(k,\tau) \, A_j(k',\tau')\rangle = \delta^4(k+k') \left[\frac{1}{k^2} \left(\delta_{ij} - \frac{k_i k_j}{k^2} \right)\left(\exp(-k^2|\tau-\tau'|) - \exp(-k^2(\tau+\tau')) \right) \right.$$

$$\left. + \frac{k_i k_j}{k^2 k_0^2} \left(\exp(-k_0^2|\tau-\tau'|) - \exp(-k_0^2(\tau+\tau')) \right) \right] \tag{5}$$

where $k:=(k_0,\mathbf{k})$. For $\tau = \tau' \to +\infty$, one obtains [7] the usual Euclidean space propagator, $D_{ij}(k)$ in the temporal gauge

$$D_{ij}(k) = \frac{1}{k^2} \left(\delta_{ij} + \frac{k_i k_j}{k_0^2} \right) \tag{6}$$

which propagator exhibits a pole at $k_0^2 = 0$. However, as noted by Landshoff [8], the correlation function (eqn. (5)) is finite at $k_0=0$ for finite τ.

It is well known that the singularity at $k_0=0$ in the longitudinal part of the propagator is due to the existence of a residual gauge freedom in the axial gauge. Therefore the divergence can be removed by eliminating the remaining gauge freedom. The maximal axial gauge is obtained by fixing one component of the field to zero on a plane at $t=t_0$. In order to fix this gauge and obtain the finite propagator of Caracciolo et. al. [1] in the framework of SQ, Lim maintained that he had to abandon the Parisi-Wu SQ scheme for the Nelson SQ scheme [7]. We will now show how to implement this gauge and obtain the propagator[1] in the framework of Parisi-Wu SQ. To this end we use the method due to Zwanziger [9].

The Zwanziger approach consists of making local, τ-dependent gauge transformations to fix the gauge [9,10]. In this way the temporal gauge propagator has been obtained [11]. We briefly sketch the derivation. A local transformation of the A_μ field produces

$$A_\mu \to A_\mu - k_\mu \, \phi(k,\tau) \tag{7}$$

This gives a Langevin equation:

$$\frac{\partial A_\mu(k,\tau)}{\partial \tau} - k_\mu \frac{\partial \phi(k,\tau)}{\partial \tau} = -k^2 \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) A_\nu(k,\tau) + \eta_\mu(k,\tau) \tag{8}$$

To implement the temporal gauge, choose ϕ such that $\partial A_0(k,\tau)/\partial \tau = 0$ which leads to

$$\frac{\partial A_i}{\partial \tau} = -k^2 A_i + \tilde{\eta}_i \qquad (9)$$

with modified noise

$$\tilde{\eta}_i = \eta_i - \frac{k_i}{k_0} \eta_0 \qquad (10)$$

with expectation values

$$<\tilde{\eta}_i> = 0 \quad ; \quad <\eta_i(k,\tau)\, \eta_j(k',\tau')> = 2\, (\, \delta_{ij} + \frac{k_i k_j}{k_0^2}) \, \delta^4(k+k')\, \delta(\tau-\tau') \qquad (11)$$

The boundary condition, $A_\mu(k,\tau=0) = 0$, guarantees the gauge condition. This Langevin equation is different to that of Lim at finite τ and leads to the two-point function:

$$< A_i(k,\tau)\, A_j(k',\tau') > = \frac{1}{k^2}(\delta_{ij} + \frac{k_i k_j}{k_0^2})\, \left[\; exp(\, -k^2|\tau-\tau'|) + exp(-k^2(\tau+\tau')) \;\right] \qquad (12)$$

In distinction to the naively fixed case of eqn. (5), this correlation function is divergent at $k_0=0$ for all values of τ,τ'. However, it too leads to the standard propagator of eqn.(6).

To maximally fix the gauge we use the remaining gauge freedom to demand the vanishing of longitudinal components on an arbitrary time slice $t=t_0$, i.e.

$$k_i\, A_i(k_i,t_0) = 0 \qquad (13)$$

which in (k,k_0)-space is equivalent to

$$k_i\!\int dk_0\, A_i(k,k_0)\, exp(ik_0 t_0) = 0 \qquad (14)$$

To impose this we make the gauge transformation

$$A_i(k,k_0) \rightarrow A_i(k,k_0) - \delta(k_0)\, k_i\, \phi(k,k_0) \qquad (15)$$

in the Langevin eqn.(9) and apply the integral operator $\int dk_0\, k_i\, exp(ik_0 t_0)$. If we demand eqn.(13) we obtain

$$k^2 \; \frac{\partial \phi(k,k_0=0,\tau)}{\partial \tau} = \int dk_0 \; (k_0^2 k_i \, A_i \; - \; k_i \, \tilde{\eta}_i \,) \exp(ik_0 t_0) \; - \; k^2 k^2 \; \phi(k,k_0=0,\tau)$$

$$(16)$$

with solution

$$\phi(k,k_0=0,\tau) = \frac{1}{k^2} \int_0^\tau d\xi \exp\left(-(\tau-\xi)k^2\right) \int dk'_0 \; (k'^2_0 k_i \, A_i \; - \; k_i \, \tilde{\eta}_i \,) \exp(ik'_0 t_0)$$

$$(17)$$

Hence we have the new Langevin equation

$$\frac{\partial A_i(k,k_0,\tau)}{\partial \tau} = - k^2 A_i(k,k_0,\tau) + \tilde{\eta}_i \, (k,k_0,\tau)$$

$$+ \; \delta(k_0) \; \frac{k_i}{k^2} \int dk'_0 \; \exp(ik'_0 t_0) \left(k_j \, A_j \, k'^2_0 - k_j \, \tilde{\eta}_j \, \right)$$

$$(18)$$

Note again that only longitudinal components of A_i are affected by this further gauge fixing. For $k_0 \neq 0$, we obtain the propagator of eqn. (6) once again; but for $k_0=0$ it is different. However, in the third term of eqn. (18), due to the $(k_0')^2$ factor, we may substitute the $k_0 \neq 0$ solution

$$A_i^L(k,\tau) = \int_0^\tau d\xi \; \exp\left(-k^2 (\tau-\xi)\right) \; \tilde{\eta}_i^L$$

$$(19)$$

back into eqn. (18). The L superscript denotes that we consider the longitudinal component which satisfies a new Langevin equation

$$\frac{\partial A_i^L(k,\tau)}{\partial \tau} = -k^2 A_i^L(k,\tau) + H_i^L (k,\tau)$$

$$(20)$$

where

$$H_i^L = \overset{\circ}{\eta}_i^L - \delta(k_0) \int dk'_0 \; \overset{\circ}{\eta}_i^L(k,k'_0,\tau) \exp(ik'_0 t_0)$$

$$(21.a)$$

$$\overset{o}{\eta}{}^{L}_{i} = \tilde{\eta}^{L}_{i} - \int_{0}^{\tau} d\xi \; k_0^2 \; \exp\left(-k^2(\tau-\xi)\right) \tilde{\eta}^{L}_{i} \qquad (21b)$$

The gauge independent transverse terms are as before (eqn. (9)). The two point correlation function is now given by

$$\langle A^{L}_{i}(k,\tau) \, A^{L}_{j}(k',\tau')\rangle = \int_{0}^{\tau} d\xi \int_{0}^{\tau'} d\xi' \; \exp\left(-k^2(\tau-\xi)\right) \exp\left(-k'^2(\tau'-\xi')\right) \langle H^{L}_{i}(k,\xi) \, H^{L}_{j}(k',\xi')\rangle$$

$$(22)$$

The expectation value $\langle H_i{}^{L} H_j{}^{L}\rangle$ can be found from eqns. (11) and (21). This and some intermediate expressions may be found in the Appendix. Substituting eqn. (A.3) into eqn.(22) we obtain after some algebra

$$\langle A^{L}_{i} A^{L}_{j}\rangle = \frac{k_i k_j}{k^2} \, \delta^3(k+k') \left[\; \frac{\delta(k_0+k'_0)}{k_0^2} \left(\exp\left(-k^2|\tau-\tau'|\right) - \exp\left(-k^2(\tau+\tau')\right) \right) \right.$$

$$- \frac{\delta(k_0)}{k'^2_0} \exp(-ik'_0 t_0) \left(\exp\left(-k'^2|\tau-\tau'|\right) - \exp\left(-k'^2(\tau+\tau')\right) \right)$$

$$- \frac{\delta(k'_0)}{k_0^2} \exp(-ik_0 t_0) \left(\exp\left(-k^2|\tau-\tau'|\right) - \exp\left(-k^2(\tau+\tau')\right) \right)$$

$$\left. + \delta(k_0) \, \delta(k'_0) \int dk''_0 \; \frac{1}{k''^2_0} \left(\exp\left(-k''^2|\tau-\tau'|\right) - \exp\left(-k''^2(\tau+\tau')\right) \right) \; \right] \qquad (23)$$

where $k''=(k''_0,k)$. The divergent last term of the above is required to ensure the boundary condition at t (or t') $=t_0$.

For $\tau=\tau' \to \infty$ this gives

$$\langle A_i^L A_j^L \rangle = \frac{k_i k_j}{k^2} \, \delta^3(k+k') \left[\frac{\delta(k_0+k'_0)}{k_0^2} - \frac{\delta(k'_0)}{k_0^2} \exp(-ik_0 t_0) \right.$$

$$\left. - \frac{\delta(k_0)}{k_0'^2} \exp(-ik'_0 t_0) + \delta(k_0)\, \delta(k'_0) \int \frac{dk''_0}{k''^2_0} \right] \qquad (24)$$

This is our final expression for the correlation function in momentum space. It must be noted that its last term is singular. This is why only the propagator in coordinate space has been obtained so far in the literature [1,2,3,4] and it has been claimed that it has no Fourier transform [3]. A more transparent expression is obtained by taking the Fourier transform of eqn. (24). In doing this , the various singularities cancel and we obtain (see Appendix)

$$\langle A_i^L(k,\infty) A_j^L(k',\infty) \rangle = (1/2) \frac{k_i k_j}{k^2} \, \delta^3(k+k') \left(|t'-t_0| + |t-t_0| - |t-t'| \right) \qquad (25)$$

This is the result of Girotti and Rothe [2] and, as noticed by them, reduces to the result of Caracciolo et.al. [1] for $t_0 \to \pm\infty$. Although a fair amount of algebra was required in order to obtain the final result, it is conceptually a remarkably straightforward derivation which follows directly from the basics of SQ.

3 - RELAXATIONS FOR AXIAL GAUGES

We saw that the propagator, whether fixed by the method of Lim [7] or that of Zwanziger [9], will relax to the same equilibrium expression, although their routes to equilibrium can differ considerably. To see this recall that for any general positive matrices $M(k,k')$, the class of Langevin equations

$$\frac{\partial A_i(k,\tau)}{\partial \tau} = -\int d^4 k' \; M_{ij}(k,k') \; \frac{\delta S}{\delta A_j(k',\tau)} + \eta_i(k,\tau) \qquad (26)$$

where

$$\langle \eta_i(k,\tau) \rangle = 0 \quad ; \quad \langle \eta_i(k,\tau)\, \eta_j(k',\tau') \rangle = 2\, M_{ij}(k,k')\, \delta(\tau-\tau') \qquad (27)$$

leads to the same equilibrium physics [6]. From eqns. (2), (11) and (27) we see that the two

Langevin equations (4) and (9) only differ by a kernel factor. For eqn. (4), M is

$$M_{ij}(k,k') = \delta_{ij}\, \delta^4(k+k') \qquad (28.a.)$$

and for eqn. (9)

$$M_{ij}(k,k') = (\delta_{ij} + \frac{k_i k_j}{k_0^2})\, \delta^4(k+k') \qquad (28.b)$$

Evidently both are positive definite, therefore the same equilibrium physics is reached, although with different relaxation rates. The relaxation rates of the propagators can be read off from eqns. (5) and (12). We immediately see that the propagator's relaxation time is $1/k^2$ for the physical modes of the unfixed case as well as for the Zwanziger axial gauge fixing. For Lim's gauge fixing, the transverse modes also relax as $1/k^2$ but the longitudinal modes relax more slowly as $1/k_0^2$.

These relaxation properties characterise not only two-point functions but all field-dependent expectation values. To see this, one must calculate the spectrum of the Fokker-Planck Hamiltonian, the eigenvalues of which determine the relaxation rates of all expectation values [6]. These relaxation rates are of importance to lattice simulations of field theories; they have been considered previously in the framework of the Langevin equation [12] and in molecular dynamics [13]. It is straightforward but instructive to show how Langevin simulations also give rise to a simple harmonic oscillator description when tackled from the Fokker-Plank angle. Splitting up eqn.(1) into a longitudinal and a transverse part gives:

$$\frac{\partial A_\mu^L(k,\tau)}{\partial \tau} = \eta_\mu^L(k,\tau) \quad ; \quad <\eta_\mu^L(k,\tau)\, \eta_\nu^L(k',\tau')> = 2\,\frac{k_\mu k_\nu}{k^2}\, \delta^4(k+k')\, \delta(\tau-\tau') \qquad (29.a)$$

$$\frac{\partial A_\mu^T(k,\tau)}{\partial \tau} = -k^2 A_\mu^T(k,\tau) + \eta_\mu^T(k,\tau) \quad ; \quad <\eta_\mu^T(k,\tau)\, \eta_\nu^T(k',\tau')> = 2(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2})\delta^4(k+k')\delta(\tau-\tau')$$

$$(29.b)$$

For the transverse modes, the Fokker-Planck Hamiltonian is then:

$$H_{FP}^{(T)} = \int d^4k\ a_\mu^{(T)\dagger}(k)\ a_\mu^{(T)}(k) \qquad (30)$$

where

$$a_\mu^{(T)}(k) = -\frac{\delta}{\delta A_\mu^T(k)} + \frac{1}{2}\, k^2 A_\mu^{T}(k) \qquad (31)$$

is a transverse annihilation operator and $a_\mu^{(T)\dagger}$ is its Hermitean conjugate. They obey the commutation relations

$$\left[\; a_\mu^{(T)}(k_1)\;,\; a_\nu^{(T)\dagger}(k_2)\;\right] \;=\; k^2\,\delta_{\mu\nu}\,\delta^4(k_1-k_2) \tag{32}$$

They are raising and lowering operators, since for an eigenstate $|\psi\rangle$ of $H_{FP}^{(T)}$, with eigenvalue E

$$H_{FP}^{(T)}\,a_\mu^{(T)\dagger}(k)\;|\psi\rangle \;=\; (E + k^2/2)\, a_\mu^{(T)\dagger}(k)\;|\psi\rangle \tag{33.a}$$

$$H_{FP}^{(T)}\,a_\mu^{(T)}(k)\;|\psi\rangle \;=\; (E - k^2/2)\, a_\mu^{(T)}(k)\;|\psi\rangle \tag{33.b}$$

The problem is therefore identical to that of a free field theory solved as a simple harmonic oscillator. The spectrum of the Hamiltonian (30) is therefore of the form

$$E^{(T)}(k_{(1)},k_{(2)},\dots,k_{(n)}) \;=\; \sum_{i=1}^{n} \frac{k_{(i)}^2}{2}\, m_i^{(T)} \tag{34}$$

where we assume discretisation of the momenta for simplicity of presentation, and we typically have $m_i^{(T)}$ excitations of the k_i mode. It is clear from eqn.(29.a) that the longitudinal components $A_\mu^L(k)$ evolve as a pure noise with infinite relaxation time. In terms of the longitudinal Fokker-Planck Hamiltonian, this means that its spectrum consists of an infinitely degenerate, k-independent ground state.

In the case of Lim's axial gauge fixing, eqn. (4) splits up as follows:

$$\frac{\partial A_i^T}{\partial\tau} = -k^2\,A_i^T + \eta_i^T \quad;\quad \langle\eta_i^T(k,\tau)\,\eta_j^T(k',\tau')\rangle = 2\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\delta^4(k+k')\,\delta(\tau-\tau') \tag{35.a}$$

$$\frac{\partial A_i^L}{\partial\tau} = -k_0^2\,A_i^L + \eta_i^L \quad;\quad \langle\eta_i^L(k,\tau)\,\eta_j^L(k',\tau')\rangle = 2\frac{k_i k_j}{k^2}\,\delta^4(k+k')\,\delta(\tau-\tau') \tag{35.b}$$

where now the longitudinal and transverse components are defined in k 3-space. In an analogous fashion to the previous unfixed case, the spectra of the Fokker-Planck Hamiltonians are

$$E^{(L)}(k_{(1)}, k_{(2)}, \ldots, k_{(n)}) = \sum_{i=1}^{n} \frac{k^2_{o\,(i)}}{2} m_i^{(L)} \tag{36.a}$$

$$E^{(T)}(k_{(1)}, k_{(2)}, \ldots, k_{(n)}) = \sum_{i=1}^{n} \frac{k^2_{(i)}}{2} m_i^{(T)} \tag{36.b}$$

The autocorrelation function of two fields is defined by

$$\rho(k,\tau,\tau') := \frac{<A_\mu(k,\tau)\, A_\mu(k,\tau')> - <A_\mu(k,\tau)>^2}{<A_\mu(k,\tau)^2> - <A_\mu(k,\tau)>^2} \tag{37}$$

Using eqn.(3) we find for the unfixed case

$$\rho_T(k,\tau,\tau') = \exp(-k^2|\tau-\tau'|) \quad ; \quad \rho_L(k,\tau,\tau') = 1 \tag{38}$$

and using eqn.(5) we find for the Lim axial gauge fixing

$$\rho_T(k,\tau,\tau') = \exp(-k^2|\tau-\tau'|) \quad ; \quad \rho_L(k,\tau,\tau') = \exp(-k_0^2|\tau-\tau'|) \tag{39}$$

Note that there are no correlations between longitudinal and transverse components of A_μ in either case.

For the Zwanziger fixed Langevin equation (9) we can similarly calculate the Fokker-Planck Hamiltonian. One finds that it is the same as the Hamiltonian of the transverse modes of the unfixed case. Therefore the spectrum is identical to that of physical modes of the unfixed theory. This is because the Zwanziger gauge fixed field-configuration generated by eqn.(9) at any τ can be obtained by gauge rotations of the unfixed gauge field-configuration generated by eqn.(1) at the same τ. (This statement is not true for gauge field configurations generated by Lim's method of eqn. (4)). We have not obtained an explicit Fokker-Planck equation for the maximal fixed Langevin equation (20).

These results lead to two conclusions: (1) When the axial gauge is fixed by the method of Lim, the relaxation slows down. We can clearly see by comparing eqns.(34) and (36) that on fixing the gauge à la Lim, the longitudinal mode in k 3-space has its eigenvalues decreased. The analogous result has already been shown on the lattice[14,15]. The Zwanziger fixed system in contrast is not slowed down and can be compared, on the lattice, with the gauge fixing method of the Cornell group[12,16]. (2) Correlations between field operators at different times t have a fall off

which is directly proportional to their relaxation rate. The same qualitative conclusions have been numerically observed on the lattice [17]. To the extent that these conclusions have some general validity and are not confined to non-interacting theories, they offer a good understanding of relaxation and correlation effects in lattice field theory simulations.

APPENDIX

From eqn.(11) we can derive the averages for the longitudinal and transverse components of the noise

$$<\tilde{\eta}_i^L(k,\tau)\ \tilde{\eta}_j^L(k',\tau')> \ = 2\frac{k_i k_j}{k^2}\frac{k^2}{k_0^2}\delta^4(k+k')\ \delta(\tau-\tau') \tag{A.1.a}$$

$$<\tilde{\eta}_i^L(k,\tau)\ \tilde{\eta}_j^T(k',\tau')> = 0 \tag{A.1.b}$$

Note that longitudinal (gauge) and transverse (physical) modes do not mix. The averages between transverse components are as in eqn.(2). These properties carry over to the rest of the noises ($\overset{o}{\eta}{}^L$ and H^L) defined in eqn.(21). We therefore concentrate on averages of the longitudinal components only. From eqns.(A.1) and (21) we derive:

$$<\overset{o}{\eta}{}_i^L(k,\tau)\ \overset{o}{\eta}{}_j^L(k',\tau')> \ = 2\frac{k_i k_j}{k^2}\frac{k^2}{k_0^2}\ \delta^4(k+k')\left[\ \delta(\tau-\tau') - k_0^2\ \exp\left(-k^2|\tau-\tau'|\right)\right.$$
$$\left. + \frac{k_0^4}{2k^2}\ \{\ \exp\left(-k^2|\tau-\tau'|\right) - \exp\left(-k^2(\tau+\tau')\right)\}\ \right] \tag{A.2}$$

from which we find

$$\langle H_i^L(k,\tau)\, H_j^L(k',\tau')\rangle \;=\; 2\,\frac{k_i k_j}{k^2}\,\delta^3(k+k')\,\Bigg[$$

$$\delta(k_0+k'_0)\,\frac{k^2}{k_0^2}\Big\{\delta(\tau-\tau')-k_0^2\exp\!\big(-k^2|\tau-\tau'|\big)+\frac{k_0^4}{2k^2}\big[\exp\!\big(-k^2|\tau-\tau'|\big)-\exp\!\big(-k^2(\tau+\tau')\big)\big]\Big\}$$

$$-\,\delta(k_0)\,\exp(-ik'_0 t_0)\,\frac{k'^2}{k'^2_0}\Big\{\delta(\tau-\tau')-k'^2_0\exp\!\big(-k'^2|\tau-\tau'|\big)$$

$$+\,\frac{k'^4_0}{2k'^2}\big[\exp\!\big(-k'^2|\tau-\tau'|\big)-\exp\!\big(-k'^2(\tau+\tau')\big)\big]\Big\}$$

$$-\,\delta(k'_0)\,\exp(-ik_0 t_0)\,\frac{k^2}{k^2_0}\Big\{\delta(\tau-\tau')-k_0^2\exp\!\big(-k^2|\tau-\tau'|\big)$$

$$+\,\frac{k_0^4}{2k^2}\big[\exp\!\big(-k^2|\tau-\tau'|\big)-\exp\!\big(-k^2(\tau+\tau')\big)\big]\Big\}$$

$$+\,\delta(k_0)\,\delta(k'_0)\int dk''_0\ \frac{k''^{\,2}}{k''^{\,2}_0}\Big\{\delta(\tau-\tau')-k_0''^{\,2}\exp\!\big(-k''^{2}|\tau-\tau'|\big)$$

$$+\,\frac{k''^{\,4}_0}{2k''^{\,2}}\big[\exp\!\big(-k''^{2}|\tau-\tau'|\big)-\exp\!\big(-k''^{2}(\tau+\tau')\big)\big]\Big\}\ \Bigg] \qquad (A.3)$$

with $k''=(k_0'',k)$. Substituting into eqn.(22) and carrying out the integrations yields the final expression for the longitudinal propagator in momentum space (eqn.(24)).

In order to obtain the finite t-space propagator, of eqn. (25), we only need to Fourier transform the k_0 propagator of eqn.(24). This is straightforward, except that in order to be able to Fourier transform expressions like $1/(k_0^2)$ of eqn. (24), we need to regularise them first as $1/(k_0^2+\varepsilon^2)$ and take the limit $\varepsilon \to 0$ *after* the Fourier integrations. This sequence of regularising and integrating is in some sense analogous to the regularisation proceedure followed in [3]. The result is that the divergences of eqn.(24) cancel and the finite propagator of [1] is obtained (eqn.(25)).

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