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S. Bellucci

INFN - Laboratori Nazionali di Frascati
Casella Postale 13, 00044 Frascati (Roma), Italy

ABSTRACT

We extend the superspace treatment of higher derivative supersymmetric Yang-Mills theories, emphasizing the role played by manifest D=10 Lorentz covariance and supersymmetry invariance. As an example, the superspace formulation of the effective action is considered for the massless fields in the open, as well as in the SO(32) or $E_8 \times E_8$ superstrings. We show that there exists a unique modification of the f-tensor supercurrent which can provide the embedding of the $O(\alpha'^3)$ string corrections in the slope parameter expansion for the Yang-Mills sector in D = 10 superspace. The new correction term analyzed here is relevant for the understanding of possible nonperturbative effects.

I. INTRODUCTION AND SUMMARY

A unified description of quantum gravity and low-energy particle physics may be achieved by suitably compactifying type-I or heterotic string theories [1,2]. The low-energy dynamics of the massless fields of such theories is consistently derived from an effective action [2 10]. It is the common belief that such effective action should exist and possess a $D = 10$, $N = 1$ supersymmetry invariance. The effective action finds an important use in the study of the structure of possible ground states which admit compactification from $D = 10$ to $D = 4$ spacetime, while preserving $N = 1$ supersymmetry [11]. The lack of a manifestly supersymmetric, Lorentz-covariant formulation of string field theory has inspired several authors to propose alternative methods to formulate superstring effective actions [12-16]. This approach, based on a superspace formalism for the massless fields of the string effective action, allows to deduce a low-energy effective action for the SO(32) or $E_8 \times E_8$ superstrings, including $O(\alpha'^3)$ string corrections to the field theory limit [17]. Previously, the use of superspace enabled the authors of Ref. [18] to derive, from the structure of superspace geometry, the lowest order modifications representing the effect of the superstring corrections to $D = 10$ supersymmetric Yang-Mills theories. A new correction term of order $O(F^5)$ is produced in the effective action, when using a formalism which makes manifest the Lorentz covariance of the theory [19].

The possibility of embedding classical superstring corrections in an on-shell superspace formalism requires to implement, at each order in the slope parameter γ', suitable modifications in the structure of the source supertensors, to reflect the effect of the elimination of the massive modes of the string. In the present work, we evaluate the next-to-lowest order modifications which describe higher derivative corrections to supersymmetric Yang-Mills theories. We found our entire analysis upon the geometrical features of superspace. We obtain an essentially unique result, corresponding to the D = 10 supersymmetric extension of the purely bosonic $O(F^5)$ term in the effective action [19]. Also, we give the $O(\alpha'^2)$ corrections for the Yang-Mills sector of type-I or heterotic superstring theories, completing the study of Ref. [17].

Our conventions and notation are as in Ref. [18]. We begin by illustrating the on-shell superspace formalism proposed in Refs. [12,13,18] for the construction of superstring corrections to the effective action. We also review results presented in the previous literature about superstring corrections in the superspace, for both the supersymmetric Yang-Mills theories and the pure supergravity sector of the SO(32) or $E_8 \times E_8$ superstring theories. In Sect. III we discuss the $O(F^5)$ term and elucidate the Lorentz-covariant structure of the effective action for the massless vector field of the open superstring. We report also about the results on the string corrections presented to date. Next, we proceed to derive our main result, considering the superspace version of the $O(F^5)$ correction to the Yang-Mills sector. We conclude with a discussion of the recent progresses toward an off-shell formulation for D = 10, N = 1 supergravity, and the possible role of auxiliary fields for the superspace description of string corrections [20].

II. SUPERSTRING EFFECTS IN D = 10 SUPERSPACE

Supersymmetric Yang-Mills theory is obtained as the $\gamma' \rightarrow 0$ limit of the open superstring [21,22]. It was shown in Ref. [18] that, in order to retain the string corrections to the effective action for the massless vector superfield of the open superstring, one has to modify the constraints on the superspace field strength. Thus, the superspace description of D = 10 super Yang-Mills theories is determined by a fifth-rank tensor $f_{\underline{a}_1\cdots\underline{a}_5}{}^{\hat{I}}$ obeying an anti-self-duality condition

$$f_{\underline{a}_1\cdots\underline{a}_5}{}^{\hat{I}} = -\frac{1}{5!}\,\varepsilon_{\underline{a}_1\cdots\underline{a}_5\underline{b}_1\cdots\underline{b}_5}\,f^{\underline{b}_1\cdots\underline{b}_5\hat{I}} \tag{1}$$

Here $\hat{I}$ is a gauge group index. The open superstring corrections to the D = 10 superspace Yang-Mills theory are given by the expansion in the string parameter γ'

$$S_{eff} = \int d^{10}x\left[\mathcal{L}_{(0)} + \sum_{n=2}^{\infty}(\gamma')^{n}\mathcal{L}_{(n)}\right] \tag{2}$$

where $\mathcal{L}_{(0)}$ is the lagrangian for D = 10 supersymmetric Yang-Mills theory. Notice the absence of a O(γ') term, due to the vanishing of $\mathcal{L}_{(1)}$, by supersymmetry [9,19]. The modifications in (2) can be realized by suitably modifying, at each order in γ', the expression of the f-tensor [18]

$$f_{[5]}{}^{\hat{I}} = \sum_{n=2}^{\infty}(\gamma')^{n} f^{(n)}_{[5]}{}^{\hat{I}} \tag{3}$$

The power series representations (2) and (3) implement the tree-level string corrections, in a way such as to reflect precisely the effect of the elimination of the massive modes of the open superstring on the massless Yang-Mills sector. The lowest order string corrections are given in terms of the superfields $W^{\hat{I}\gamma}$, $F_{\underline{a}\underline{b}}{}^{\hat{I}}$ (see Ref. [18])

$$\begin{aligned} f_{\underline{c}_1\cdots\underline{c}_5}{}^{\hat{I}} &= i\,(g\gamma')^2\, b^{\hat{I}}{}_{\hat{k}_1\hat{k}_2\hat{k}_3}\, F_{\underline{a}\underline{b}}{}^{\hat{k}_3} \\ &\times\left(W^{\hat{k}_1}\sigma^{\underline{a}}\sigma_{\underline{c}_1\cdots\underline{c}_5}\sigma^{\underline{b}}W^{\hat{k}_2}\right) + O\left[(g\gamma')^3\right] \end{aligned} \tag{4}$$

where $b_{\hat{I}\hat{K}_1\hat{K}_2\hat{K}_3}$ is a constant, totally symmetric tensor in the gauge group and g is a Yang-Mills coupling constant. It was proved in Ref. [23] that (4) implies, at the component level, the $O(F^4)$ modifications to the effective action derived by Tseytlin [9]. This fact supports the validity of the superspace approach we are adopting.

For the closed superstring or heterotic string corrections to D = 10 supergravity theory, the following effective action has been proposed

$$\tilde{S}_{eff} = \frac{1}{\kappa^2}\int d^{10}x\; e^{-1}\Big[\tilde{\mathcal{L}}_{(0)} + \sum_{n=1}^{\infty}(\gamma')^n \tilde{\mathcal{L}}_{(n)}\Big] \tag{5}$$

In (5) κ is the gravitational coupling constant and $\tilde{\mathcal{L}}_{(0)}$ represents the lagrangian for the coupled D = 10, N = 1 supergravity and supersymmetric Yang-Mills theory. The classical superstring corrections in (5) can be represented by a power series expansion of the supergravity supercurrent $A_{\underline{a}\underline{b}\underline{c}}$ [13]

$$A_{\underline{a}\underline{b}\underline{c}} = \frac{1}{32\sqrt{2}}\, i\big(\lambda^{\hat{I}}\sigma_{\underline{a}\underline{b}\underline{c}}\lambda^{\hat{I}}\big) + \sum_{n=1}^{\infty}(\gamma')^n A^{(n)}{}_{\underline{a}\underline{b}\underline{c}} \tag{6}$$

In order to implement additional self-coupling in the Yang-Mills sector, the following modifications are needed

$$f_{\underline{a}_1\cdots\underline{a}_5}{}^{\hat{I}} = \sum_{n=1}^{\infty}(\gamma')^n B^{(n)}{}_{\underline{a}_1\cdots\underline{a}_5}{}^{\hat{I}} \tag{7}$$

For the expression of the order γ' and γ'^2 Yang-Mills contributions to $A^{(1)}{}_{\underline{a}\underline{b}\underline{c}}$ and $A^{(2)}{}_{\underline{a}\underline{b}\underline{c}}$, as well as for the $O(\gamma'^3)$ string corrections for the pure supergravity sector, we refer the reader to Ref. [17]. There, results are provided for the order γ' and γ'^2 contributions to $f^{(1)}{}_{\underline{a}_1\cdots\underline{a}_5}{}^{\hat{I}}$ and $f^{(2)}{}_{\underline{a}_1\cdots\underline{a}_5}{}^{\hat{I}}$, in the case of the SO(32) or $E_8\times E_8$ superstring theories

$$B_{[5]}^{(1)\ I} = i\ C_1 b^{\hat{I}}{}_{\hat{k}_1\hat{k}_2\hat{k}_3}\ e^{2\Phi}(\lambda^{k_1}\sigma^{\underline{a}}\sigma_{[5]}\sigma^{\underline{b}}\lambda^{k_2})F_{\underline{a}\underline{b}}{}^{k_3} \tag{8}$$

$$B_{[5]}^{(2)\ \hat{I}} = i\ C_2\, e^{3\Phi}(\tilde{\rho}^{\underline{d}\underline{e}}\sigma^{\underline{a}}\sigma_{[5]}\sigma^{\underline{b}}\tilde{\rho}_{\underline{d}\underline{e}})F_{\underline{a}\underline{b}}{}^{\hat{I}} \tag{9}$$

where

$$\tilde{\rho}^{\alpha\,\underline{d}\underline{e}} = i\,\sigma^{\underline{c}\alpha\beta}\,\tilde{R}_{\beta\underline{c}}{}^{\underline{d}\underline{e}}$$

and the modified curvature tensor $\tilde{R}_{\beta\underline{c}}{}^{\underline{d}\underline{e}}$ corresponds to the modified fermionic field strength $\tilde{T}_{\underline{a}\underline{b}}{}^{\gamma}$ [17]. In Ref. [17] it is stressed that a difference could arise between the two effective actions in the $E_8 \times E_8$ versus SO(32) string theories. A string calculation is required in order to elucidate such possibility, which is implied by the different relationships among the gravitational coupling constant, the slope parameter and the Yang-Mills coupling constant, in a type-I or heterotic superstring.

While (8) is a natural generalization, for the closed superstring or heterotic string corrections to D = 10 supergravity theory, of the modifications in D = 10 supersymmetric Yang-Mills theory constructed in Ref. [18], it is clear that the result (9) can be only considered incomplete, as long as corrections of order γ'^2 to the pure Yang-Mills sector are not taken into account. In Sect. IV we shall carry out such task. We first look closely at the Lorentz-covariant structure, in D = 10 ordinary spacetime, of the effective action for the massless vector field of the open superstring.

III. D = 10 LORENTZ-COVARIANT EFFECTIVE ACTION

An unpleasant and dissatisfactory situation afflicts the present knowledge of the effective action for the massless vector field of the open superstring. In the formalism of component fields the superstring corrections to supersymmetric Yang-Mills theories have been computed in Refs. [9,24], with contrasting results. The effective action method suggested in Ref. [7], based on the covariant Polyakov path integral, allowed Tseytlin to produce a closed expression for the abelian constant field-strength limit of the tree level effective lagrangian for the open superstring [9]

$$\mathcal{L}_s = \frac{2}{3}\left(L_s + \frac{3}{8}\check{F}^2 - 1\right) \tag{10}$$

where

$$\begin{aligned} L_s &= \{\det(\delta_{ij} + \check{F}_{ij})/\det[\eta_{\alpha\beta} + \tfrac{1}{4}(\gamma^{ij})_{\alpha\beta}\check{F}_{ij}]\}^{1/2} \\ &\longrightarrow \{\det(\eta_{\underline{ab}} + \check{F}_{\underline{ab}})/\det[\eta_{\alpha\beta} + \tfrac{1}{4}(\sigma^{\underline{ab}})_{\alpha\beta}\check{F}_{\underline{ab}}]\}^{1/2} \end{aligned} \tag{11}$$

We use the checks above each component field strength to avoid confusion with the corresponding superfield strength. In the last line of (11) we have indicated that we are obtaining a natural SO(10) invariant generalization of the result of Ref. [9], by making the replacement

$$(\delta_{ij}\ ,\gamma_{ij}\ ,\ F_{ij}) \longrightarrow (\eta_{\underline{ab}}\ ,\ \sigma_{\underline{ab}}\ ,\ \check{F}_{\underline{ab}})$$

of the SO(8) tensors, with the corresponding SO(10) covariant objects [19]. The low-energy effective lagrangian has the power series representation, in terms of the string parameter α'

$$\mathcal{L}_s = \frac{1}{4}\check{F}^2 - \frac{1}{8}\check{F}^4 + \frac{1}{32}\left(\check{F}^2\right)^2 - \\ - \frac{1}{1920}\,\varepsilon_{\underline{a}_1\cdots\underline{a}_{10}}\,\check{F}^{\underline{a}_1\underline{a}_2}\cdots\check{F}^{\underline{a}_9\underline{a}_{10}} + O\left(F^6\right) \tag{12}$$

The $O(F^5)$ term in (12) constitutes the original result of Ref. [19]. In Ref. [9] one cannot have a nonvanishing $O(F^5)$ term, since there the theory is formulated in D = 8, where the tensor $\varepsilon_{[10]}$ vanishes. The existence of a $O(F^5)$ contribution to the low-energy effective action can be proved only when turning, from the light-cone gauge approach, to the fully Lorentz-covariant formulation.

The application of worldsheet superspace methods to the open superstring in the NSR formalism leads the authors of Ref. [24] to claim that the low-energy effective action corresponding to the condition of vanishing beta-function is the Born-Infeld action

$$L = \left[\det\left(\eta_{\underline{a}\underline{b}} + \check{F}_{\underline{a}\underline{b}}\right)\right]^{1/2} \tag{13}$$

Thus, according to the findings of Ref. [24], the string corrections to the field theory limit are not sensitive to D = 10 spacetime supersymmetry

$$\tilde{\mathcal{L}}_s = \frac{1}{4}\check{F}^2 - \frac{1}{8}\check{F}^4 + \frac{1}{32}\left(\check{F}^2\right)^2 + O\left(F^6\right) \tag{14}$$

Notice that there are no $O(F^5)$ contributions to (14). Some time after the appearence of Ref. [24], Tseytlin and collaborators [25] amended the result presented in Ref. [9]. They claim that the effective action of the open superstring theory coincides with the one of the purely bosonic open string, i.e. the Born-Infeld action obtained previously in Ref. [24]. However, the issue of the effective action of the open superstring cannot be considered as settled. Evidence is provided in Ref. [26] that neither candidates proposed as effective actions for the massless field sector of the open superstring can be conclusively considered as the correct answer. In fact, after dimensional reduction and truncation to N=1, D=4 supersymmetric nonlinear Maxwell theory, one cannot match the bosonic part of those effective actions, unless supersymmetry is spontaneously broken. This indicates that such matters are precisely to be considered as unsettled, and furtherinvestigations are in order before deciding what is the correct expression for the effective action of the open superstring.

For higher derivative supersymmetric Yang-Mills theories in topologically nontrivial gauge field configurations, as well as for compactified string theories on manifolds with topologically nontrivial properties, the variation of the $O(F^5)$ term in (12) is nonvanishing under "large" gauge transformations.

The importance of such term can become manifest in connection to nonperturbative effects.

There are well known examples,related to instanton effects, where terms which are total divergences play a fundamental role. For the closed string case, one expects to produce such $O(F^5)$ term multiplied by some function of the dilaton field ϕ (see (43) below). Contributions to the effective action of the type $\varepsilon_{[10]}(F^{[2]})^5 f(\phi)$ cannot be expressed in general as total divergence terms. As an application of the formalism of D=10 superspace and for the purpose of exposing the connection between the effective action and the $f_{[5]}$-supertensor expansions, we establish the explicit correspondence of different sets of coefficients in $f_{[5]}$ to different proposals for the effective action, to higher orders. The supertensor (4) agrees with both $\mathcal{L}_s$ and $\tilde{\mathcal{L}}_s$. The expansions (12) and (14) of the effective actions start differing at the level of the $O(F^5)$ terms. It turns out that a vanishing coefficient for $f^{(3)}_{[5]}$ in (3) must be chosen, in order to reproduce, for the purely bosonic part of the component projection, the expansion of the Born-Infeld action. On the other hand, a nonvanishing choice of such coefficient of the expansion of the supertensor (see (42) below) corresponds to the supersymmetric completion of the effective lagrangian (10), to the order (γ'^3). Supersymmetry dictates the form of the terms of the expansions (3), (6), (7), but does not fix the coefficients of these expansions. It is only by performing string calculations that those coefficients can be determined. Our analysis in the present and in the next section shows how different choices of the coefficients of the expansions lead to different expressions for the effective action. Even if such expressions do not represent the effective action of the open superstring, yet they are valid and correct in a more general context than only for calculating string theory effective actions. The D=10 superspace feature we elucidate in the present work, i.e. the presence of a $O(F^5)$ term in the bosonic part of the Lorentz-covariant higher-derivative effective action, as well as the proof that there exists a unique supersymmetric completion of such contribution, can be directly applied to the construction and the analysis of higher derivative supersymmetric Yang-Mills theories.

IV. SUPERSYMMETRIC EXTENSION

The D = 10 supersymmetric completion of the $O(F^4)$ string modification in the low-energy lagrangian (12) was provided by Refs. [18,23]. An interesting task is the supersymmetrization of the $O(F^5)$ correction term. In order to elucidate the existence of such contribution, as well as the role it plays in testing the method for construction of the low-energy effective action, we have to maintain Lorentz covariance in D = 10 spacetime. This is most naturally accomplished by embedding the superstring modifications in D = 10 superspace, an approach which, as we have seen, makes both supersymmetry and Lorentz covariance manifest in D = 10. Let us first concentrate on the open superstring.

At each order in γ', higher order string corrections to the effective action (2) correspond to the power series representation (3) for $f_{[5]}{}^{\hat{I}}$, where the terms $f^{(n)}{}_{[5]}{}^{\hat{I}}$ must be constructed from only the superfields $W^{\hat{I}\gamma}$, $F_{\underline{ab}}{}^{\hat{I}}$ obeying the equations [18]

$$\nabla_{\alpha} W^{\beta \hat{I}} = -\frac{1}{2\sqrt{2}} (\sigma^{\underline{bc}})_{\alpha}{}^{\beta} F_{\underline{bc}}{}^{\hat{I}} \qquad (15)$$

$$\nabla_\alpha F_{\underline{b}\underline{c}}{}^{\hat{I}} = \frac{1}{\sqrt{2}} i\, \sigma_{[\underline{b}\alpha\gamma} \nabla_{\underline{c}]} W^{\gamma\hat{I}} \tag{16}$$

In (15) and (16) we are neglecting higher order terms in γ', since they are irrelevant to our purposes. The string modifications to the equation of motion for the superfield $F_{\underline{a}\underline{b}}{}^{\hat{I}}$ can be obtained considering the term $\nabla_\alpha \nabla_\beta f_{[5]}{}^{\hat{I}}$. The $O(\gamma'^3)$ modification $f^{(3)}{}_{[5]}{}^{\hat{I}}$ must have the structure

$$f^{(3)}{}_{\underline{c}_1\cdots\underline{c}_5}{}^{\hat{I}} = i g^3 C_3\, d^{\hat{I}}{}_{\hat{k}_1\hat{k}_2\hat{k}_3\hat{k}_4} \times \left(W^{\hat{k}_1} \Sigma^{\underline{a}\underline{b}\underline{d}\underline{e}}{}_{\underline{c}_1\cdots\underline{c}_5} W^{\hat{k}_2}\right) F_{\underline{a}\underline{b}}{}^{\hat{k}_3} F_{\underline{d}\underline{e}}{}^{\hat{k}_4} \tag{17}$$

if it is to produce the $O(F^5)$ string correction in the effective action (12), which corresponds to modifications of order $O(F^4)$ in the equation of motion for $F_{\underline{a}\underline{b}}{}^{\hat{I}}$. In fact, using (15) and (17), one sees that $\nabla_\alpha \nabla_\beta f^{(3)}{}_{[5]}{}^{\hat{I}}$ contains terms with the structure $F_{[2]}{}^{\hat{k}_1} F_{[2]}{}^{\hat{k}_2} F_{[2]}{}^{\hat{k}_3} F_{[2]}{}^{\hat{k}_4}$, etc.. Since we choose canonical dimensions, we have, in mass units

$$[W^{\hat{I}\gamma}] = 9/2 \quad , \quad [F_{[2]}{}^{\hat{I}}] = 5 \quad , \quad [f_{[5]}{}^{\hat{I}}] = 4 \,,$$

$$[g] = -3 \quad , \quad [\gamma'] = -2$$

Recalling (3), it is clear that (17) is the only possibility, also on the basis of mere dimensional analysis. The component level modifications implied by the superfield expression (17) can be worked out with the help of (15) and (16). In order to reproduce, at the component level, the $O(F^5)$ term in (12), it is clearly necessary to assume total symmetry in the gauge group indices of the tensor $d_{\hat{I}\hat{k}_1\hat{k}_2\hat{k}_3\hat{k}_4}$ appearing in (17). Indeed, reintroducing the gauge group indices in the component level string corrections to the effective action (12), one gets, for the order $O(F^5)$ contribution $\mathcal{L}_S^{(5)}$ to the low-energy lagrangian (12)

$$\mathcal{L}_S^{(5)} = -\frac{1}{1920}\, d_{\hat{k}_1\hat{k}_2\hat{k}_3\hat{k}_4\hat{k}_5}\, \varepsilon^{\underline{a}_1\cdots\underline{a}_{10}}\, \check{F}_{\underline{a}_1\underline{a}_2}{}^{\hat{k}_1} \cdots \check{F}_{\underline{a}_9\underline{a}_{10}}{}^{\hat{k}_5} \qquad (18)$$

where $d_{\hat{k}_1\hat{k}_2\hat{k}_3\hat{k}_4\hat{k}_5}$ is a constant, totally symmetric tensor in the gauge group. Notice that the value (-1/1920) of the constant coefficient in r.h.s. of (18) is specified, within the context of the effective action method [7], by the calculations of Refs. [9, 19]. It is only through "stringy" calculations that we can determine the coefficient of (18). The geometry of superspace dictates the form of the higher order contributions to the action, but cannot fix the corresponding set of coefficients. The choice of such coefficients remains unspecified, within the context of higher derivative supersymmetric Yang-Mills theories. Next, we carry out the classification of all possible products of the type $\Sigma^{\underline{a}\underline{b}\underline{d}\underline{e}}{}_{\underline{c}_1\cdots\underline{c}_5}$ of generalized Pauli matrices, which contribute to the spinor superfield bilinear in (17).

Taking into account the anti-self-duality condition (1) that the field strength (17) must satisfy, we have

$$\left(\Sigma^{\underline{a}\underline{b}\underline{d}\underline{e}}{}_{\underline{c}_1\cdots\underline{c}_5}\right)_\alpha{}^\beta =$$

$$= \left[\left(\sigma^{[1]}\right)^{2p+1} \sigma_{\underline{c}_1\cdots\underline{c}_5} \left(\sigma^{[1]}\right)^{2q+1}\right]_\alpha{}^\beta \left(\eta_{[2]}\right)^{p+q-1} \qquad (19)$$

i.e. the bispinor Σ must contain an odd number of factors of the type $\sigma_{\underline{a}}{}^{\alpha\beta}$ on both sides of the matrix $\sigma_{[5]}{}^{\gamma\delta}$. The first possibility we want to analyze is the product

$$\left(\sigma^{\underline{a}}\sigma_{[5]}\sigma^{\underline{b}}\sigma^{\underline{d}}\sigma^{\underline{e}}\right)_\alpha{}^\beta = \left(\sigma^{\underline{a}}\sigma_{[5]}\sigma^{\underline{b}\underline{d}\underline{e}}\right)_\alpha{}^\beta$$

$$+\,\eta^{\underline{d}\underline{e}}\left(\sigma^{\underline{a}}\sigma_{[5]}\sigma^{\underline{b}}\right)_\alpha{}^\beta + \eta^{\underline{b}[\underline{d}|}\left(\sigma^{\underline{a}}\sigma_{[5]}\sigma^{|\underline{e}]}\right)_\alpha{}^\beta \qquad (20)$$

Using the alge braic relation

$$\left(W^{\hat{k}_1}\sigma^{\underline{a}}\sigma_{[5]}\sigma^{\underline{b}}W^{\hat{k}_2}\right) = -\left(W^{\hat{k}_2}\sigma^{\underline{b}}\sigma_{[5]}\sigma^{\underline{a}}W^{\hat{k}_1}\right) \tag{21}$$

and recalling that the tensor $d_{\hat{1}\hat{k}_1\hat{k}_2\hat{k}_3\hat{k}_4}$ is completely symmetric, so that $F_{[\underline{a}|\underline{b}}{}^{\hat{k}_3}F^{\underline{b}}{}_{|\underline{e}]}{}^{\hat{k}_4}$ gives zero, it is easy to see that a nonvanishing contribution to (17) can be obtained only from the first term in the r.h.s. of (20). Since the same kind of argument applies to all products of the type (20), whatever the order of the indices $\underline{a}$, $\underline{b}$, $\underline{d}$, $\underline{e}$ considered, we have

$$\sigma^{[1]}\sigma_{[5]}\left(\sigma^{[1]}\right)^3 = D_1\,\sigma^{\underline{a}}\sigma_{[5]}\sigma^{\underline{bde}} + \dots \tag{22}$$

where D_1 is a real number and the ellipsis indicates terms which do not contribute to (17). The other possibility

$$\left(\sigma^{[1]}\right)^3\sigma_{[5]}\sigma^{[1]} = \tilde{D}_1\,\sigma^{\underline{bde}}\sigma_{[5]}\sigma^{\underline{a}} + \dots \tag{23}$$

reduces to the one given in (22), since we can use the relationship

$$\left(W^{\hat{k}_1}\sigma^{\underline{bde}}\sigma_{[5]}\sigma^{\underline{a}}W^{\hat{k}_2}\right) = \left(W^{\hat{k}_2}\sigma^{\underline{a}}\sigma_{[5]}\sigma^{\underline{bde}}W^{\hat{k}_1}\right) \tag{24}$$

where we have taken into account the antisymmetry of the bispinor $(\sigma_{[3]})_{\alpha\beta}$ on the interchange of spinor indices α, β.

The next possibility we analyze is

$$\left(\sigma^{\underline{a}}\sigma^{\underline{d}}\sigma^{\underline{h}}\sigma_{[5]}\sigma^{\underline{b}}\sigma^{\underline{i}}\sigma^{\underline{e}}\right)_\alpha{}^\beta\,\eta_{\underline{hi}}$$
$$= 2\left(\sigma^{\underline{a}}\sigma^{\underline{d}}\sigma^{\underline{b}}\sigma_{[5]}\sigma^{\underline{e}}\right)_\alpha{}^\beta \tag{25}$$

In (25) we used the expression

$$\sigma_{\underline{a}\alpha\beta}\, \sigma_{[5]}{}^{\beta\gamma}\, \sigma^{\underline{a}}{}_{\gamma\delta} = 0 \tag{26}$$

which is a consequence of the *Fierz* **identity**

$$\sigma^{\underline{a}}{}_{\alpha\beta}\, \sigma_{\underline{a}\gamma\delta} = -\sigma^{\underline{a}}{}_{\alpha(\gamma}\, \sigma_{\underline{a}\,\delta)\beta} \tag{27}$$

Recalling the property (21), and indicating by ellipsis vanishing contributions tp (17), we get from (25)

$$\begin{aligned} &(\sigma^{\underline{a}}\sigma^{\underline{d}}\sigma^{\underline{h}}\, \sigma_{[5]}\, \sigma^{\underline{b}}\sigma^{\underline{i}}\sigma^{\underline{e}})_{\alpha}{}^{\beta}\, \eta_{\underline{hi}} \\ &= (\sigma^{\underline{adb}}\, \sigma_{[5]}\, \sigma^{\underline{e}})_{\alpha}{}^{\beta} + \ldots \end{aligned} \tag{28}$$

Plugging (28) into (17) one gets the result

$$\begin{aligned} &(W^{\hat{k}_1}\, \sigma^{\underline{a}}\sigma^{\underline{d}}\sigma^{\underline{h}}\, \sigma_{[5]}\, \sigma^{\underline{b}}\sigma^{\underline{i}}\sigma^{\underline{e}}\, W^{\hat{k}_2}) F_{\underline{ab}}{}^{\hat{k}_3}\, F_{\underline{de}}{}^{\hat{k}_4} \\ &= (W^{\hat{k}_2}\, \sigma^{\underline{a}}\, \sigma_{[5]}\, \sigma^{\underline{bde}}\, W^{\hat{k}_1}) F_{\underline{ab}}{}^{\hat{k}_4}\, F_{\underline{de}}{}^{\hat{k}_3} \end{aligned} \tag{29}$$

where we made use of (24). Considering the new term

$$
\begin{aligned}
&(\sigma^{\underline{a}}\sigma^{\underline{d}}\sigma^{\underline{h}}\sigma_{[5]}\sigma^{\underline{b}}\sigma^{\underline{e}}\sigma^{\underline{i}})_\alpha{}^\beta\,\eta_{\underline{hi}} \\
&= -(\sigma^{\underline{a}}\sigma^{\underline{d}}\sigma^{\underline{h}}\sigma_{[5]}\sigma^{\underline{b}}\sigma^{\underline{i}}\sigma^{\underline{e}})_\alpha{}^\beta\,\eta_{\underline{hi}} \\
&\quad + 2(\sigma^{\underline{a}}\sigma^{\underline{d}}\sigma^{\underline{e}}\sigma_{[5]}\sigma^{\underline{b}})_\alpha{}^\beta
\end{aligned}
\tag{30}
$$

we see that it reduces to the term analyzed previously in (25), plus a term similar to the r.h.s. of (22). Terms of the type

$$
\begin{aligned}
&(\sigma^{\underline{a}}\sigma^{\underline{h}}\sigma^{\underline{d}}\sigma_{[5]}\sigma^{\underline{b}}\sigma^{\underline{e}}\sigma^{\underline{i}})_\alpha{}^\beta\,\eta_{\underline{hi}} \\
&= -(\sigma^{\underline{a}}\sigma^{\underline{d}}\sigma^{\underline{h}}\sigma_{[5]}\sigma^{\underline{b}}\sigma^{\underline{e}}\sigma^{\underline{i}})_\alpha{}^\beta\,\eta_{\underline{hi}} \\
&\quad + 2\,(\sigma^{\underline{a}}\sigma_{[5]}\sigma^{\underline{b}}\sigma^{\underline{e}}\sigma^{\underline{d}})_\alpha{}^\beta
\end{aligned}
\tag{31}
$$

reduce to (30), plus a term proportional to the first term in the r.h.s. of (22). Such arguments allow us to prove that

$$
(\sigma^{[1]})^3\,\sigma_{[5]}(\sigma^{[1]})^3\,\eta_{[2]} = D_2\,\sigma^{\underline{a}}\,\sigma_{[5]}\,\sigma^{\underline{bde}} + \ldots \tag{32}
$$

We must consider another possibility

$$
\begin{aligned}
&(\sigma^{\underline{m}}\sigma^{\underline{a}}\sigma^{\underline{h}}\sigma_{[5]}\sigma^{\underline{d}}\sigma^{\underline{i}}\sigma^{\underline{b}}\sigma^{\underline{e}}\sigma^{\underline{n}})_\alpha{}^\beta\,\eta_{\underline{hi}}\,\eta_{\underline{mn}} \\
&= 2(\sigma^{\underline{m}}\sigma^{\underline{a}}\sigma^{\underline{d}}\sigma_{[5]}\sigma^{\underline{b}}\sigma^{\underline{e}}\sigma^{\underline{n}})_\alpha{}^\beta\,\eta_{\underline{mn}}
\end{aligned}
\tag{33}
$$

where (26) is used. Then, the results of our previous analysis, given in (32), tell us that

$$\left(\sigma^{\underline{m}}\sigma^{\underline{a}}\sigma^{\underline{h}}\sigma_{[5]}\sigma^{\underline{d}}\sigma^{\underline{i}}\sigma^{\underline{b}}\sigma^{\underline{e}}\sigma^{\underline{n}}\right)_\alpha{}^\beta\eta_{\underline{hi}}\eta_{\underline{mn}}$$
$$= D_2'\,\sigma^{\underline{a}}\sigma_{[5]}\sigma^{\underline{bde}} + \ldots \tag{34}$$

for some value of the numerical coefficient D_2'. The ordering of the vector indices $\underline{m}$, $\underline{a}$, etc. considered in (33) and (34) is generic. Hence, the reader will be easily convinced that, by the arguments used to reduce (33) to the form (34), one can prove the general statement

$$\left(\sigma^{[1]}\right)^3\sigma_{[5]}\left(\sigma^{[1]}\right)^5\left(\eta_{[2]}\right)^2 = D_3\,\sigma^{\underline{a}}\sigma_{[5]}\sigma^{\underline{bde}} + \ldots \tag{35}$$

It is obvious that the remaining possibility

$$\left(\sigma^{[1]}\right)^5\sigma_{[5]}\left(\sigma^{[1]}\right)^3\left(\eta_{[2]}\right)^2 = \tilde{D}_3\,\sigma^{\underline{bde}}\sigma_{[5]}\sigma^{\underline{a}} + \ldots \tag{36}$$

reduces to the result (35), after using the property (24).

Finally, we must take into account the term

$$\left(\sigma^{\underline{r}}\sigma^{\underline{a}}\sigma^{\underline{m}}\sigma^{\underline{d}}\sigma^{\underline{h}}\sigma_{[5]}\sigma^{\underline{k}}\sigma^{\underline{i}}\sigma^{\underline{e}}\sigma^{\underline{n}}\sigma^{\underline{s}}\right)_\alpha{}^\beta\eta_{\underline{hi}}\eta_{\underline{mn}}\eta_{\underline{rs}}$$
$$= 2\left(\sigma^{\underline{r}}\sigma^{\underline{a}}\sigma^{\underline{m}}\sigma^{\underline{d}}\sigma^{\underline{b}}\sigma_{[5]}\sigma^{\underline{e}}\sigma^{\underline{n}}\sigma^{\underline{s}}\right)_\alpha{}^\beta\eta_{\underline{mn}}\eta_{\underline{rs}} \tag{37}$$

We conclude that, because of our general result (36)

$$\left(\sigma^{\underline{r}}\sigma^{\underline{a}}\sigma^{\underline{m}}\sigma^{\underline{d}}\sigma^{\underline{h}}\sigma_{[5]}\sigma^{\underline{k}}\sigma^{\underline{i}}\sigma^{\underline{e}}\sigma^{\underline{n}}\sigma^{\underline{s}}\right)_\alpha{}^\beta\eta_{\underline{hi}}\eta_{\underline{mn}}\eta_{\underline{rs}}$$
$$= D_3'\,\sigma^{\underline{a}}\sigma_{[5]}\sigma^{\underline{bde}} + \ldots \tag{38}$$

for some real number D_3'. The same kind of reasoning, applied to (37) above, allows us to prove in general

$$(\sigma^{[1]})^5 \sigma_{[5]} (\sigma^{[1]})^5 (\eta_{[2]})^3 = D_4 \sigma^{\underline{a}} \sigma_{[5]} \sigma^{\underline{bde}} + \ldots \tag{39}$$

The σ-matrices

$$\sigma^{\underline{a}}{}_{\alpha\beta} \; , \quad (\sigma^{\underline{abc}})_{\alpha\beta} \; , \quad (\sigma^{\underline{a}_1 \cdots \underline{a}_5})_{\alpha\beta} \tag{40}$$

form a basis for the purely left-handed bispinors. Any structure of the type (19) can be expressed as the product of left-handed bispinors of the type (40). Therefore, our analisys can stop with products having the structure given on the l.h.s. of (39). This allows us to conclude that all the products of σ-matrices, which could conceivably contribute to (19), are indeed proportional to the matrix

$$(\Sigma^{\underline{abde}}{}_{\underline{c}_1 \cdots \underline{c}_5})_\alpha{}^\beta = (\sigma^{\underline{a}} \sigma_{\underline{c}_1 \cdots \underline{c}_5} \sigma^{\underline{bde}})_\alpha{}^\beta \; . \tag{41}$$

Thus, the geometrical features of superspace dictate the unique possibility for $f^{(3)}{}_{[5]}{}^{\hat{I}}$ in (3)

$$f^{(3)}{}_{[5]}{}^{\hat{I}} = i g^3 C_3 \; d^{\hat{I}}{}_{\hat{k}_1 \hat{k}_2 \hat{k}_3 \hat{k}_4} \times (W^{\hat{k}_1} \sigma^{\underline{a}} \sigma_{[5]} \sigma^{\underline{bde}} W^{\hat{k}_2}) F_{\underline{ab}}{}^{\hat{k}_3} F_{\underline{de}}{}^{\hat{k}_4} \tag{42}$$

Since it represents a unique candidate in superspace, (42) is expected to produce the component level modifications proportional to $\varepsilon_{[10]}(\check{F}_{[2]})^5$ given in (12). Work on the component level modifications implied by (42) is presently in progress.

By generalizing the superspace formulation of the higher order open superstring correction (42), we find that, within the context of the type-I or heterotic superstrings, the following result

$$B^{(2)}{}_{[5]}{}^{\hat{I}} = i\, C_2\, e^{3\Phi} (\tilde{\rho}^{\underline{de}}\, \sigma^{\underline{a}} \sigma_{[5]} \sigma^{\underline{b}}\, \tilde{\rho}_{\underline{de}})\, F_{\underline{ab}}{}^{\hat{I}}$$
$$+\, i\, \tilde{C}_2\, e^{3\Phi}\, d^{\hat{I}}{}_{\hat{k}_1 \hat{k}_2 \hat{k}_3 \hat{k}_4} (\lambda^{\hat{k}_1} \sigma^{\underline{a}} \sigma_{[5]} \sigma^{\underline{bde}} \lambda^{\hat{k}_2})$$
$$\times\, F_{\underline{ab}}{}^{\hat{k}_3}\, F_{\underline{de}}{}^{\hat{k}_4} \qquad (43)$$

replaces the incomplete expression (9) provided by Ref. [17]. In the result (43) we are concentrating on the case of the heterotic string, since we are aware that a difference, possibly arising between the heterotic and type-I string effective actions, can only be elucidated performing certain

"stringy" calculations [17]. It is clear, by inspection of (43), that the terms of order $O(F^5)$ obtained by component projection are expected to depend from Φ, as well. Therefore, in general, they cannot be cast as total derivative terms and it is not correct to drop them out from the effective action.

V. DISCUSSION AND OUTLOOK

If we read (2) in the broader context of higher derivative supersymmetric Yang-Mills theories, a contribution to the effective action of the form $\varepsilon_{[10]}(F^{[2]})^5$ cannot be ruled out on the basis of supersymmetry invariance. In fact, the corresponding modification induced in the effective action can be realized most elegantly in superspace, by modifying suitably the expression of the $f_{[5]}$-supertensor in the unique way given by (42). Naturally, such contribution to the effective action is important for the theory on a manifold which allows for topologically nontrivial configurations of the (nonabelian) gauge fields. When dealing with the restricted case of string corrections, the coefficients of the expansions of the supertensors (3), (6), (7) in the string parameter become specified by "stringy" calculations, e.g. in the path integral or in the β-function approaches for the effective action. The result (42) provides an application of the D=10 superspace formalism which shows that, corresponding to different choices of those coefficients, one gets different expressions for the effective action. In particular, a vanishing coefficient for the $O(\gamma'^3)$ term of the expansion (3) reproduces, up to the same order, the expansion of the Born-Infeld lagrangian (14). On the other hand, fixing the value of the coefficient given by (42), corresponds to the effective lagrangian (12). In the compactification of superstring theories on a manifold with nontrivial topological properties of the gauge fields, such contribution to the effective lagrangian may find an interesting application in connection to nonperturbative effects.

It is conjectured that all classical superstring corrections to the massless sector can be embedded in an on-shell superspace formulation [12,13,18]. Having obtained the higher order results (42) and (43) in this framework, by advocating properties of the geometrical structure of superspace, it provides evidence of the efficacy and validity of the proposal suggested by Refs. [12,13,18] for the treatment of effective actions for massless string states, in all superstring theories possessing spacetime supersymmetry, to arbitrary order in the string slope parameter. However, no proof of such statement exists and it is conceivable that, in order to incorporate higher order string modifications, the use of only the A and f-tensors may be not enough. Then, a natural extension of the superspace approach is to resort to the off-shell components of the D = 10, N = 1 supergravity multiplet, which would become relevant for $E_8 \times E_8$ or SO(32) strings.

In a previous work we have pointed out that some insight on the structure of the off-shell D = 10, N = 1 supergravity multiplet can be gained by studying the consequences of the geometrical requirement of integrability on a light-line in superspace [20]. This has led the authors of Ref. [27] to propose a set of constraints which, although equivalent on-shell to the standard formulation [20], contain a vector-spinor auxiliary superfield. We wish to suggest that the auxiliary components of the D = 10, N = 1 supergravity multiplet can play a role in the embedding of superstring corrections in superspace, providing a more general framework to incorporate the effects of higher string modes on the massless supergravity string states.

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