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IONIZING PARTICLE DETECTORS**

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**SIGNAL TO NOISE RATIO IN SUPERCONDUCTING TUNNEL JUNCTIONS AS
IONIZING PARTICLE DETECTORS**

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ABSTRACT

The reasons for the disagreement between expected and measured performances of Superconducting Tunnel junctions (STJ) used as high resolution nuclear detector are discussed. In particular, the optimization of signal to noise (S/N) ratio for devices having low and high tunnel probability (p) versus the bath temperature (T) and the front-end electronics used i.e. voltage and charge amplifier has been analyzed and discussed. Calculations have been done about the intrinsic recombination time versus T for some conventional superconducting materials. It was found the time for which the maximum of the voltage signal from STJ versus p and T occurs. It is shown that for a charge amplifier coupled to the STJ the S/N ratio indefinitely increases decreasing T .

1. - INTRODUCTION

The potential extremely high energy resolution of Superconducting Tunnel Junction (STJ) (1, 2, 3, 4) as ionizing particle detectors arises from the energy gap (Δ) value which in conventional superconductors is of the order of 0.18 ± 1.5 meV.

The number N of produced charges, in any type of detector is, theoretically, at least,

$N = E_{rel}/\epsilon$ where ϵ , proportional to Δ , is the minimal energy required to produce one "free charge" useful for the detection and E_{rel} is the energy released from ionizing particle in the detector. Since the nominal energy resolution ($\Delta E/E$) is just proportional to $\sqrt{\epsilon}$, we can plan to get a detector 30+50 times better than the best semiconductor device operating at any temperature. Most of the experiments performed up to now make use of STJ operating in tunnel regime where, at fixed bias conditions, the excess current, or related voltage variations, induced by the external perturbation, is measured.

In Table I the main features of the performed experiments are reported. The obtained performances were rather disappointing in comparison with the expected results. In our opinion, to get a really good and practical nuclear detector, it should be necessary to analyse in a deep way the excitation and relaxation phenomena involved in the STJ under ionizing particle excitations.

Because it is not completely clear if there is some intrinsic limit to energy resolution coming from noise of detector itself, in this paper we discuss the signal to noise S/N ratio as a function of both the temperature T and the tunnel probability p, by assuming the complete junction uniformly perturbed. For this reason, in sec. II we briefly analyse the general features of the I-V characteristic both in the unperturbed state and under excitations generated by ionizing particles, and the usual expression of the current noise of a STJ is recalled. In both cases ideal junctions without true leakage currents are supposed. In sec. III we analyse the S/N ratio of the STJ coupled to a voltage or charge amplifier. In sec. IV we give conclusions and some discussion is performed.

2. - STJ DETECTION SYSTEM

We analyse a STJ connected to a bias network and to a preamplifier followed by an optional filter as shown in Fig. 1a. The equivalent circuit is shown in Fig. 1b, where the junction is described by a noise source, a capacitor C_D , and a resistance $R(V)$, which is changed by the ionizing particle and generates the signal.

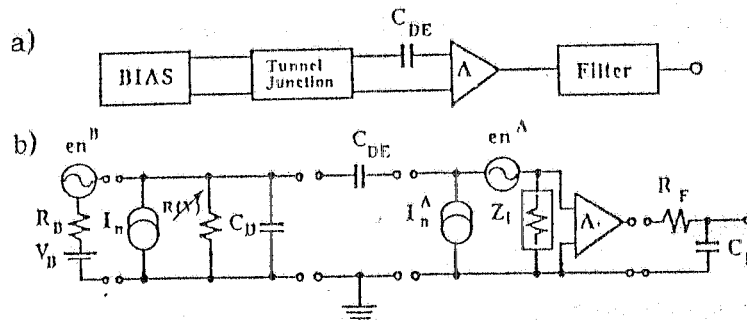


FIG. 1 - (a) Block diagram of the STJ detection system and (b) equivalent circuit: C_{DE} is a decoupling capacity, Z_i is the input impedance of the amplifier, I_n and e_n are the usual noise sources.

With this circuit we investigate the intrinsic signal to noise ratio.

External noise source here are neglected and both ideal bias network and amplifiers are assumed. The possibility of using pulse amplifier will not be considered.

TABLE - I - Experimental results and operating conditions for the experiments performed with STJ as nuclear detector.

Authors	White Wood (1)	Kurakado (2)	Barone (3)	Barone (3)	Barone (3)	Twerenbold (4)
Year	1973	1984	1985	1985	1985	1986
Junctions Electrodes	Sn-SnO-Sn	Sn-SnO-Sn	Nb-NbO-Pb	Nb-NbO-Pb	Nb-NbO-Pb	Sn-SnO-Sn
Geometry	cross	cross	cross	island	cross	island
Film thickness (Å)	2000; 1000	1550; 1500	2900; 35000	1000; 20000	1000; 20000	1500; 6500
$R_{NN}(m\Omega)$	77	—	20	1+2	1+2	470
Surface (cm ²)	$7 \cdot 10^{-4}$	$12 \cdot 10^{-4}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-4}$
Temp.(K)	1.2	0.32	1.4	1.4	1.4	0.38
Source	$^{239}\text{Pu};\alpha$	$^{210}\text{Po};\alpha$	$^{241}\text{Am};\alpha$	$^{241}\text{Am};\alpha$	$^{241}\text{Am};\alpha$	$^{55}\text{Fe};X$
$E_{rel}(\text{keV})$	500	70	900	3800	3800	5.89
Amplifier	current	charge	voltage	voltage	voltage	charge
$\epsilon(\text{eV}), \text{exp.}$	0.41	—	0.07	0.18	0.25	0.03
$\Delta E/E, \text{exp.} (\%)$	—	6.9	37	22	31	0.7

2.1. - Characteristic times

The detection system is characterized by four characteristic times as following:

- the detector time $\tau_D = C_D R_D = \tau_D(p, T)$;
- the tunnel time $\tau_t = 1/p$;
- the recombination time $\tau_r(T)$;
- the filter time τ_f .

The detector time τ_D is the product of detector resistance R_D for detector capacitance C_D . The C_D is generally large: the intrinsic value is of the order of $30 \mu\text{F}/\text{cm}^2$.

The tunnel time τ_t is the "middle" transit time of the excess quasi particles, generated by the excitation, causing the charge collection process and is given by the following expression (1)

$$\tau_t = R_{NN} e^2 N_0 A \cdot d \quad (1)$$

where R_{NN} is the normal state resistance at $V \gg (\Delta_1 + \Delta_2)/e$, e is the electron charge, N_0 is the density of states at Fermi energy, A is the junction area and d is the thickness of the film. The typical order of magnitude for τ_t is: $2 \text{ ns } R_{NN}(\Omega) A(\mu\text{m}^2) d(\mu\text{m})$.

The recombination time τ_r is the "middle time" of quasi particle recombination; it limits the charge collection and in quasi-equilibrium state can be approximated by

$$\tau_r = \tau_0 \exp(\Delta/K_B T) \pi^{-1/2} (2\Delta/K_B T_c)^{-5/2} (T/T_c)^{-1/2} \quad (2)$$

where τ_0 and T_c are respectively the characteristic quasi particle time and the transition temperature, which depend on the superconducting materials, as shown in Fig. 2.

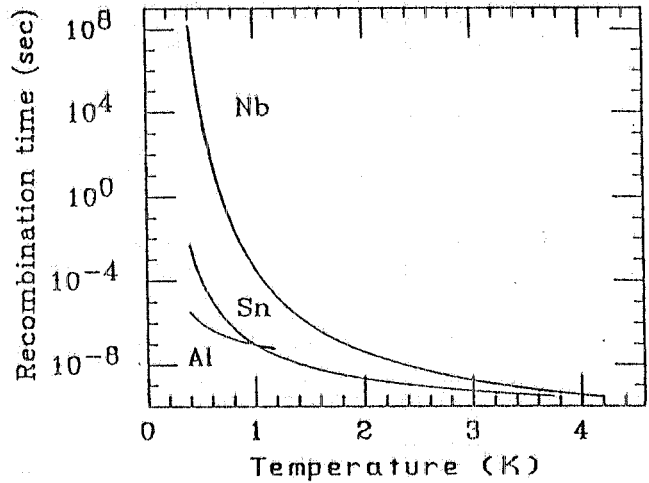


FIG. 2 - Temperature dependance of the recombination time for Al, Sn and Nb.

As far as the filter is concerned, in this paper we consider only a single low pass filter with a characteristic time τ_F which determines the bandwidth (BW) of the amplifier. In a first analysis τ_F can be taken equal to τ_R that is the effective quasi particle relaxation time. In this way we have

$$\text{BW} = 1/\tau_R = 1/\tau_r + 1/\tau_t \quad (3)$$

Consequently, the characteristic detector time $\tau_D = C_D R_D$ can be comparable with the signal times τ_r and τ_t . By a comparison of eq. 1 and 2 it is shown that the condition $\tau_R \gg \tau_t$, usual for conventional detectors, is satisfied only when R_{NN} or T are low.

2.2. - Equivalent circuit of STJ

Neglecting true leakage current, in the limit of low p , that is $\tau_t \gg \tau_r$, the tunnel current, unperturbed state, has the following expression

$$I = f(V,T)/R_{NN} = g(V,T)p \quad (4)$$

where, following eq. 1, both R_{NN} , the static resistance $R_S = V/I = 1/G_S$, and the dynamic resistance $R_D = dV/dI = 1/G_D$ are inversely proportional to the tunnel probability $p = 1/\tau_t$ and the functions $f(V,T)$, $g(V,T)$ depend on the junction electrodes.

When the ionizing particle hits the junction the quasi-static I-V characteristic can be written by the following expression:

$$I' = g'(V,T,n'(E,t)) \cdot p \quad (5)$$

where $n'(E,t)$ describes the time dependence of the energy distribution of the excess quasi particles with energy E . In a first approximation we can neglect the details of the energy distribution, and the current I' is only a function of the total number of quasi particles $n'(t) = \int_0^{\infty} n'(E,t) dE$.

If the thermal quasi particle current is independent of the excess quasi particle current (i.e. for a voltage bias), in some intermediate voltage range between zero and the gap sum $\Delta_1 + \Delta_2$, the signal current $\Delta I = I' - I$ is given by:

$$\Delta I = pn'(0) \exp(-t/\tau_p) = \Delta I_0 \exp(-t/\tau_p) \quad (6)$$

Following eq. 6, the excess current is proportional to p , i.e. to the inverse of the resistance. In eq. (6) we have neglected gap reductions caused by the excess quasi particles. These phenomena and others that can enable multiplication or "avalanche like" processes are neglected in the present paper.

In this approximation, the circuit of STJ is shown in Fig. 3.

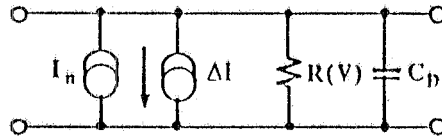


FIG. 3 - Equivalent circuit of the STJ.

Due to the presence of the bias network and amplifier the effective current or voltage signals are strictly related to the intersection of the load line with the perturbed and the unperturbed I-V characteristics. In dynamic conditions the true dynamic load line must be considered, taking into account the presence of cables, input impedance of the amplifier, junction capacitance and possible matching impedance network.

3. - S/N RATIO VERSUS p AND T WITH VOLTAGE AND CHARGE AMPLIFIER

3.1. - Voltage Signal

From the model of Fig. 3, the voltage measured with a voltage amplifier is maximized by a current polarization and is given as following:

$$\Delta V = R_D \Delta I_0 [\exp(-t/\tau_R) - \exp(-t/\tau_D)] (1 - \tau_D/\tau_R) \quad (7)$$

From eq. (7), for very low values of the tunnel probability p , i.e. for R_D values high enough to get $\tau_D \gg \tau_R$, the detector time τ_D cuts the voltage signal. As p increases the dynamic resistance decreases; as we get $\tau_D < \tau_R$, the capacitor effects can be neglected and the voltage signal is:

$$\Delta V = \Delta I R_D \quad (8)$$

From eq. (7), the maximum voltage signal $V(t)$ is:

$$\Delta V(t^*) = R_D \Delta I_D (\tau_R / \tau_D) \exp \tau_D / (\tau_D - \tau_R) \quad (9)$$

in the limit of $\tau_D \rightarrow \infty$ (i.e. for $\tau_D \ll \tau_R$) we get

$$\Delta V(t^*) = \Delta I_0 / C_D \tau_R \quad (10)$$

In the opposite case of $\tau_D \rightarrow 0$ we get

$$\Delta V(t^*) = R_D \Delta I_0 \quad (11)$$

3.2. - Voltage noise

The spectral distribution of the current noise in a STJ is given by (5):

$$I_n^2(\omega) = (e/\pi) I(v) \coth(eV/2K_B T) \quad (12)$$

which can be written as:

$$I_n^2(\omega) = F(V, T) / (\pi R_S) \quad (13)$$

where $F(V, T)$ in the two limits $eV/K_B T \gg 1$ and $eV/K_B T \ll 1$ is respectively equal to eV and

$2K_B T$, and is respectively equivalent to a shotky noise and to a Johnson noise generated by the static resistance R_S .

The spectral density of the voltage noise is given by:

$$V_n^2(\omega) = I_n^2(\omega) R_D^2 = F(V, T) / \pi R_S \cdot R_D^2 \quad (14)$$

From eqs. (13) and (14) $V_n^2(\omega)$ is proportional to $1/p$.

The S/N ratio is given just combining eqs. (9), (13), (14) and we will get:

$$S/N = \Delta I_0 \frac{\sqrt{R_S} \pi}{\sqrt{F(V, T)}} \left[(\tau_R / \tau_D)^{\tau_D / (\tau_D - \tau_R)} \right] \sqrt{\tau_R} \quad (15)$$

In the case of $\tau_D \rightarrow 0$ we will get:

$$S/N = pn'(0) \frac{\sqrt{\pi R_S}}{\sqrt{F(V, T)}} \cdot \tau_R^{1/2} \quad (16)$$

In the case of $\tau_D \rightarrow \infty$, we will get:

$$S/N = \frac{pn'(0)}{C_D R_D} \frac{\sqrt{\pi R_S}}{\sqrt{F(V, T)}} \cdot \tau_R^{3/2} \quad (17)$$

At a fixed T value, starting from low values of p, by eq. (17) the S/N increases with p so that the optimum conditions are out of the high τ_D limit.

Following eq. (16), the S/N increases with p but it saturates as $\tau_D > \tau_R$.

For a fixed high value of p, as the temperature decreases, following eq. (16), the S/N decreases but after it saturates. Moreover, as R_D increases we will go in the high τ_D limit and, by eq. (17), the S/N decreases with the temperature.

3.3. - Charge Amplifier

In the case of a charge amplifier coupled to the junction, the dynamic load line is determined by the low value of the input impedance of the amplifier. The current signal will be optimized under the condition of a voltage dynamic polarization, i.e. by a dynamic load $R_L^D \ll R_D$. Moreover, the detector plus the bias network must be a current source in comparison with the amplifier. As a consequence it holds: $R_B > R_S(V) \gg |Z_i|$.

By simple integration of eq. (6) the charge signal is equal to:

$$\Delta Q = pn'(0) \tau_R \quad (18)$$

Then, as p increases, ΔQ at the beginning is proportional to p , but as $\tau_i < \tau_r$, ΔQ saturates to the limit value $\Delta Q = n'(0)$. On the other hand, by eq. (13), the spectral density of the current noise increases indefinitely with the increase of p . The S/N ratio has the following expression:

$$S/N = pn'(0)\tau_R[\pi R_S/F(V,T)BW]^{1/2} = pn'(0)[\pi R_S/F(V,T)]^{1/2}\tau_R^{3/2} \quad (19)$$

which shows its maximal value for $p=1/2\tau_r$. By substitution in eq. (18) for $eV/K_B T \gg 1$, we obtain:

$$(S/N)_{\max} = n'(0)[2\tau_r\pi/(eI)]^{1/2}/3^{3/2} \quad (20)$$

In the limit $eV/K_B T \ll 1$ we get:

$$(S/N)_{\max} = n'(0)[\tau_r\pi R_S/(K_B T)]^{1/2}/3^{3/2} \quad (21)$$

so that in the two limits the S/N ratio increases respectively by decreasing the current bias value and by increasing the value of the static resistance of the working point.

In the case of a charge amplifier coupled to the junction the characteristic detector time τ_D doesn't play any role. Then, with the decrease of the temperature we get:

- a) an increase of the signal, which saturates to its limit value $\Delta Q=n'(0)$;
- b) a decrease of the spectral density of the current noise;
- c) a decrease of the BW, which saturates to its limit value $1/\tau_r$.

In this way, following eq. (19), the S/N increases indefinitely with the decrease of the temperature.

4. - CONCLUSIONS

In conclusion, we have supposed ideal STJ, without true leakage currents, perturbed in an uniform way by ionizing particles, coupled to a noiseless bias network and to noiseless ideal voltage or charge amplifiers. By using a voltage amplifier the signal to noise ratio has the following behaviour:

- a) the S/N increases as the tunnel probability increases but it saturates;
- b) the S/N shows a maximum with the decrease of the temperature.

In the case of a charge amplifier coupled to the junction we have:

- a) the S/N shows a maximum as a function of p ;
- b) the S/N increases indefinitely as the temperature decreases.

We believe that our analysis in ideal conditions is only the starting point to optimize the measurements conditions in practical experimental set up. In order to obtain more significative results, the effects of non ideal amplifiers has to be considered.

Moreover, the optimization of the S/N leads to recombination times longer or of the same order of magnitude of the tunnel time.

If the condition $\tau_1 \ll \tau_r$ is reached, other problems can arise. In fact, in the usual lowest order perturbative analysis of the tunnel current, the tunnel process doesn't change the equilibrium state of the two films. On the contrary, if $\tau_1 \gg \tau_r$ the tunnel itself perturbs the thermal equilibrium in the superconducting films. In this limit some anomalies in the I-V characteristics should be observed and a higher order perturbative analysis should be done, so that the usual semiconducting model could not be still valid. Neglecting these problems, it should be taken into account that at the arrival of the ionizing particle only a small portion of the junction is strongly excited and the influence of such variation on the global current-voltage (I-V) characteristic should be studied in more details.

As far as the time and spatial evolution of the perturbed state is concerned, the usual non-equilibrium theory has been developed for systems near the equilibrium, homogeneous in the space and not explicitly time dependent. The experimental condition of a STJ under the excitation of a nuclear particle is quite different and just opposite to the three previous items: a state very far from the equilibrium is generated in a very limited spatial region. Later on, the quasi particles diffuse in the surrounding film generating a smaller perturbation into a larger area. Other excitations such as surface plasmons could play some role in the diffusion process, but, up to now, they have not been considered. In particular it should be analysed:

- a) the recombination time for a superconducting state far from the equilibrium;
- b) quasi-particle diffusion dynamics;
- c) effects of space inhomogeneity on the effective quasi-particle recombination time.

Moreover, the presence of disuniformities in the STJ and of true leakage currents can play some important role to the realistic value of intrinsic resolution.

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