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A DIFFRACTION RADIATION MODEL FOR ENERGY LOSSES

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ABSTRACT

The electromagnetic interaction of a bunch of particles with the surrounding equipment is cause of a radiation process leading to energy losses. The radiation may substantially reduce the energy gain of an accelerating structure therefore more power has to be supplied to the beam. Radiation laws derived by experimental observations and by some theoretical models predict an unrealistic infinite energy loss for infinitely short bunches. Here we present an analytical approach developed for an ideal simple structure which allows for deriving a radiation law for any bunch length and particle energy.

1. INTRODUCTION

A bunch of particles moving within an accelerating structure (e.g. a storage ring) generates induced currents in the surrounding walls which in turn produce "secondary" electromagnetic fields in addition to the "primary" fields carried by the bunch itself.

These fields may act back to the bunch giving rise to instability phenomena and lead to energy losses. The e.m. interaction between beam and surrounding walls is successfully described in terms of the Coupling Impedance which is strictly related to the parasitic losses and which sets the current thresholds for the beam instabilities.

The energy lost by a bunch of particles passing through an accelerating structure can be calculated as [1]

$$U = 2 \int_0^{\infty} Z_r(\omega) |I(\omega)|^2 d\omega \quad (1)$$

where $Z_r(\omega)$ is the real part of the longitudinal impedance and $I(\omega)$ is the bunch spectrum. It is worth noting that the integrand term has the dimension of a power spectrum density (Joule sec).

Putting in eq. 1) the impedance of a storage ring we obtain the energy per turn lost in all the components of the machine.

The energy lost in a given component is usually determined by the specific loss factor k_l defined by:

$$k_l = \frac{U}{Q^2} \quad (2)$$

where Q is the circulating charge. In case of gaussian bunches the loss factor and the real part of the impedance are related by :

$$k_l = \frac{1}{\pi} \int_0^{\infty} Z_r(\omega) e^{-(\omega\sigma/\beta c)^2} d\omega \quad (3)$$

where σ is the r.m.s. bunch length and βc is the particle velocity. It is apparent from the above relation that the shorter is the bunch the wider is the frequency range where the impedance should be known.

The loss factor can be calculated for ideal structures by means of numerical codes which, when dealing with very short bunches, become too time consuming and impracticable.

In the case of a linear accelerating structure, using theoretical models, H. Henke [2] predicts for the loss factor a power dependence on the bunch length of the following type:

$$k_l(\sigma) = k_0 \sigma^{-1} \quad (4)$$

On the other hand the behaviour of the measured losses at SPEAR has suggested a law [3]:

$$k_l(\sigma) = k_0 \sigma^{-1.23} \quad (5)$$

This law seems to be confirmed by measurements at PETRA and PEP.

In case of infinitely short bunches, the power laws 4) and 5) would lead to unrealistic infinite losses, while the bunch cannot lose more than its own energy.

In this paper we present the analytical results of a study done for an ideal geometry where the the loss factor can be derived for any bunch length and particle energy.

2. DIFFRACTION RADIATION MODEL

Our model consists of the simple discontinuity shown in Fig.1.

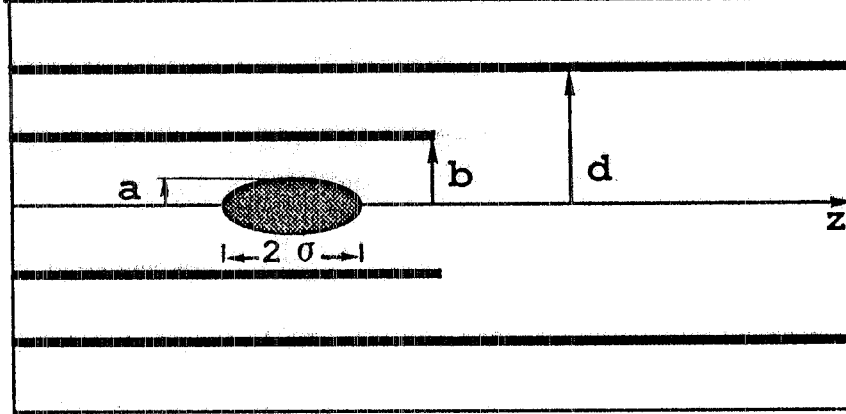


Fig.1 The relevant geometry (cylindrical symmetry.)

We consider a bunch of particles travelling at constant velocity βc on the axis of the structure and we are interested on the fields excited within the structure when the bunch crosses the discontinuity edge.

For this problem a fully analytical approach has been developed [4] making use of the diffraction radiation theory applied to discontinuous waveguides [5].

We expect that, as long as the bunch is far from the edge, the field configuration is practically that of a bunch moving on the axis of an infinite waveguide; then the radiation would mainly occur when the fields traveling with the bunch lights the edge and suddenly new boundary conditions have to be restored.

Let the bunch length be 2σ , then the radiation process lasts a time

$$\Delta T \sim \frac{2}{\beta c} \left(\sigma + \frac{b}{\gamma} \right) \quad (6)$$

during which radiated fields are excited within the structure. Accordingly, the diffracted field spectrum has a bandwidth :

$$\Delta f \sim \frac{1}{\Delta T} \sim \frac{\beta c}{2(\sigma + b/\gamma)} \quad (7)$$

We may easily recognize two regimes : i) the bunch length is longer than a "critical length" b/γ , then the spectrum width is independent of the particle energy; ii) bunches are much shorter than b/γ , they behave like point charges and the radiation spectrum extends over a frequency range which is proportional to the relativistic factor γ .

3. THE LONGITUDINAL IMPEDANCE

The analytical expression of the longitudinal Green function of our geometry has been worked out by reducing the e.m. problem to a pair of integral equations of Wiener-Hopf type, and solving them by means of standard techniques [4].

The longitudinal impedance for such infinite structures is defined by [6]

$$Z(\omega) = -\frac{2\pi}{Q} \int_{-\infty}^{+\infty} E_z(\omega) e^{-ikz/\beta} d\omega \quad (8)$$

where $E_z(\omega)$ is the frequency spectrum of the electric field on the axis. It is convenient, in order to single-out the impedance due to the discontinuity, to write the impedance as the sum of two terms

$$Z(\omega) = Z_1(\omega) + Z_2(\omega) \quad (9)$$

$Z_1(\omega)$ is the impedance of a charge moving on the axis of an infinite perfectly conducting waveguide, which in the limits $\chi b \ll 1$ and $\chi d \ll 1$, with $\chi = \kappa/(\beta\gamma)$, $\kappa = \omega/c$, is given by:

$$Z_1(\omega) = \frac{i\kappa Z_0}{(\beta\gamma)^2} \begin{cases} [1 + 2\ln(b/a)] & z < 0 \\ [1 + 2\ln(d/a)] & z > 0 \end{cases} \quad (10)$$

where Z_0 is the free space impedance (377 ohms). This term is purely reactive and vanishes as $1/\gamma^2$.

The "discontinuity" term is :

$$Z_2(\omega) = \frac{Z_0 k}{2\pi(\beta\gamma)^2} \left[\frac{K_0(\chi b)}{I_0(\chi b)} - \frac{K_0(\chi d)}{I_0(\chi d)} \right] \left[\frac{(1+\beta)(\beta\gamma)^2}{\beta\kappa} - \gamma b \frac{I_1(\chi b)}{I_0(\chi b)} - \sum \right] \quad (11)$$

where K_0, I_0, I_1 are the modified Bessel functions and

$$\sum = \sum_{n=1}^{\infty} \left(\frac{1}{\alpha_n^b - k/\beta} + \frac{1}{\alpha_n^c - k/\beta} - \frac{1}{\alpha_n^d - \kappa/\beta} \right) \quad (12)$$

α_n^b, α_n^c , and α_n^d being the zeroes of the Bessel functions $J_0(\Omega b), J_0(\Omega d)$, and of the cross product $J_0(\Omega b)Y_0(\Omega d) - J_0(\Omega d)Y_0(\Omega b)$ respectively, with $\Omega = \sqrt{\kappa^2 - \alpha^2}$

In the limits $\chi b \ll 1$ and $\chi d \ll 1$, with $\beta \sim 1$, we get:

$$Z_2(\omega) \sim \frac{Z_0}{\pi} \ln\left(\frac{d}{b}\right) \left[1 + \frac{i\kappa}{2\gamma^2} \text{Im} \sum \right] \quad (13)$$

We can see that the imaginary part of Z_2 vanishes as $1/\gamma^2$ while its real part is energy-independent.

The impedance for the case of a charge moving to the opposite direction is also given by eq. 11) where one has to replace β with $-\beta$.

The behaviour of the real part of the impedance, for both cases of charge leaving and entering the inner semiinfinite waveguide, is shown in Fig.2. After a practically constant value the impedance eventually drops down exponentially at the frequency where $\kappa b \sim \gamma$.

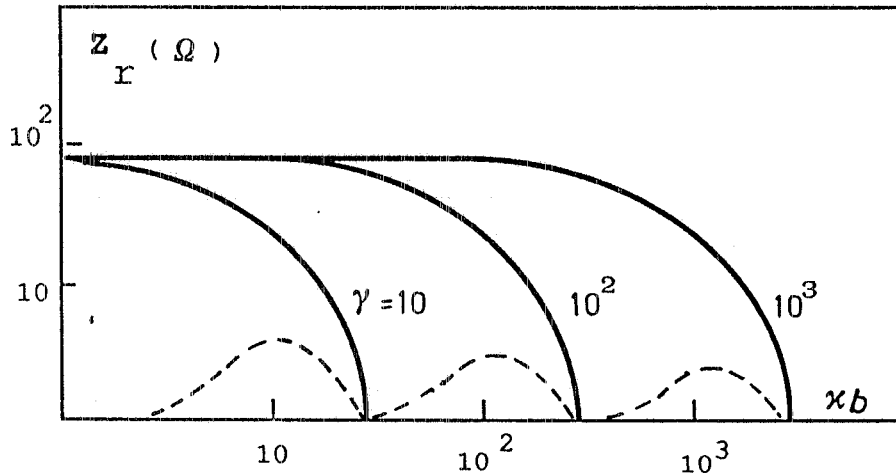


Fig.2 Real part of the impedance for entering charge (dashed line) and exiting charge (solid line).

These curves confirm the results obtained for a similar structure [7], (only in the frequency range $\kappa b \leq 100$), by means of the fields matching method.

A first consequence of these calculations is that the radiation process is not independent on the motion direction. This is not really surprising if we bear in mind that the radiation process mainly occurs because new boundary conditions have to be restored. A charge exiting from the inner tube has to fill more space with field, and part of its kinetic energy is transferred to e.m. energy.

In the other case, fields traveling with the charge already satisfy the new boundary conditions (this is exactly true when $\beta = 1$) so that fields are swept off while the charge holds its kinetic energy.

3. LOSS FACTOR $k_l(\sigma)$

An approximate analytical evaluation of the loss factor can be obtained by plugging eq.13) into the impedance definition 3) and limiting the integration range to the frequency $\bar{\omega} = \beta\gamma c/b$, beyond which $Z_r(\omega)$ vanishes exponentially. We obtain the following law (plotted in Fig.3) :

$$k_l(\sigma) \sim \frac{\beta c Z_0}{2\sigma\pi^{3/2}} \ln\left(\frac{d}{b}\right) \Phi\left(\frac{\gamma\sigma}{b}\right) \quad (14)$$

where $\Phi(x)$ is the Fresnel integral [8].

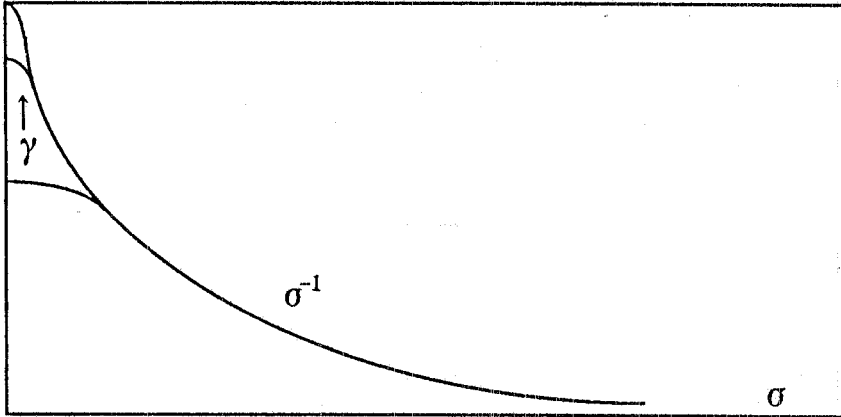


Fig.3. The loss factor k versus the bunch length.

For the realistic case where $\gamma\sigma/b \gg 1$, we have $\Phi \sim 1$ and the loss factor becomes:

$$k_l(\sigma) \sim \frac{\beta c Z_0}{2\sigma\pi^{3/2}} \ln\left(\frac{d}{b}\right) \quad (15)$$

Expressing $k_l(\sigma)$ in the common form $k_0 \cdot \sigma^{-1}$ with σ measured in centimeters, for typical values of $d/b \sim 1.5$ we obtain $k_0 \sim 1$ V/pC. In the complementary limit $\sigma\gamma/b \ll 1$, $\Phi(x) \sim 2x/\sqrt{\pi}$, then we get:

$$k_l(\sigma) \sim \frac{2\beta c Z_0 \gamma}{\pi^2 b} \ln\left(\frac{d}{b}\right) \quad (16)$$

which is the limit value for the loss factor in case of extremely short bunches; in this regime the loss factor becomes proportional to the particle energy.

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