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CONFINEMENT AND VACUUM STRUCTURE IN COMPACT QED ON THE LATTICE

V. AZCOITI
INFN, I-00044 Frascati, Italy
and Departamento de Fisica Teorica, Universidad de Zaragoza, E-50009 Zaragoza, Spain

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Using an effective action and assuming that the physical ground state has no structure, we demonstrate that the spectrum of the compact U(1) gauge model has massless (vector and scalar) bosons.

Over the last few years an important amount of effort has been devoted to the study of the dynamical mass generation mechanism in gauge theories with scalar matter fields (Higgs mechanism). Indeed this is at present one of the most interesting problems of theoretical physics since the Higgs mechanism is an essential ingredient of the standard model of electroweak interactions. On the other hand, the study of gauge-Higgs systems on the lattice has shown that the Higgs phase confines in the sense that no free charge can be found in this phase. This result suggests that "gauge" bosons are composite objects [1] and then the confining dynamics would be responsible for the nonvanishing mass of these objects.

The lattice approach provides us with a strong nonperturbative method to compute nonperturbative quantities such as masses of composite objects (hadrons in QCD, Higgs and vector boson masses in gauge-Higgs systems, ...). Using this technique the phase structure of abelian and nonabelian lattice-gauge systems has been explored by Monte Carlo simultions. A confining phase in the strong coupling region appears for all these models. This confining phase seems to be the only phase in the nonabelian pure SU(2) and SU(3) gauge models at zero temperature. In the abelian case, a deconfining transition from the strong coupling to the weak coupling region has been found and numerical evidence for the existency of massless (massive) vector states in the nonconfining (confining) phase has been reported [2]. In this paper we will show how in the most simple pure gauge system with a continuum local gauge symmetry, the mass spectrum of the model contains massless vector and scalar bosons if the physical vacuum is completely symmetric. Our starting model is the compact U(1) lattice-gauge model. The action for this model is

$$S = -\beta \sum_n \sum_{\mu<\nu} \mathrm{Re}\, U_{n\mu\nu} ,$$

$$U_{n\mu\nu} = U_{n\mu} U_{n+\hat{\mu}\nu} U^{\dagger}_{n+\hat{\nu}\mu} U^{\dagger}_{n\nu} , \quad (1)$$

where the summation is over all oriented plaquettes and $\beta = 1/g^2$.

In order to define vacuum expectation values of quantum operators we need to assign an integration measure for the sum over the field configurations. The integration measure associated to action (1) is the Haar measure which is, as is well known, gauge invariant.

The theory given by action (1) has, in addition to the global spacetime and local U(1) symmetries, a global Z_2 symmetry $U_{n\mu} \to U^{*}_{n\mu}$ which is the reflection of the charge conjugation symmetry in the pure gauge formulation.

Now, let us do the following unitary variables change in (1):

$$U_{n\mu} \to \exp(i\alpha_{n\mu})\, U_{n\mu} , \quad (2)$$

where $\alpha_{n\mu}$ are arbitrarily fixed phases. The integration measure is invariant under this change and therefore the partition function Z will be $\alpha_{n\mu}$ independent. Furthermore, we can choose the $\alpha_{n\mu}$

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parameters in such a way that the sum of the four phases around any oriented plaquette in the $\mu\nu$ plane is

$$\alpha_{n\mu}+\alpha_{n+\hat{\mu}\nu}-\alpha_{n+\hat{\nu}\mu}-\alpha_{n\nu}=a_{\mu\nu}\,, \tag{3}$$

and then the action (1) can be written in the new variables as

$$S(a_{\mu\nu})=-\beta\sum_{n}\sum_{\mu<\nu}\mathrm{Re}(\exp(\mathrm{i}a_{\mu\nu})\,U_{n\mu\nu}) \tag{4}$$

with the integration measure $\prod_{n,\mu}\mathrm{d}U_{n\mu}$.

In a finite lattice with periodic boundary conditions, the possible values of the $a_{\mu\nu}$ plaquette phases are restricted by the conditions

$$Na_{\mu\nu}=2\pi m\,,$$

where N is the number of lattice sites in each direction and m an integer number. However, in the thermodynamic limit ($N\to\infty$) we can approximate $a_{\mu\nu}$ by any real number $0\leqslant a_{\mu\nu}\leqslant 2\pi$.

The independence of the partition function on $a_{\mu\nu}$ allows us to integrate $Z(a_{\mu\nu})$ over $a_{\mu\nu}$ since the transfer matrix formalism guarantees the physical equivalence of this new model with the old one. This integration can be made analytically and we get the new effective action

$$\exp(-S_{\mathrm{eff}})=\prod_{\substack{\mu,\nu=1\\ \mu<\nu}}^{4}\sum_{k=0}^{\infty}\frac{\beta^{2k}}{4^k(k!)^2}\left|\sum_n U_{n\mu\nu}\right|^{2k}\,,$$

$$\left|\sum_n U_{n\mu\nu}\right|^2=\left(\sum_n \mathrm{Re}\,U_{n\mu\nu}\right)^2+\left(\sum_n \mathrm{Im}\,U_{n\mu\nu}\right)^2\,. \tag{5}$$

Action (5) has in addition to the spacetime, charge conjugation and local gauge symmetries, an extra "global" symmetry given by the transformation rules

$$U_{n\mu}\to\exp(\mathrm{i}\alpha_{n\mu})\,U_{n\mu}\,, \tag{6}$$

where the $\alpha_{n\mu}$ parameters are restricted to the conditions (3). We call this symmetry "global" in the sense that a symmetry transformation acts over an infinite number of degrees of freedom. The transformations (6) with the conditions (3) have a group structure. The transformed plaquette variable can be written in a simple way as

$$U_{n\mu\nu}\to\exp(\mathrm{i}a_{\mu\nu})\,U_{n\mu\nu}\,. \tag{7}$$

Using this "global" symmetry one can prove that the connected correlations of the Im $U_{n\mu\nu}$ and Re $U_{n\mu\nu}$ operators are identical

$$\langle \mathrm{Im}\,U_{n\mu\nu}\,\mathrm{Im}\,U_{m\mu\nu}\rangle_{\mathrm{c}}=\langle \mathrm{Im}\,U_{n\mu\nu}\,\mathrm{Im}\,U_{m\mu\nu}\rangle$$
$$=\langle \mathrm{Re}\,U_{n\mu\nu}\,\mathrm{Re}\,U_{m\mu\nu}\rangle_{\mathrm{c}}$$
$$=\langle \mathrm{Re}\,U_{n\mu\nu}\,\mathrm{Re}\,U_{m\mu\nu}\rangle\,. \tag{8}$$

Therefore we get a 0^{++} state for each vector state which has a nonzero overlap with the Im $U_{n\mu\nu}$ operator and vice versa. Additionally, an infinite correlation length is associated with action (5). This can be easily seen by doing the strong coupling expansion of the $\langle \mathrm{Re}\,U_{n\mu\nu}\,\mathrm{Re}\,U_{m\mu\nu}\rangle$, $\langle \mathrm{Im}\,U_{n\mu\nu}\,\mathrm{Im}\,U_{m\mu\nu}\rangle$ correlations. The first nontrivial term in this expansion $\frac{1}{8}\beta^2$ does not depend on the distance between the two operators.

In conclusion we construct an effective action for the U(1) lattice-gauge theory which has a "global" symmetry and then we prove that the connected correlations between scalar and vector operators are identical. We also show how this model generates an infinite correlation length so that massless scalar and massless vector states will appear. On the other hand we want to remark that the only way to avoid this result is to kill the step from (4) to (5). This is possible if the physical ground state lacks some of the symmetries of action (1) since in such a case the integration region in (4) will be divided in several disconnected regions and then the step from (4) to (5) is not guaranteed. This result combined with the numerical proof on the existency of massive (massless) vector states in the confining (nonconfining) phase of this model [2], strongly suggests a relation between confinement and spontaneous breaking of some of the global symmetries of action (1).

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