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**NUMERICAL EVIDENCE FOR SPONTANEOUS BREAKING OF CHARGE CONJUGATION
IN THE HIGGS PHASE OF THE STANDARD U(1)-HIGGS MODEL**

V. Azcoiti

INFN, Laboratori Nazionali di Frascati, P.O.Box 13, 00044 Frascati, Italy
and Departamento de Fisica Teorica, Universidad de Zaragoza, 50009 Zaragoza, Spain

J.L. Cortes

Departamento de Fisica Teorica, Universidad de Zaragoza, 50009 Zaragoza, Spain

ABSTRACT

We present numerical evidence for the spontaneous breaking of the charge conjugation symmetry in the Higgs phase of the standard U(1)-Higgs model in the infinite self-coupling limit.

C, P and CP violations in electroweak interactions have been experimentally observed and phenomenologically included in the Standard Model. The origin of such violations is not very clear and this is one of the most unpleasant features of this model. However, it is not excluded that this phenomenon be related to some non trivial structure of the physical ground state and in such a case, non perturbative methods are needed in order to check this possibility.

The lattice approach to Quantum Field Theories (QFT) combined with Monte Carlo simulations are the most readily available mathematical tool for a quantitative study of non perturbative phenomena in QFT. This, together with the fact that the Higgs mechanism is an essential ingredient in the Standard Model for electroweak interactions makes the lattice U(1) gauge-Higgs model¹ the simplest non trivial model where spontaneous breaking of discrete symmetries can be studied.

We present in this paper the first results from a Monte Carlo simulation of the standard U(1)-Higgs model with strong evidence for spontaneous breaking of the charge conjugation symmetry C in the Higgs phase.

The euclidean continuum action for this model is

$$S_c = \int d^4x \{ (D_\mu \phi_c)^* (D_\mu \phi_c) + m_c^2 \phi_c^* \phi_c + \lambda_c (\phi_c^* \phi_c)^2 + 1/4 F_{\mu\nu} F_{\mu\nu} \} \quad (1)$$

where $D_\mu = \partial_\mu + igA_\mu$ is the covariant derivative and $F_{\mu\nu}$ the electromagnetic tensor. This model has a local U(1) symmetry

$$\begin{aligned} \phi_c(x) &\longrightarrow e^{i\beta(x)} \phi_c(x) \\ A_\mu(x) &\longrightarrow A_\mu(x) - 1/g \partial_\mu \beta(x) \end{aligned} \quad (2)$$

and a global Z_2 symmetry

$$\begin{aligned} \phi_c(x) &\longrightarrow \phi_c^*(x) \\ A_\mu(x) &\longrightarrow -A_\mu(x) \end{aligned} \quad (3)$$

which is the C symmetry.

The lattice version of action (1) is

$$\begin{aligned} S = & -\beta \sum_{\text{plaq.}} \text{Re } U_{\text{plaq.}} + \sum_n \{ \rho_n^2 - \log \rho_n + \lambda(\rho_n^2 - 1)^2 \} \\ & - (\kappa/2) \sum_{n,\mu} \rho_n \rho_{n+\mu} \{ \xi_n^+ U_{n\mu} \xi_{n+\mu} + \xi_{n+\mu} U_{n\mu}^+ \xi_n \} \end{aligned} \quad (4)$$

The first term in (4) is the standard Wilson action for the pure gauge theory ($\beta = 1/g^2$) and the scalar field at site n is given by $\phi_n = \rho_n \xi_n$, ξ_n being an element of the gauge group and ρ_n a real number running from zero to infinity. The integration measure for the action (4) is $\prod_n d\rho_n d\xi_n \prod_\mu dU_{n\mu}$ with the Haar measure for the gauge group.

Other useful relations between continuum and lattice variables are

$$\begin{aligned} \phi_c(x) &= (\kappa^{1/2}/a) \phi_n & \lambda_c &= \lambda / \kappa^2 \\ m_c^2 &= (1 - 2\lambda - 8\kappa) / \kappa a^2 \end{aligned} \quad (5)$$

One of the advantages in working with the lattice regularization scheme is that we don't need to fix the gauge. In fact, action (4) can be written using only gauge invariant variables in this way

$$S = -\beta \sum_{\text{plaq.}} \text{Re } W_{\text{plaq.}} + \sum_n \{ \rho_n^2 - \log \rho_n + \lambda(\rho_n^2 - 1)^2 \} \\ - \kappa \sum_{n,\mu} \rho_n \rho_{n+\mu} \text{Re } W_{n\mu} \quad (6)$$

where $W_{n\mu} = \xi_n + U_{n\mu} \xi_{n+\mu}$. Notice that the integration measure is invariant under this change of variables because of the gauge invariance of the Haar measure.

The equivalent to the action (6) in the continuum formulation is

$$S_c = \int d^4x \{ m_c^2 \rho_c^2 + \lambda_c \rho_c^4 + \partial_\mu \rho_c \partial_\mu \rho_c + g^2 \rho_c^2 B_\mu^2 + 1/4 F_{\mu\nu} F_{\mu\nu} \} \quad (7)$$

where $B_\mu = A_\mu + 1/g \partial_\mu \alpha$ is the gauge invariant vector field, $\phi_c(x) = \rho_c(x) \exp(i\alpha(x))$ and $F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$. The action (6) can be regarded as the discretized non perturbative regularization of (7) with $W_{n\mu} = \exp(igaB_\mu(n))$.

The only internal symmetry when we work in the gauge invariant formulations (6), (7) is $W_{n\mu} \rightarrow W_{n\mu}^*$ and $B_\mu \rightarrow -B_\mu$ respectively, which is just the reflection of the C symmetry (and of all combinations of C with a gauge transformation) in this formulation. If we call this symmetry transformation C, physical states will be C-invariant if and only if they are C-invariant.

In the numerical simulation we have approximated U(1) by Z(12) and we work in the $\lambda \rightarrow \infty$ limit. The effective action in this limit can be written as

$$S = -\beta \sum_{\text{plaq.}} \text{Re } W_{\text{plaq.}} - \kappa \sum_{n,\mu} \text{Re } W_{n\mu}$$

Adding to this action a term of the form $-h \sum_{n,m} \text{Im } W_{nm}$ which breaks explicitly the C symmetry we have computed the vacuum expectation values $\langle \text{Im } \prod_\mu W_{n\mu} \rangle$, $\langle \text{Im } W_{n\mu} \rangle$ and $\langle \text{Re } W_{n\mu} \rangle$ for different values of h , β and κ doing eventually a linear extrapolation to $h = 0$. We use the $\text{Im } \prod_\mu W_{n\mu}$ operator as order parameter instead of $\text{Im } W_{n\mu}$ because of two reasons: i) the $\text{Im } W_{n\mu}$ operator has no scalar projection, ii) it can be analytically proved that $\langle \text{Im } W_{n\mu} \rangle$ and $\langle \text{Re } W_{n\mu} \rangle$ are related by the equation

$$\langle \text{Im } W_{n\mu} \rangle = h / \kappa \langle \text{Re } W_{n\mu} \rangle$$

and therefore, $\lim_{h \rightarrow 0} \langle \text{Im } W_{n\mu} \rangle = 0$.

In Fig. 1 we show the numerical results for $\langle \text{Im } \prod_\mu W_{n\mu} \rangle$ at $h = 0$, $\beta = 1.15$ and several values of κ . All the points in this figure have been obtained from a linear extrapolation of the results at $h = 0.08$ and $h = 0.15$. The dotted points in the figure are results from an 8^4 lattice and each point is the extrapolated value of $\langle \text{Im } \prod_\mu W_{n\mu} \rangle$ averaged over 22000 MC iterations with 2000 thermalization sweeps. The two crossed points are the corresponding extrapolated values in a 4^4 lattice with 100000 MC iterations.

The observation of Fig. 1 strongly suggests the existence of two phases: i) a broken phase in

the large k region and ii) an unbroken phase in the small k region. Again the results in the 4^4 lattice support this conclusion in the sense that in the unbroken phase finite size effects are negligible whereas in the broken phase they become quite relevant with a high increase of the expectation value of the order parameter when we go from the 4^4 to the 8^4 lattice.

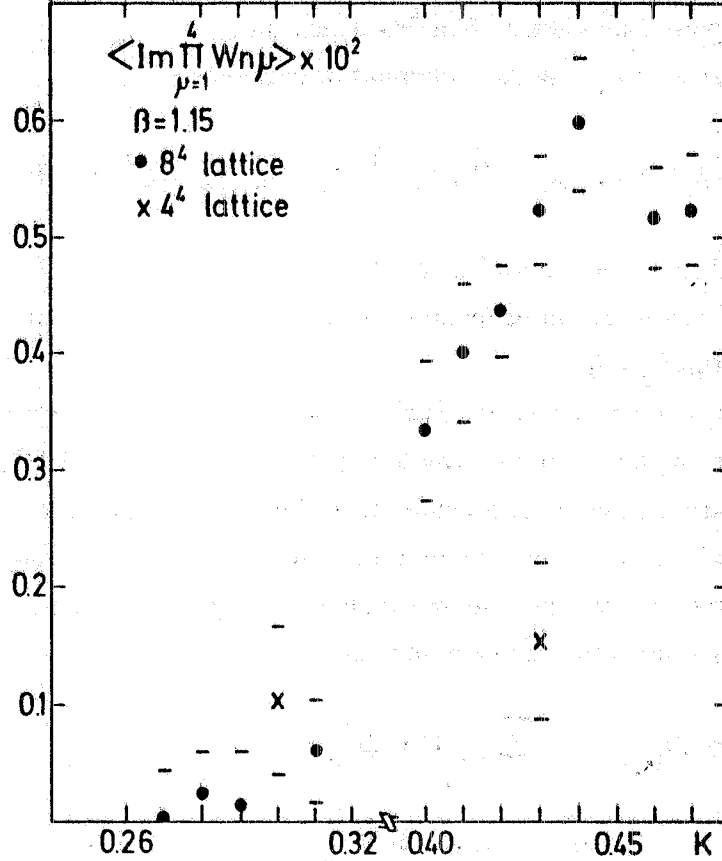


FIG. 1- Numerical results for $\langle \text{Im} \prod_{\mu} W_{n\mu} \rangle$ at $h = 0$, $\beta = 1.15$ against κ . The dotted, crossed points have been obtained from 8^4 , 4^4 lattices respectively.

We have also computed the expectation value of $\langle \text{Im} \prod_{\mu} W_{n\mu} \rangle$ at $\beta=0.75$, $\kappa = 0.3$ and $h = 0.08, 0.15$ extrapolating it to $h = 0$. The surprising result was that the 4^4 and 8^4 predictions were compatible and both indistinguishable from zero. The exciting thing about this result lies in the fact that it suggests a different realization of the C symmetry in the confined and Higgs phases of the model ¹, giving us a criterion to distinguish both phases.

Our numerical results for $\langle \text{Im} W_{n\mu} \rangle$ extrapolated by a linear fit to $h=0$ were always compatible with zero in the Higgs and confined phases but an anomalous behavior (negative extrapolated values) was observed for this quantity in the Coulomb phase. The origin of this anomalous behavior in the Coulomb phase can be easily understood as a consequence of the abrupt change in the slope of $\langle \text{Im} W_{n\mu} \rangle$ at $h = 0$ when we cross the transition line from the Higgs to the Coulomb phase. Indeed, the existence of massless vector states in the Coulomb phase ² implies that $a_0 = \partial/\partial h \langle \text{Im} W_{n\mu} \rangle_{h=0}$ will be divergent in the thermodynamic limit. In Table I we report the numerical values of a_0 obtained from a fit of $\langle \text{Im} W_{n\mu} \rangle = a_0 h + a_1 h^3$. The first and second

rows in this table are the results from 4^4 , 8^4 lattices respectively. One can see from these data that the change in a_0 when we go from the 4^4 to the 8^4 lattice is a 10, 20 percent in the Higgs and confined phases against a 100 percent in the Coulomb phase.

Finally and as a further check of our program, we summarize in Table II some of our numerical results for R_{exp} against the theoretical value R , where R is the ratio

$$R = \langle \text{Im } W_{\eta\mu} \rangle / \langle \text{Re } W_{\eta\mu} \rangle = h / \kappa.$$

The agreement between theoretical and experimental results is quite good.

TABLE I - Numerical results for the slope a_0 of $\langle \text{Im } W_{\eta\mu} \rangle$. The first and second rows are results from a 4^4 , 8^4 lattice respectively.

	$\beta = 1.15, \kappa = 0.43$	$\beta = 1.15, \kappa = 0.30$	$\beta = 0.75, \kappa = 0.30$
a_0	1.17(2)	0.56(2)	0.52(2)
	1.24(1)	1.09(1)	0.65(1)

TABLE II - Comparison of the experimental results for R against the theoretical results at several values of h and κ .

	$h = 0.08, \kappa = 0.47$	$0.15, 0.47$	$0.08, 0.32$	$0.08, 0.30$
$R_{\text{exp.}}$	0.1702(8)	0.3184(9)	0.2505(15)	0.2662(21)
R	0.1702	0.3191	0.2500	0.2667

In conclusion, we present numerical evidence for spontaneous breaking of the charge conjugation symmetry in the Higgs phase of the standard U(1)-Higgs model in the infinite self-coupling limit. Our numerical results suggest also a different realization of the C symmetry in the confined and Higgs phases of the model and in such a case, this would provide us with an order parameter to differentiate these two phases. On the other hand, it should be noticed that our result strongly contrasts with the expectations based on the semiclassical approach where no spontaneous breaking of the C symmetry is expected in a theory with only one scalar field.

In our opinion it would be very interesting to check how our results depend on l . If similar results are obtained, then the dynamical mechanism we have studied could have implications on the understanding of discrete symmetries in the Standard Model.

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