Laboratori Nazionali di Frascati

To be submitted to Phys. Letters B

LNF-87/86(PT)
17 Settembre 1987

V.Azcoiti, A.Cruz, G.Di Carlo, A.F.Grillo and A.Tarancón:

STUDY OF CONFINEMENT IN THE ADJOINT SU(2)-HIGGS MODEL BY MEANS OF THE FREDENHAGEN AND MARCU CRITERION

Servizio Documentazione
dei Laboratori Nazionali di Frascati
P.O. Box, 13 - 00044 Frascati (Italy)

INFN - Laboratori Nazionali di Frascati
Servizio Documentazione

LNF-87/86(PT)
17 Settembre 1987

STUDY OF CONFINEMENT IN THE ADJOINT SU(2)-HIGGS MODEL BY MEANS OF THE FREDENHAGEN AND MARCU CRITERION

V.Azcoiti*, G.Di Carlo, A.F.Grillo
INFN, Laboratori Nazionali di Frascati, P.O.Box 13, 00044 Frascati, (Italy)

A. Cruz, A.Tarancón
Departamento de Física Teórica, Facultad de Ciencias. 50009-Zaragoza, (Spain)

ABSTRACT

We report on numerical results of a simulation of the Standard Adjoint SU(2)-Higgs model with clear evidence for charge confinement in the Higgs and confined phases. We report also on some numerical results on the accuracy of the Icosahedron in describing the full SU(2) gauge group.

The study of the non-perturbative dynamics of Gauge-Higgs models is a matter of great interest in field theory. Despite the extraordinary success of the standard SU(3)xSU(2)xU(1) model in describing the phenomenology of elementary particle interactions, open questions still remain, mainly related with non perturbative dynamics. The lattice formulation of Quantum Chromodynamics together with Monte Carlo simulations have provided us with a good understanding of some of these questions such as confinement of colour charge and chiral symmetry breaking. The success of these techniques when applied to Q.C.D. has motivated an increasing interest in applying them to the Gauge-Higgs model in order to understand the dymamical mass generation mechanism.

---

* Permanent Address: Departamento de Física Teórica, Facultad de Ciencias. 50009-Zaragoza, (Spain)

The understanding of Gauge-Higgs systems is at present much less complete than that of Q.C.D.; however, a few important and, we believe, surprising results have been obtained. The phase structure of the U(1) and SU(2)-Higgs models has been explored [1], and charge confinement in the Higgs and confined phases has been found for the U(1) [2] and SU(2) models with the scalar field in the fundamental representation of SU(2) [3].These results, if we assume that the inclusion of fermionic matter fields will not change qualitatively the confining property, point in the direction of compositeness of the gauge and Higgs bosons [4].

On the other hand, the understanding of the Gauge-Higgs system with the scalar field in the adjoint representation of the gauge group is crucial in the context of any grand-unified model and their implications about magnetic monopoles.

In this letter we report the numerical results of a simulation of the SU(2)-Higgs model with the scalar field in the adjoint representation of SU(2), with clear evidence for charge confinement in both Higgs and confined phases. We will present also results on a check about the accuracy in describing the SU(2)-Higgs model by its discrete version (icosahedron). The partition function of the lattice SU(2)-Higgs model with the scalar field in the adjoint representation of SU(2) is:

$$Z = \int \Pi_{\mu,\nu}\, dU_\mu(n)\, \Pi_n\, d\phi(n)\, \exp(-\beta\, \Sigma_{\mu\nu n,\mu<\nu} [1-(1/2)\mathrm{Tr}\, U_{\mu\nu}(n)]$$

$$-J\, \Sigma_{\mu,n} [1-\phi^+(n) U_\mu(n) \phi\, (n+\mu)]) \qquad (1)$$

where $\mu,\nu = 0,1,2,3$, μ is a unit vector in the positive μ direction, $n \in Z$ labels the site, $\phi(n)$ is a three component real vector, $U_\mu(n)$ an element of SU(2) in the adjoint representation, and $U_{\mu\nu}(n)$ is the plaquette variable defined by the links (n, μ) and (n, ν) and costructed whith the group elements in the fundamental representation of SU(2). To simplify the problem without throwing away any important physics, we freeze out the radial mode of the Higgs fields [1]. In these conditions and making use of the one to one correspondence

$$\phi^a(n) \iff \bar{\phi}(n) = i\, \sigma^a\, \phi^a(n)$$

we can write the action associated to (1) as:

$$S = \beta\, \Sigma_{\mu\nu n,\mu<\, n} [1-(1/2)\mathrm{Tr}\, U_{\mu\nu}(n)] +$$

$$+ \quad \kappa\, \Sigma_{\nu,\mu} [1-(1/2)\, \mathrm{Tr}\, (\bar{\phi}_n U_{n\mu}\, \bar{\phi}^+{}_{n+\mu} U^+{}_{n\mu})] \qquad (2)$$

where now all the gauge and matter field variables are taken in the fundamental representation of the gauge group.

In order to study confinement of static charges, the Wilson loop is a good order parameter since it is well known that from its asymptotic behaviour we can extract the coefficient of the linear dependence of the interquark potential. However, the Wilson loop criterion cannot be used to test the existence of free dynamical charges in gauge theories with dynamical matter fields, since the string can be broken if there is enough energy to create a charge-anticharge pair.

A short time ago Fredenhagen and Marcu [5] proposed a criterion which apparently can be used in these theories.The Fredenhagen and Marcu (F-M) order parameter σ is defined as:

$$\sigma = \lim_{\substack{L,T \to \infty \\ L/T \neq 0 \text{ fixed}}} \frac{| G(L,T) |^2}{R(L, 2T)} \qquad (3)$$

with

$$G(L,T) = \langle \phi^+(0,0) \prod_{n \in PG(L,T)} U^+_\mu(n) \, \phi(\Lambda,0) \rangle$$

$$R(L,T) = \langle \mathrm{Tr} \prod_{n \in PR(L,T)} U^+_\mu(n) \rangle \qquad (4)$$

where P_G (L,T) , P_R (L,T) are plotted in Fig.1 . The (F-M) criterion is based on the fact that if free charge can be isolated, σ should be zero because the operator in the numerator of (3) will have no projection on the vacuum state.

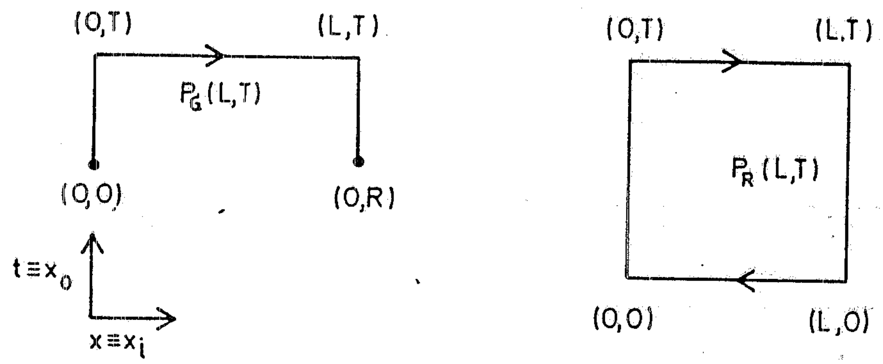


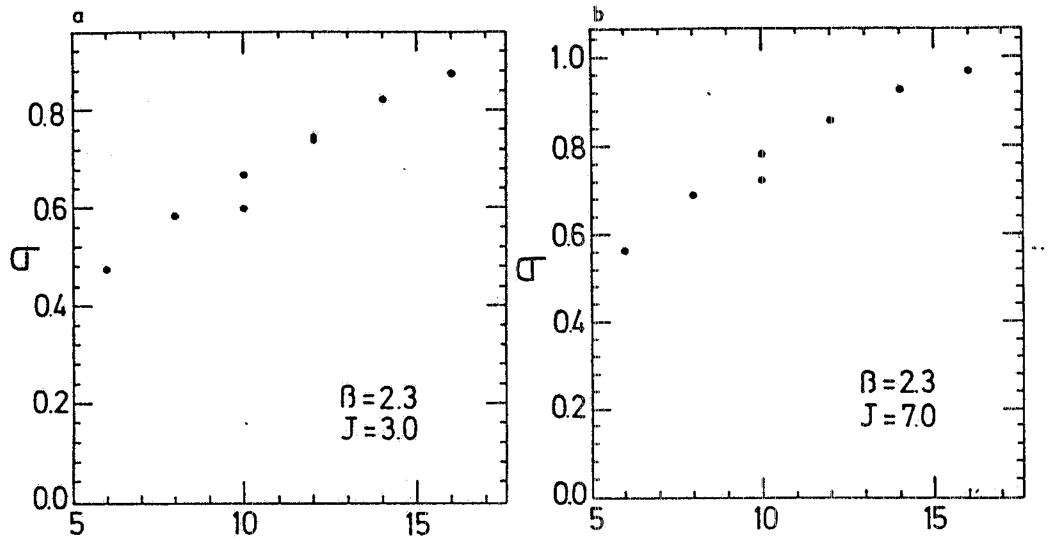
FIG. 1 - The paths P_G(L,T) and P_R(L,T) for the operators entering σ.

In the numerical calculation we have used an 8^4 lattice with periodic boundary conditions and, to avoid finite size effects, we consider rectangular paths with a perimeter ranging from 6 to 16. Using the discrete version of SU(2), the icosahedron, we have explored several points in the confining and Higgs phases of the model. To reduce the complexity of the numerical calculations, we work in the unitary gauge. When working with the discrete version of SU(2), the unitary gauge must be constructed with care. In fact, when we work with the icosahedron, the residual local gauge symmetry, which in the continuum version is always U(1), depends on the direction the scalar field points to. In order to get the maximum residual gauge symmetry Z(10), we have constructed the unitary gauge by fixing the scalar field ϕ in all the sites to

$$\bar{\phi} = \cos(\pi/5) + i\,(v \cdot \sigma)\ \sin(\pi/5)$$

$$v = (\,-0.52573,\ 0,\ 0.85065\,) \qquad (5)$$

In Fig. 2 we present the numerical results for σ as a function of the perimeter of the different paths and for several points in the Higgs and confining phases. All the points have been thermalized with 1000 M.C. iterations and the mean values have been taken over 2000-10000 M.C. iterations. These results give clear evidence of charge confinement in all the studied points. On the other hand, from the observation of these results, one can see that σ depends asymptotically only on the perimeter of the paths, in agreement with the assumption of an exponential decay with the perimeter for G(L,T) and R(L,T) in (3), as noted in [5].



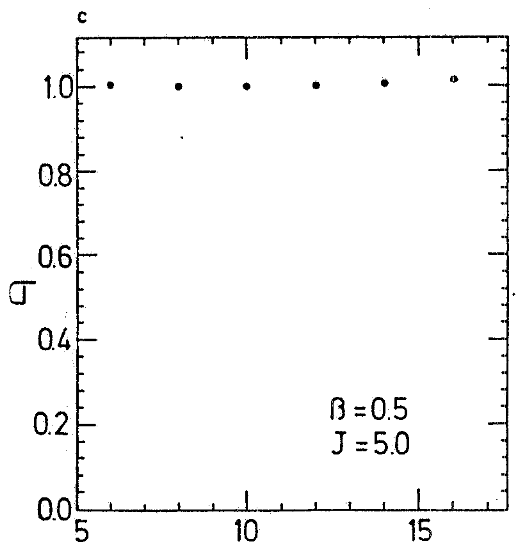


FIG. 2 - Numerical values for the ratio $|G(L,T)|^2 \cdot R^{-1}(L,2T)$ entering (3) versus P=2(L+T). Two symbols at the same perimeter correspond to a different combination of L and T. The size of the symbols is also the error size.

While the non existence of charge is expected in the confining phase, the presence of a massless vector boson in the Higgs phase [6] might be taken as an argument against our numerical result in that phase. Yet, what can be proven in this respect is that, in an abelian theory, whenever the photon acquires a mass, charge gets screened [7]. The fact that a theory without a mass gap does not confine escapes the proof.

On the other hand, the theorems by S. Weinberg and E. Witten [8], on the impossibility of the existence of composite massless vector particles, do not apply in our case, since our massless vector boson has no conserved charge associated.

It is interesting to compare these numerical results for σ in the Higgs phase with those one can obtain by using mean field techniques.

The use of the mean field in axial gauge is the most practical choice to describe the Higgs transition both qualitatively and quantitavely [9], and give a good value for σ in the U(1)-Higgs model.

With a treatment and notation similar to those of reference [2], we obtain, in the mean field approximation to zero order, in the axial gauge, for the action (2):

$$F = \beta d(d-1)V^4 + \beta d(d-1)V^2 + J(d-1)B^2V^2 + JB^2 - (d-1)\alpha V - \gamma B +$$

$$+ (d-1)\omega(\alpha) + W(\gamma) \qquad (6)$$

with

$$\omega(\alpha) = \int [dU] \exp[-\frac{i}{4} \mathrm{Tr}(\alpha^+ U + U^+ \alpha)] = \ln \frac{2 I_1(\alpha)}{\alpha} \qquad (7.1)$$

$$W(\alpha) = \int [d\phi] \exp[-i\bar{\gamma}\,\bar{\phi}] = \ln \frac{sh(\gamma)}{\gamma} \qquad (7.2)$$

where $I_1(\alpha)$ is the modified Bessel function of order 1.

In these conditions, the equations of motion, or saddle point equations, are

$$4\beta V^3 + 2\beta V + 2JB^2V = \alpha \qquad (8.1)$$

$$V = I_2(\alpha)/I_1(\alpha) \qquad (8.2)$$

$$6JV^2B + 2JB = \gamma \qquad (8.3)$$

$$B = \coth(\gamma) - (1/\gamma) \qquad (8.4)$$

In the Higgs phase, these equations present a non trivial solution with B and V different from zero.

For α and γ large we can solve analytically the equations, by considering the asymptotic behaviour of (8.2) and (8.4),

$$B = 1 - 1/\gamma \qquad V = 1 - 3/(2\alpha) \tag{9}$$

and using (8.1) and (8.3)

$$B = 1 - 1/(8J) \ , \qquad V = 1 - 3/(12\beta+4J) \tag{10}$$

in the mean field approximation to zero order we obtain for σ

$$\sigma = B^4$$

and using (10) we obtain for β=2.3 (in the Higgs phase)

$$\sigma(J=3.0) = 0.84$$

$$\sigma(J=7.0) = 0.93$$

in excellent agreement with the Monte-Carlo results of Figs. 2a and 2b.

The use of the icosahedron instead of the full SU(2) group has, as is well known, many advantages from the technical point of view in the numerical simulations. The phase diagram of the Icosahedron-Higgs model was studied accurately in ref. [10]. From this study it follows that the icosahedron has, in addition to the phase transition which separates the Higgs and the confining phases completely (in the adjoint representation), another phase transition due to the discrete character of the gauge group. Due to the prejudices aquired in the numerical simulations of the pure SU(2) gauge theory, one is tempted to think that the icosahedron should be a good approximation of SU(2) if we are far enough from the spurious transition. In order to check this prejudice we have made the numerical simulation of the full SU(2)-Higgs theory. In Table I we report the numerical results for the mean plaquette energy and the mean link energy against the corresponding values for the icosahedron gauge group for several values of β and J. The surprising evidence which follows from the results reported in Table 1 is that the icosahedron and the SU(2) results are different even in the confining phase: 34% for the mean link energy at β =0.5, J=5.0, 90% at β=0.5, J=10.0.The reason for this anomalous behavior of the icosahedron at large J lies in the fact that the equilibrium configurations for the Icosahedron-Higgs theory in the large J region approach the unbroken symmetry group Z(10) much faster than in the full SU(2)-Higgs theory [11]. On the other hand, these results are in contrast with the correspoding values that we have obtained in a simulation of the fundamental SU(2)-Higgs model. In this case no difference between the icosahedron and the full SU(2) gauge group have been found for $\beta < 4$ in agreement with the results of ref. [12].

TABLE I - Numerical results for the mean plaquette energy <1-¡Tr$U_{\mu\nu}$> and the mean link energy <1-¡Tr($f_n U_{n\mu} f^{+}_{n+\mu} U^{+}_{n\mu}$)> for the icosahedron and SU(2) Higgs model. All the points have been obtained in 4^4 lattices.

		SU(2)		ICOSAHEDRON	
β	J	<P>	<L>	<P>	<L>
0.5	1.0	0.874±0.001	0.685±0.001	0.875±0.002	0.686±0.002
0.5	5.0	0.823±0.001	0.196±0.001	0.803±0.002	0.129±0.001
0.5	10.0	0.796±0.001	0.098±0.001	0.758±0.002	0.010±0.001
2.0	2.25	0.317±0.001	0.271±0.001	0.300±0.001	0.227±0.001
2.3	3.0	0.258±0.002	0.205±0.002	0.216±0.001	0.118±0.001
3.0	0.6355	0.273±0.001	0.726±0.003	0.273±0.001	0.725±0.003
2.0	1.0	0.480±0.003	0.636±0.004	0.476±0.001	0.655±0.002

About the results on the Fredenhagen and Marcu order parameter, we believe that the use of the full SU(2) group will change, especially in the Higgs phase, the numerical results, but not the qualitative property of confinement. Indeed, the effect of the use of the discrete group can be thought of as a renormalization in the J coupling costant, as suggested by the results reported in Table I .

This work has been carried out in the context of an INFN-CAICYT collaboration. We thank both institutions for their finantial support.

REFERENCES

1) E. Fradkin and S.J. Shenker, Phys. Rev. D19 (1979) 3682; J. Ranft, J. Kripfganz and G. Ranft, Phys. Rev. D28 (1983) 360; K.Jansen, J. Jersak, C.B. Lang, T. Neuhaus and G. Vones, Phys. Lett. 155B (1985) 268.
2) V. Azcoiti and A. Tarancon, Phys. Lett. 176B (1986) 153 and references therein.
3) H.G. Evertz et al. , PITHA 85/24 .
4) Abbot and Farhi, Phys. Lett. 101B (1981) 69 .
5) K. Fredenhagen and M. Marcu, Commun. Math. Phys. 92 (1983) 81; Phys. Rev. Lett. 56 (1986) 223 .
6) I-H. Lee and J. Shigemitsu, Nucl. Phys. B263 (1986) 280.
7) J.A. Swieca, Phys. Rev. D13 (1976) 312.
8) S. Weinberg and E. Witten, Phys. Lett. 96B (1980) 59.
9) V. Alessandrini, J.L. Alonso, A. Cruz and A. Tarancón, Nucl. Phys. B281 (1987) 445-469.
8 - C.B. Lang , C. Rebbi and M. Virasoro, Phys. Lett. 104B (1981) 294 .
10) C.B.Lang, C. Rebbi and M. Virasoro, Phys. Lett. 104B (1981) 294.
11) V. Azcoiti, A. Cruz, G. Di Carlo, A.F. Grillo and A. Tarancon, in preparation .
12) J. Jersak " Advances in Lattice Gauge Theory " ed. by D.W. Duke and J.F. Owens, p.241 .