INFN - Laboratori Nazionali di Frascati

To be Submitted to Nuclear Phys. B

LNF-87/76(PT)

9 Luglio 1987

S. Bellucci:

ON-SHELL EQUIVALENCE OF LIGHT-LIKE INTEGRABILITY AND CANONICAL CONSTRAINTS FOR D=10, N=1 SUPERGRAVITY

Servizio Documentazione
dei Laboratori Nazionali di Frascati
P.O. Box, 13 - 00044 Frascati (Italy)

July 1987

LNF-87/76(PT)

ON-SHELL EQUIVALENCE OF LIGHT-LIKE INTEGRABILITY AND CANONICAL CONSTRAINTS FOR D = 10, N = 1 SUPERGRAVITY

S. Bellucci

INFN - Laboratori Nazionali di Frascati
Casella Postale 13, 00044 Frascati (Roma), Italy

ABSTRACT

Using Weyl transformations in superspace, we find that the nonminimal version of D = 10, N = 1 supergravity based on geometrical constraints obtained by Chau and Milewski, is equivalent on-shell to the canonical formulation. We interpret the appearence of auxiliary components for the off-shell D = 10, N = 1 supergravity multiplet as a feature of the geometrical formulation. The analysis of the implications of the Bianchi identities in the geometrical version of the theory is extended to the nonlinear case.

I. INTRODUCTION

Type-I or heterotic superstrings provide suitable candidates for a unified description of quantum gravity and low-energy particle physics [1,2]. The massless field effective action which should arise as low-energy limit of the SO(32) or $E_8 \times E_8$ superstrings is expected to possess a D = 10, N = 1 supersymmetry. The form of the effective action may be crucial in determining the ground state(s) which allow to preserve N = 1 supersymmetry in the compactification to D = 4 spacetime [3]. Superstring field theory exhibiting manifest supersymmetry and Lorentz covariance seems still far from being constructed. Thus, the use of superspace has been suggested as a method to formulate superstring effective actions [4]. This approach allows to derive an effective action for the massless fields in type-I or heterotic superstrings, including $O(\alpha'^3)$ string corrections to the field theory limit [5]. The use of a formalism which makes manifest the Lorentz covariance of the open superstrin produces a new string correction term proportional to $\epsilon_{\underline{a}_1\underline{b}_1\cdots\underline{a}_5\underline{b}_5}F^{\underline{a}_1\underline{b}_1}\cdots F^{\underline{a}_5\underline{b}_5}$ in the low-energy effective action [6].

The above examples elucidate how, in order to achieve the description of the effective supersymmetric D = 10 field theory in the most efficient way, it is necessary to have a complete understanding of N = 1 supergravity and supersymmetric Yang-Mills theories in D = 10 spacetime [7-11]. The geometrical requirement of integrability on a light-line in D = 10, N = 1 superspace leads to a vanishing spinor-spinor field-strength tensor and to supersymmetric Yang-Mills equations of motion [12]. Recently, a new set of constraints for D = 10, N = 1 supergravity has been derived in Ref. [13], imposing the condition of light-like integrability.

In the literature several constraints have been imposed in order to describe D = 10, N = 1 supergravity in superspace [12,14-18]. In Ref. [4] it is shown that relating those different sets of constraints requires the use of generalized conformal transformations in superspace. In the present work we sho that the new set of constraints presented in Ref. [13] describes essentially th same theory as those appearing in the previous literature. Using the general formalism of Weyl-scale transformations in superspace [19,20], we derive the field dependent transformations allowing to relate the theory of Ref. [13] to more standard versions of D = 10, N = 1 supergravity.

We begin by reviewing different standard formulations of D = 10, N = 1 supergravity. Section IIIcontains a discussion of the geometrical constraints imposed in Ref. [13]. The next section is devoted to the analysis of the generalized superspace conformal transformations. We prove the equivalence between the nonminimal and the canonical formulations of D = 10, N = 1 supergravity, giving, at the same time, the redefinitions of the supergravity fields required by the superspace Weyl transformations. The corrections to the linearized theory are presented in Sect. V. We end our discussion commenting on light-like integrability conditions and possible off-shell formulations of supergravity in D = 10 superspace.

II. CANONICAL CONSTRAINTS

Both new and old D = 10, N = 1 supergravity were originally constructed using the formulation of component fields [7,21]. The two supergravity theories are related by a duality transformation. The corresponding superspace geometry has been investigated in Ref. [18]. Torsions and curvatures are introduced by the definition

$$[\nabla_{\underline{A}},\nabla_{\underline{B}}\} = T_{\underline{AB}}{}^{\underline{C}}\nabla_{\underline{C}} + \frac{1}{2} R_{\underline{ABd}}{}^{\underline{e}}M_{\underline{e}}{}^{\underline{d}} \tag{1}$$

Notation and conventions are as in Ref. [4]. To our purposes, i.e. in order to show the equivalence of the different choices of constraints for D = 10, N = 1 supergravity, it is sufficient to quote the conditions imposed on the torsions [22]. The following choice of constraints, which we refer to as "canonical" [18]

$$N_{\alpha_1\ldots\alpha_7} = N_{\alpha_1\ldots\alpha_6\underline{b}_1} = N_{\alpha_1\ldots\alpha_5\underline{b}_1\underline{b}_2} = \ldots = N_{\alpha_1\alpha_2\alpha_3\underline{b}_1\ldots\underline{b}_4} = 0 \ ,$$

$$T_{\alpha\beta}{}^{\underline{c}} = i\sigma^{\underline{c}}{}_{\alpha\beta} \ , \quad N_{\alpha\beta\underline{c}_1\ldots\underline{c}_5} = i\,\frac{1}{2}\,e^{\Phi}(\sigma_{\underline{c}_1\ldots\underline{c}_5})_{\alpha\beta} \ ,$$

$$T_{\alpha\beta}{}^{\gamma} = -\frac{1}{2\sqrt{2}}\,[\delta^{\gamma}_{(\alpha}\delta^{\varepsilon}_{\beta)} + \sigma^{\underline{a}}{}_{\alpha\beta}\sigma_{\underline{a}}{}^{\gamma\varepsilon}]\chi_{\varepsilon} \quad , \quad T_{\alpha\underline{b}}{}^{\underline{c}} = 0 \quad ,$$

$$N_{\alpha\underline{b}_1\ldots\underline{b}_6} = -\frac{1}{2\sqrt{2}}\, e^{\Phi}(\sigma_{\underline{b}_1\ldots\underline{b}_6})_{\alpha}{}^{\beta}\chi_{\beta} \quad ,$$

$$T_{\alpha\underline{b}}{}^{\gamma} = \frac{1}{24}\,(\sigma_{\underline{b}}\sigma^{\underline{cde}})_{\alpha}{}^{\gamma}[e^{-\Phi}\,\tilde{N}_{\underline{cde}} + i\,\frac{1}{8}\,(\chi\sigma_{\underline{cde}}\chi)]$$

$$+\frac{1}{48}\,(\sigma^{\underline{cde}}\sigma_{\underline{b}})_{\alpha}{}^{\gamma}\,[e^{-\Phi}\tilde{N}_{\underline{cde}} + i\,\frac{1}{16}\,(\chi\sigma_{\underline{cde}}\chi)] \tag{2}$$

describe the new theory in superspace. In (2) $\tilde{N}_{\underline{abc}}$ is the dualized seven-form field strength. In the superspace description of the old version of D = 10, N = 1 supergravity some of the constraints are modified with respect to (2)

$$T_{\alpha\beta}{}^{\underline{c}} = i\sigma^{\underline{c}}{}_{\alpha\beta} \quad ,$$

$$T_{\alpha\beta}{}^{\gamma} = -\frac{1}{2\sqrt{2}}\,[\delta^{\gamma}_{(\alpha}\delta^{\varepsilon}_{\beta)} + \sigma^{\underline{a}}{}_{\alpha\beta}\sigma_{\underline{a}}{}^{\gamma\varepsilon}]\chi_{\varepsilon} \quad , \quad T_{\alpha\underline{b}}{}^{\underline{c}} = 0 \quad ,$$

$$T_{\alpha\underline{b}}{}^{\gamma} = -\frac{1}{24}\,(\sigma_{\underline{b}}\sigma^{\underline{cde}})_{\alpha}{}^{\gamma}[e^{\Phi}H_{\underline{cde}} - i\,\frac{1}{8}\,(\chi\sigma_{\underline{cde}}\chi)]$$

$$-\frac{1}{48}\,(\sigma^{\underline{cde}}\sigma_{\underline{b}})_{\alpha}{}^{\gamma}[e^{\Phi}H_{\underline{cde}} - i\,\frac{1}{16}\,(\chi\sigma_{\underline{cde}}\chi)] \tag{3}$$

In (3) the modified superspace field strength $H_{\underline{abc}}$ appears. The Bianchi identities, together with (2), imply the equations of motion

$$\nabla_\alpha \Phi = -\frac{1}{\sqrt{2}} \chi_\alpha \quad ,$$

$$\nabla_\alpha \chi_\beta = -i \frac{1}{12\sqrt{2}} (\sigma^{\underline{cde}})_{\alpha\beta} \; e^{-\Phi} \tilde{N}_{\underline{cde}} - i \frac{1}{\sqrt{2}} \sigma^{\underline{a}}{}_{\alpha\beta} \nabla_{\underline{a}} \Phi \tag{4}$$

Consistency of the constraints (3) with the Bianchi identities implies

$$\nabla_\alpha \Phi = -\frac{1}{\sqrt{2}} \chi_\alpha \quad ,$$

$$\nabla_\alpha \chi_\beta = \frac{i}{12\sqrt{2}} (\sigma^{\underline{cde}})_{\alpha\beta} \; e^{\Phi} H_{\underline{cde}} - i \frac{1}{\sqrt{2}} \sigma^{\underline{c}}{}_{\alpha\beta} \nabla_{\underline{c}} \Phi \tag{5}$$

The superfield Φ is the fundamental field strength of the on-shell theory [14]. The old version of D = 10, N = 1 supergravity is directly related to the low-energy limit of type-I or heterotic superstrings. On the other hand, the superspace formulation of the dual theory allows to embed superstring corrections in a considerably simpler way [5,23,24]. The two field theories are indistinguishable in the light-cone formulation of superstring theories [25,26].

III. A NONMINIMAL FORMULATION

The consequences of light-like integrability conditions in D = 10, N = 1 supergravity have been investigated by the authors of Ref. [13]. Those conditions lead to [13]

$$\tilde{T}_{\alpha\beta}{}^{\underline{c}} = i \sigma^{\underline{c}}{}_{\alpha\beta} \; , \quad \tilde{T}_{\alpha\beta}{}^{\gamma} = 0 \; , \quad \tilde{R}_{\alpha\beta\gamma}{}^{\delta} = 0 \tag{6}$$

In (6), with respect to more standard formulations, torsion constraints are replaced by a constraint on the curvature. For reference, we quote here the set of constraints derived by Witten [12]

$$T_{\alpha\beta}{}^{\underline{c}} = i\,\sigma^{\underline{c}}{}_{\alpha\beta} \ , \quad T_{\alpha\beta}{}^{\gamma} = 0 \ ,$$

$$T_{\alpha\underline{b}}{}^{\gamma} = i\,\sigma_{\underline{b}\alpha\beta}\, f^{\beta\gamma} \ , \quad T_{\alpha\underline{b}}{}^{\underline{c}} = 0 \tag{7}$$

The first Bianchi identity

$$[[\,\tilde{\nabla}_{(\alpha}\,,\,\tilde{\nabla}_{\beta}\}\,,\,\tilde{\nabla}_{\gamma)}\} = 0 \tag{8}$$

has the solution

$$[\tilde{\nabla}_{\alpha}\,,\,\tilde{\nabla}_{\underline{b}}\} = i\,\sigma_{\underline{b}\alpha\beta}\, V^{\beta} \tag{9}$$

in terms of some operator V^{β}. This implies, in addition to the expressions found in Ref. [13]

$$\tilde{T}_{\alpha\underline{b}}{}^{\underline{c}} = i\,\sigma_{\underline{b}\alpha\beta}\,\psi^{\underline{c}\beta} \tag{10}$$

$$\tilde{T}_{\alpha\underline{b}}{}^{\gamma} = i\,\sigma_{\underline{b}\alpha\beta}\, f^{\beta\gamma} \tag{11}$$

an explicit solution for the curvature

$$R_{\alpha \underline{b} \delta} = i \sigma_{\underline{b} \alpha\beta} \rho^{\Gamma}{}_{\delta}{}^{\varepsilon} \tag{12}$$

The solution of the remaining Bianchi identities is carried out only partially in Ref. [13]. Such a task (i.e. solving <u>all</u> Bianchi identities at the nonlinear level) can be simplified by making use of (12). We shall not attempt to pursue such equations here. At this point we turn our attention to the structure of the vector-spinor superfield which appears in (10).

The geometrical constraints are not sufficient to eliminate all unphysical degrees of freedom for the description of D = 4, N $\geq$ 4 extended supergravity theories. One is forced to enlarge the original set of constraints, setting certain fields equal to zero, in order to recover Poincaré supergravity [27]. For D = 10, N = 1 supergravity the situation is similar and the torsion constraint

$$\tilde{T}_{\alpha(\underline{a}}{}^{\underline{b})} - \frac{1}{10} \delta_{\underline{a}}{}^{\underline{b}} \tilde{T}_{\alpha \underline{c}}{}^{\underline{c}} = 0 \tag{13}$$

is proposed in Ref. [13], in order to reduce $\psi^{\underline{c}\beta}$ to the pure contribution of the σ-trace part

$$\psi^{\underline{c}\beta} = i \sigma^{\underline{c}\beta\gamma} \lambda_{\gamma} \tag{14}$$

In alternative to (13), we can add to the set (6) a different torsion constraint which does not involve explicit symmetrization as in (13) and can be expressed in terms of the product of σ-matrices

$$\rho^{\underline{c}\alpha} + \frac{1}{10} \sigma^{\underline{c}\alpha\delta} \sigma^{\underline{d}}{}_{\delta\varepsilon} \rho_{\underline{d}}{}^{\varepsilon} = 0 \tag{15}$$

where

$$\rho^{\underline{c}\delta} \equiv \sigma^{\underline{b}\alpha\delta}\,\tilde{T}_{\alpha\underline{b}}{}^{\underline{c}} \tag{16}$$

It is not difficult to prove that (14) follows from (15), (16) and (10). One can use (15) to put the theory derived from the set of constraints (6) on mass-shell. In the off-shell formulation of D = 10 supergravity expressed by the constraints (6), the spinor-vector $\hat{\psi}^{\underline{c}\beta}$ defined as

$$\hat{\psi}^{\underline{c}\beta} = \psi^{\underline{c}\beta} - i\,\sigma^{\underline{c}\beta\gamma}\lambda_{\gamma} \tag{17}$$

with

$$i\,\sigma_{\underline{c}\alpha\beta}\,\hat{\psi}^{\underline{c}\beta} = 0 \tag{18}$$

is an auxiliary superfield. In fact, requiring light-like integrability in superspace provides for the first time a set of constraints where such object appears. The existence of a spinor-vector as part of the off-shell D = 10 supergravity multiplet was suggested in Ref. [4], as a way of interpreting the three-form $V^{\underline{abc}}$. The authors of Ref. [28] proposed the latter as the fundamental prepotential of D = 10, N = 1 supergravity. Assuming universality of the D = 10, N = 1 and D = 4, N = 4 supergravity theories (the D = 4, N = 4 supergravity prepotential corresponds to the SO(6) components of the D = 10, N = 1 prepotential), one is led to conclude that $V^{\underline{abc}}$ cannot be the basic prepotential for D = 10, N = 1 because it cannot accommodate the entire vierbein. This suggests that the irreducible representation of SO(1,9) described by a spinor-vector superfield constrained by (18) is the actual prepotential for D = 10, N = 1 supergravity. In terms of the basic prepotential, the three form reads [4]

$$V^{\underline{a}\underline{b}\underline{c}} = \frac{1}{3!} (\sigma^{[\underline{a}\underline{b}})_{\beta}{}^{\gamma} D_{\gamma} \hat{\psi}^{\underline{c}]\beta} \tag{19}$$

The constraints (6) contribute to shed some light on the structure of the off-shell theory, through the identification of the σ-traceless part of the ψ-superfield appearing in their solution (10), as the fundamental prepotential of D = 10, N = 1 supergravity. We believe that, using definitions of the differential operators more general than the set of operators picked in Ref. [13], and requiring that they define integral lines in superspace, one will gair further insight for the construction of the complete off-shell D = 10, N = 1 supergravity multiplet.

Making the assumption

$$\lambda_{\alpha} = \tilde{\nabla}_{\alpha} \tilde{\Phi} \tag{20}$$

where $\tilde{\Phi}$ is a scalar superfield, which defines also an antisymmetric tensor field

$$F_{\underline{a}\underline{b}\underline{c}} = i (\sigma_{\underline{a}\underline{b}\underline{c}})^{\alpha\beta} \tilde{\nabla}_{\alpha} \tilde{\nabla}_{\beta} \tilde{\Phi} \tag{21}$$

the authors of Ref. [13] express $f^{\beta\gamma}$ in (11) as

$$f^{\alpha\beta} = -\frac{1}{2} i (\sigma_{\underline{a}})^{\alpha\beta} \tilde{\nabla}^{\underline{a}} \tilde{\Phi} - \frac{1}{16} i \frac{1}{3!} (\sigma_{\underline{a}\underline{b}\underline{c}})^{\alpha\beta} F^{\underline{a}\underline{b}\underline{c}} \tag{22}$$

in the linearized approximation. Among the physical fields in the theory defined by the constraints (6) and (13), or (15), equations of motion have been derived in Ref. [13], again only in the linearized approximation, for both the Rarita-Schwinger field strength $\tilde{T}_{\underline{a}\underline{b}}{}^{\alpha}$ and the spinor superfield λ_{α}. From our point of view, one does not really need bother solving all Bianchi identities at the nonlinear level, to confirm the expectation that the remaining equations of motion for the dilaton field $\tilde{\Phi}$, the antisymmetric field strength $F_{\underline{abc}}$ and the Einstein equation for the Riemann tensor $\tilde{R}_{\underline{abcd}}$ are so obtained. In fact, such task has been carried out for the canonical formulation of new D = 10, N = 1 supergravity [18] (see (4)). After showing that the nonminimal geometrical formulation of on-shell D = 10, N = 1 supergravity described in this Section can be conformally transformed into the minimal canonical formulation, one can simply read the equations for $\tilde{\Phi}$, λ_α, etc. from well-known results [4]. We proceed next to derive the redefinitions of the physical fields introduced in this Section, in terms of the fields Φ, χ_α, $\tilde{N}_{\underline{abc}}$, etc. of Sect. II.

IV. WEYL TRANSFORMATIONS IN SUPERSPACE AND THE REDEFINITION OF THE SUPERGRAVITY FIELDS

Any set of constraints describing D = 10, N = 1 supergravity in superspace can be related to any other, e.g. to the set of canonical constraints introduced in Ref. [18], by field redefinitions. The proof of this statement is given in Ref. [4], using the general formalism of Weyl-scale transformations in superspace [19,20]. The super-Weyl redefinitions of the covariant derivatives, corresponding to a superspace scale parameter $L = K\Phi$ (for some constant K)

$$\nabla'_\alpha = e^{K\Phi}\left(\nabla_\alpha + \tfrac{1}{2} f_{\alpha\underline{d}}{}^{\underline{e}} M_{\underline{e}}{}^{\underline{d}}\right)$$

$$\nabla'_{\underline{a}} = e^{2K\Phi}\left(\nabla_{\underline{a}} + f_{\underline{a}}{}^{\gamma}\nabla_\gamma + \tfrac{1}{2} f_{\underline{a}\underline{d}}{}^{\underline{e}} M_{\underline{e}}{}^{\underline{d}}\right) \qquad (23)$$

where

$$f_{\alpha \underline{d}}{}^{\underline{e}} = -\frac{1}{\sqrt{2}} k A (\sigma_{\underline{d}}{}^{\underline{e}})_\alpha{}^\beta \chi_\beta$$

$$f_{\underline{a}}{}^\gamma = -i\frac{1}{\sqrt{2}} k B \sigma_{\underline{a}}{}^{\gamma\delta}\chi_\delta \tag{24}$$

implies that the torsion $\tilde{T}_{\underline{A}\underline{B}}{}^{\underline{C}}$ is related to the canonical torsion $T_{\underline{A}\underline{B}}{}^{\underline{C}}$ by [4]

$$T'_{\alpha\beta}{}^{\underline{c}} = T_{\alpha\beta}{}^{\underline{c}} \quad ,$$

$$T'_{\alpha\beta}{}^{\gamma} = e^{k\Phi}(T_{\alpha\beta}{}^{\gamma} - T_{\alpha\beta}{}^{\underline{c}} f_{\underline{c}}{}^{\gamma} - \frac{k}{\sqrt{2}}\chi_{(\alpha}\delta_{\beta)}{}^{\gamma} + \frac{1}{4} f_{(\alpha\underline{d}}{}^{\underline{e}}(\sigma_{\underline{e}}{}^{\underline{d}})_{\beta)}{}^{\gamma}) \quad ,$$

$$\begin{aligned} T'_{\alpha\underline{b}}{}^{\gamma} = e^{2k\Phi}(&T_{\alpha\underline{b}}{}^{\gamma} - f_{\underline{b}}{}^{\beta}T_{\alpha\beta}{}^{\gamma} + f_{\underline{b}}{}^{\beta}T_{\alpha\beta}{}^{\underline{c}}f_{\underline{c}}{}^{\gamma} - T_{\alpha\underline{b}}{}^{\underline{c}}f_{\underline{c}}{}^{\gamma} \\ &+ \nabla_\alpha f_{\underline{b}}{}^{\gamma} - k(\nabla_{\underline{b}}\Phi)\delta_\alpha{}^{\gamma} - \frac{1}{4} f_{\underline{b}\underline{d}}{}^{\underline{e}}(\sigma_{\underline{e}}{}^{\underline{d}})_\alpha{}^{\gamma} \\ &+ k\,(\frac{1}{\sqrt{2}})(f_{\underline{b}}{}^{\beta}\chi_\beta)\delta_\alpha{}^{\gamma}) \quad , \end{aligned}$$

$$T'_{\alpha\underline{b}}{}^{\underline{c}} = e^{k\Phi}(T_{\alpha\underline{b}}{}^{\underline{c}} - f_{\underline{b}}{}^{\beta}T_{\alpha\beta}{}^{\underline{c}} - k\sqrt{2}\,\chi_\alpha\delta_{\underline{b}}{}^{\underline{c}} - f_{\alpha\underline{b}}{}^{\underline{c}}) \quad ,$$

$$\begin{aligned} T'_{\underline{a}\underline{b}}{}^{\underline{c}} = e^{2k\Phi}(&T_{\underline{a}\underline{b}}{}^{\underline{c}} + f_{[\underline{a}}{}^{\alpha}T_{\alpha\underline{b}]}{}^{\underline{c}} - f_{\underline{a}}{}^{\alpha}f_{\underline{b}}{}^{\beta}T_{\alpha\beta}{}^{\underline{c}} \\ &- f_{[\underline{a}\underline{b}]}{}^{\underline{c}} + 2k(\nabla_{[\underline{a}}\Phi - \frac{1}{\sqrt{2}} f_{[\underline{a}}{}^{\alpha}\chi_\alpha)\delta_{\underline{b}]}{}^{\underline{c}}) \quad , \end{aligned}$$

$$\begin{aligned} T'_{\underline{a}\underline{b}}{}^{\gamma} = e^{3k\Phi}(&T_{\underline{a}\underline{b}}{}^{\gamma} + f_{[\underline{a}}{}^{\alpha}T_{\alpha\underline{b}]}{}^{\gamma} - f_{\underline{a}}{}^{\alpha}f_{\underline{b}}{}^{\beta}\,T_{\alpha\beta}{}^{\gamma} \\ &- T_{\underline{a}\underline{b}}{}^{\underline{c}}f_{\underline{c}}{}^{\gamma} - f_{[\underline{a}}{}^{\alpha}T_{\alpha\underline{b}]}{}^{\underline{c}}f_{\underline{c}}{}^{\gamma} + f_{\underline{a}}{}^{\alpha}f_{\underline{b}}{}^{\beta}T_{\alpha\beta}{}^{\underline{c}}f_{\underline{c}}{}^{\gamma} \\ &+ \nabla_{[\underline{a}}f_{\underline{b}]}{}^{\gamma} + f_{[\underline{a}}{}^{\alpha}(\nabla_\alpha f_{\underline{b}]}{}^{\gamma})) \end{aligned} \tag{25}$$

Similar transformations for $\tilde{R}_{\underline{abde}}$ are obtained from (23), but we will not need to make use of them. The constraints (6) and (13), or (15), are a consistent choice to describe D = 10, N = 1 Poincaré supergravity if and only if one can find a unique redefinition of the type (25), in order to connect them to canonical constraints.

Indeed, this is the case, and we can obtain (10) and (11) from (2), using the transformations defined by

$$K = -7/4$$

$$A = -\frac{1}{5}\left(\frac{1}{K} + 2\right) = -2/7$$

$$B = \frac{1}{10}\left(8 - \frac{1}{K}\right) = 6/7$$

$$\begin{aligned} f_{\underline{abc}} &= -\frac{1}{5}(2K+1)\,\eta_{\underline{a}[\underline{b}}\nabla_{\underline{c}]}\Phi - \frac{2}{5}(2K+1)e^{-\Phi}\tilde{N}_{\underline{abc}} - \\ &\quad - \frac{4}{25}\left(\frac{19}{64} - \frac{11}{16}K - K^2\right)i(\chi\sigma_{\underline{abc}}\chi) \\ &= \frac{1}{2}\eta_{\underline{a}[\underline{b}}\nabla_{\underline{c}]}\Phi + e^{-\Phi}\tilde{N}_{\underline{abc}} + \frac{1}{4}i(\chi\sigma_{\underline{abc}}\chi) \end{aligned} \tag{26}$$

These imply

$$\begin{aligned} \psi^{\underline{c}\beta} &= i\,e^{K\Phi}\sigma^{\underline{c}\beta\delta}\nabla_\delta\Phi\left(\frac{6}{5}K + \frac{1}{10}\right) \\ &= -2i\,e^{-\frac{7}{4}\Phi}\sigma^{\underline{c}\beta\delta}\nabla_\delta\Phi \end{aligned}$$

$$
\begin{aligned}
f^{\beta\gamma} &= -i e^{\ldots} \{ [\tfrac{1}{10} e^{\ldots} N_{\underline{cde}} (\tfrac{1}{3}K - \tfrac{1}{4}) \\
&\quad + \tfrac{1}{150} i (\chi \sigma_{\underline{cde}} \chi)(K^2 + \tfrac{1}{16}K - \tfrac{39}{64})] (\sigma^{\underline{cde}})^{\beta\gamma} \\
&\quad + \tfrac{1}{5}(3K + \tfrac{1}{4}) \sigma_{\underline{a}}{}^{\beta\gamma} \nabla^{\underline{a}} \Phi \} \\
&= -i e^{-\frac{7}{2}\Phi} \{ [-\tfrac{1}{12} e^{-\Phi} \tilde{N}_{\underline{cde}} + \tfrac{1}{64} i (\chi \sigma_{\underline{cde}} \chi)] (\sigma^{\underline{cde}})^{\beta\gamma} - \\
&\quad - \sigma_{\underline{a}}{}^{\beta\gamma} \nabla^{\underline{a}} \Phi \}
\end{aligned}
\tag{27}
$$

The light-like integrability constraints (6) can describe the old theory as well. The redefinitions (25), with

$$
\begin{aligned}
K &= -7/4 \\
A &= -2/7 \\
B &= 6/7
\end{aligned}
$$

$$
\begin{aligned}
f_{\underline{abc}} &= \tfrac{1}{2} \eta_{\underline{a}[\underline{b}} \nabla_{\underline{c}]} \Phi - e^{-\Phi} H_{\underline{abc}} \\
&\quad + \tfrac{1}{4} i (\chi \sigma_{\underline{abc}} \chi)
\end{aligned}
\tag{28}
$$

transform the set of constraints for standard D = 10, N = 1 supergravity (3) into (6). In the case in which (10) and (11) describe the solution to the first Bianchi identity, we get

$$\psi^{\underline{c}\beta} = -2i\,e \qquad \sigma^{\underline{c}\beta}\,\nabla_\delta \Phi$$

$$f^{\beta\gamma} = -i\,e^{-\frac{7}{2}\Phi}\Big\{\Big[\frac{1}{12}e^{-\Phi}H_{\underline{c}\underline{d}\underline{e}} + \frac{1}{64}i(\chi\sigma_{\underline{c}\underline{d}\underline{e}}\chi)\Big](\sigma^{\underline{c}\underline{d}\underline{e}})^{\beta\gamma} - \sigma_{\underline{a}}{}^{\beta\gamma}\nabla^{\underline{a}}\Phi\Big\}$$

(29)

From (26)-(29) we see that the set of constraints (6), obtained imposing integrability on light-like lines, are suitable for the description of both the standard and the dual D = 10, N = 1 supergravity theories.

As a check of our formulas (26) and (27), we can recover the redefinition which allows to obtain Witten's constraints (7) in terms of canonical ones. In this case, from (25) we get K = -1/12. Thus, from (26) and (27) one reads

$$A = B = 2$$

$$f_{\underline{a}\underline{b}\underline{c}} = -\frac{1}{6}\eta_{\underline{a}[\underline{b}}\nabla_{\underline{c}]}\Phi - \frac{1}{18}i(\chi\sigma_{\underline{a}\underline{b}\underline{c}}\chi) - \frac{1}{3}e^{-\Phi}\tilde{N}_{\underline{a}\underline{b}\underline{c}}$$

$$\psi^{\underline{c}\beta} = 0$$

$$f^{\beta\gamma} = \frac{1}{36}i\,e^{-\frac{1}{6}\Phi}\Big[e^{-\Phi}\tilde{N}_{\underline{c}\underline{d}\underline{e}} + \frac{7}{48}i(\chi\sigma_{\underline{c}\underline{d}\underline{e}}\chi)\Big](\sigma^{\underline{c}\underline{d}\underline{e}})^{\beta\gamma}$$

in agreement with the findings of Ref. [4]. At this point, it is obvious that Witten's set can be as well related directly to light-like integrability constraints, where a curvature constraint is introduced, replacing torsion constraints. The Weyl-redefinition, transforming (7) into (10) and (11), corresponds to setting in (25)

$$K = 5/6 \quad , \quad A = -2/5 \quad , \quad B = 4/5$$

The expression of $f_{\underline{abc}}$ is determined in analogy to (26).

V. NONLINEAR CORRECTIONS

As we mentioned earlier, the implications of the Bianchi identities have been discussed only for the linearized case [13]. We can take full advantage of the powerful technique of super-Weyl transformations to compute the nonlinear corrections to (22), as well as to the other formulas of Ref. [13] obtained from the Bianchi identities up to dimension 1. Recalling (4), (20), (21) and using the results of Sect. IV, as well as the relation for σ-matrices

$$(\sigma^{\underline{de}})_{\alpha}{}^{\gamma}(\sigma_{\underline{ed}})_{\beta}{}^{\delta}(\sigma_{\underline{abc}})^{\alpha\beta}\,\nabla_\gamma\Phi\,\nabla_\delta\Phi$$

$$= -6(\sigma_{\underline{abc}})^{\alpha\beta}\,\nabla_\alpha\Phi\,\nabla_\beta\Phi$$

one can show that

$$\tilde{\nabla}_\delta\tilde{\Phi} = -2e^{-\frac{7}{4}\Phi}\,\nabla_\delta\Phi$$

$$\tilde{\nabla}_{\underline{a}}\tilde{\Phi} = -2e^{-\frac{7}{2}\Phi}\,\nabla_{\underline{a}}\Phi$$

$$F_{\underline{abc}} = -2i\,e^{-\frac{7}{2}\Phi}\left[-4ie^{-\Phi}\tilde{N}_{\underline{abc}} - \frac{5}{2}(\sigma_{\underline{abc}})^{\alpha\beta}\,\nabla_\alpha\Phi\,\nabla_\beta\Phi\right] \qquad (30)$$

Equations (30) relate the physical fields of the D = 10, N = 1 supergravity multiplet in the formulation of Ref. [13] to those of the canonical formulation of Ref. [4]. Plugging (30) into (27) we get

$$f^{\beta\gamma} = -\frac{1}{2} i \sigma_{\underline{a}}{}^{\beta\gamma} \tilde{\nabla}^{\underline{a}} \tilde{\Phi} -$$
$$- \frac{1}{96} i \left(\sigma^{\underline{cde}}\right)^{\beta\gamma} \left[F_{\underline{cde}} - 2i \left(\sigma_{\underline{cde}}\right)^{\alpha\delta} \tilde{\nabla}_{\alpha} \tilde{\Phi} \tilde{\nabla}_{\delta} \tilde{\Phi} \right] \tag{31}$$

This provides the nonlinear corrections to (22). Let us consider the remaining Bianchi identity of dimension 1

$$\sigma^{\underline{a}}{}_{\alpha\beta} \tilde{T}_{\underline{a}\underline{c}}{}^{\underline{b}} - i \sigma_{\underline{c}\gamma(\alpha} \sigma^{\underline{b}}{}_{\beta)\delta} f^{\gamma\delta} -$$
$$- \sigma_{\underline{c}\gamma(\alpha} \nabla_{\beta)} \psi^{\underline{b}\gamma} + i \sigma_{\underline{c}\gamma(\alpha} \psi^{\underline{a}\gamma} \sigma_{\underline{a}\beta)\delta} \psi^{\underline{b}\delta} = 0 \tag{32}$$

We notice that the sign of the nonlinear correction, given by the last term in (32), is opposite with respect to the corresponding term found in Eq. (4.6) of Ref. [13]. Tracing (32) with $\sigma_{\underline{d}\alpha\beta}$ and using the algebraic relation

$$(\sigma_{\underline{ac}})_{\gamma}{}^{\beta} \sigma_{\underline{d}\beta\delta} \sigma^{\underline{b}\delta\mu} \sigma^{\underline{d}\gamma\lambda} \nabla_{\lambda} \Phi \nabla_{\mu} \Phi$$
$$= -6 \left(\sigma_{\underline{ac}}{}^{\underline{b}}\right)^{\lambda\mu} \nabla_{\lambda} \Phi \nabla_{\mu} \Phi$$

we obtain, recalling (14), (20) and (31)

$$\tilde{T}_{\underline{a}\underline{c}}{}^{\underline{b}} = \frac{1}{4} F_{\underline{a}\underline{c}}{}^{\underline{b}} + 4\, \tilde{\nabla}_{[\underline{a}} \tilde{\Phi}\, \eta_{\underline{c}]}{}^{\underline{b}} + \frac{1}{2} i \left(\sigma_{\underline{a}\underline{c}}{}^{\underline{b}}\right)^{\lambda\mu} \tilde{\nabla}_{\lambda} \tilde{\Phi}\, \tilde{\nabla}_{\mu} \tilde{\Phi} \tag{33}$$

Equation (33) extends to the nonlinear case the result in Eq. (4.26) of Ref. [13]. From (25), (26) and (30), we can derive the expression of the torsion $T_{\underline{a}\underline{b}}{}^{\underline{c}}$ implied by the Bianchi identities in the canonical formulation of the theory given by (2)

$$T_{\underline{a}\underline{b}}{}^{\underline{c}} = -4\, \nabla_{[\underline{a}} \Phi\, \eta_{\underline{b}]}{}^{\underline{c}} - \frac{1}{4} i \left(\chi\, \sigma_{\underline{a}\underline{b}}{}^{\underline{c}}\, \chi\right) \tag{34}$$

This result is not given in Ref. [4] and we believe it will prove helpful in analyzing the equations of motion of the spin-3/2 and spin-2 components of the supergravity multiplet in the dual formulation of the theory [19]. Notice that the dual form $\tilde{N}_{\underline{a}\underline{b}}{}^{\underline{c}}$ does not contribute to (34). The use of Weyl transformations in superspace outlined in this Section will greatly simplify the study of the implications of the Bianchi identities of dimension higher than 1, in terms of the equations of motion for the physical field strengths $F_{\underline{a}\underline{b}\underline{c}}$, $T_{\underline{a}\underline{b}}{}^{\alpha}$ and $R_{\underline{a}\underline{b}\underline{c}\underline{d}}$.

VI. DISCUSSION

Weyl transformations in superspace allow us to prove the equivalence on-shell of the D = 10, N = 1 supergravity theory defined by the formulation of Ref. [13], to the formulations described in the previous literature. This provides a check of the consistency of the results of Ref. [13]. In addition, the use of super-Weyl transformations enables us to extend the analysis of the Bianchi identities to the fully nonlinear theory, in its geometrical formulation. A more fundamental feature of the geometrical approach is the appearence, in a natural way, of the off-shell field $\hat{\psi}_{\underline{a}}{}^{\alpha}$. The presence of this auxiliary component of the D = 10, N = 1 supergravity multiplet was suggested already in Ref. [4]. We can imagine easily a mechanism to build further components of such multiplet. Recalling the transformations (23), one can introduce the set of differential operators, for a given light-like vector $\lambda_{\underline{a}}$

$$X(\lambda) = \lambda^{\underline{a}} \nabla'_{\underline{a}}$$
$$Y^{\alpha}(\lambda) = i \lambda^{\underline{a}} \sigma_{\underline{a}}{}^{\alpha\beta} \nabla'_{\beta}$$
$$\lambda_{\underline{a}} \lambda^{\underline{a}} = 0 \tag{35}$$

and then impose that they form a closed algebra

$$[X , Y^{\alpha}\} = 0$$
$$[Y^{\alpha}, Y^{\beta}\} = 2i \lambda^{\underline{a}} \sigma_{\underline{a}}{}^{\alpha\beta} X \tag{36}$$

Such treatment is more general than the one in Ref. [13], since it takes into account covariance under the transformations (23). A transformation of the type (25) suggests that the field $\tilde{\nabla}_\alpha \hat{\Psi}_{\underline{b}}{}^{\gamma}$ will contribute to the solution of the set of geometrical constraints defined by (35) and (36). We wish to suggests to interpret $\tilde{\nabla}_\alpha \hat{\Psi}_{\underline{b}}{}^{\gamma}$ as yet another auxiliary component of the off-shell D = 10, N = 1 supergravity multiplet. Imposing integrability conditions in superspace should provide even further insight about the structure of the off-shell formulation of the theory, provided general superspace transformations are considered, by introducing an unrestricted superspace scale parameter L [19,20]. The geometrical constraints required by the integrability conditions in superspace may find in this scenario a both natural and exciting application.

Acknowledgement

It is a pleasure to thank Professor Jim Gates for many helpful discussions. We wish to acknowledge Professor Ling-Lie Chau for the kind hospitality at the Physics Department of the University of California at Davis while this work was undertaken.

REFERENCES

[1] M. B. Green and J. H. Schwarz, Nucl. Phys. B181 (1981) 502;
ibid. Nucl. Phys. B198 (1982) 252, 441;
ibid. Phys. Lett. 109B (1982) 444.

[2] D. J. Gross, J. A. Harvey, E. Martinec and R. Rohm, Phys. Rev. Lett. 54 (1985) 502;
ibid. Nucl. Phys. B256 (1985) 253.

[3] P. Candelas, G. Horowitz, A. Strominger and E. Witten, Nucl. Phys. B258 (1985) 46.

[4] S. J. Gates and S. Vashakidze, Nucl. Phys. B291 (1987) 173.

[5] S. J. Gates and H. Nishino, Nucl. Phys. B291 (1987) 205.

[6] S. Bellucci, University of Maryland Preprint UMDEPP 87-178.

[7] A. H.Chamseddine, Nucl. Phys. B185 (1981) 403.

[8] E. Bergshoeff, M. de Roo, B. de Wit and P. van Nieuwenhuizen, Nucl. Phys. B195 (1982) 97.

[9] G. F. Chapline and N. S. Manton, Phys. Lett. 120B (1983) 105.

[10] F. Gliozzi, J. Scherk and D. I. Olive, Nucl. Phys. B122 (1977) 253.

[11] L. Brink, J. H. Schwarz and J. Scherk, Nucl. Phys. B121(1977) 77.

[12] E. Witten, Nucl. Phys. B266 (1986) 245.

[13] L. L. Chau and B. Milewski, University of California, Davis Preprint UCD-87-05 (revised).

[14] B. E. W. Nilsson, Nucl. Phys. B188 (1981) 176.

[15] S. W. MacDowell and M. T. Rakowski, Nucl. Phys. B274 (1986) 589.

[16] J. J. Atick, A. Dhar and B. Ratra, Phys. Rev. Lett. 33 (1986) 2824.

[17] B. E. W. Nilsson and A. K. Tollsten, Phys. Lett. 170B (1986) 212.

[18] S. J. Gates and H. Nishino, Phys. Lett. 173B (1986) 46.

[19] S. J. Gates, in Supergravity, Eds. P. van Nieuwenhuizen and D. Z. Freedman (1979) North-Holland, p. 215;
ibid. Nucl. Phys. B176 (1980) 397.

[20] P. Howe, Nucl. Phys. B199 (1982) 309.

[21] A. H. Chamseddine, Phys. Rev. D24 (1981) 3065.

[22] N. Dragon, Z. Phys. C2 (1979) 29.

[23] S. J. Gates and H. Nishino, Phys. Lett. 173B (1986) 52.

[24] S. J. Gates and H. Nishino, Nucl. Phys. B282 (1987) 1.

[25] P. Goddard, J. Goldstone, C. Rebbi and C. B. Thorn, Nucl. Phys. B56 (1973) 109.

[26] M. B. Green and J. H. Schwarz, Phys. Lett. 136B (1984) 367.

[27] L. L. Chau and C. S. Lim, Phys. Rev. Lett. 56 (1986) 294.
[28] P. Howe, H. Nicolai and A. Van Proeyen, Phys. Lett. 112B (1982) 446.