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RADIATIVE CORRECTIONS TO $e^+ e^-$ REACTIONS AT LEP/SLC ENERGIES

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ABSTRACT

Precision tests of the standard electro-weak model at LEP/SLC energies need the precise evaluation of e.m. radiative corrections, well beyond the one-loop level. The theoretical situation concerning the main processes of the experimental interest is discussed in detail.

In addition to exact formulae to $O(\alpha)$ from virtual and real photon emission, a thorough treatment of higher order effects is presented which sums up the full series of double leading logarithms and some classes of single logarithms. This includes the vacuum polarization effects, the multiple soft photon effects and the hard and collinear terms associated to the detection of charged particles in electromagnetic calorimeters. The results are presented by means of simple analytic formulae of immediate phenomenological application. Some numerical results are also given.

1. - INTRODUCTION

The standard model of electroweak interactions of Glashow, Salam and Weinberg(1) is in agreement with all the experimental information which is available so far. It has had dramatic success in predicting the features of the weak neutral current and the correct values of the masses of the charged and neutral intermediate vector bosons, recently discovered(2) at the CERN proton-antiproton collider. Yet the full experimental proof of the theory is far from being completed. Furthermore a large number of fundamental parameters is not determined by theory, including the mass of the Higgs boson. Indeed the Higgs mechanism of spontaneous symmetry breaking is the most peculiar aspect of the model which has to be tested, particularly in connection with the high energy behaviour of the theory. Many theoretical ideas have been developed(3) to extend the standard model to distances much shorter than those we currently explore. In spite of the fact that none of the proposed extensions provides an overall solution to the various problems of the standard model, there is a common belief that new physics is to expected in the range of energy (0.1-1) TeV.

Next operation of the new colliders LEP and SLC will provide a unique opportunity for testing the properties of the electroweak model beyond the tree-level[4]. Indeed the measurement of various asymmetries in $e^+e^- \rightarrow \mu^+\mu^-$, $\tau^+\tau^-$, ..., in particular in the vicinity of the Z_o where statistics are expected to be good, and/or the observation of longitudinal polarization-either in initial-state electron beams or in final state τ polarization - will test the standard model at the one loop level and possibly reveal the existence of a new level of physics. The latter would give observable effects via radiative corrections if the associated new particles are too heavy to be produced directly.

Experimental investigation of these effects will provide, however, precision tests of the theory only if QED radiative corrections, associated with radiation of real or virtual photons and dependent upon the details of the experimental arrangements, are under control at the level of $\lesssim 1\%$[5]. In fact this is the order of magnitude of the weak effects which are expected to be measured. Indeed the most relevant higher order corrections can be directly reabsorbed into a redefinition of the basic parameters (M_Z, Γ_Z, $\sin^2\theta_W$) and the renormalization of the fine structure constant. The leftover corrections lead then to observable effects of order 1%.

On the other hand pure e. m. corrections, which add no additional theoretical information, are expected[6] to be rather important, especially in the vicinity of the Z_o boson and change sizably the naive expectations. Indeed first-order corrections, for example, reduce the Z_o peak cross section by more than 50%, or shift the zero in the forward-backward asymmetry by about ($\pm$300) MeV, for an energy resolution of (10^{-1}-10^{-2}).

In addition, one must evaluate the corrections due to soft photon emission, which become increasingly important as the energy increases, to all orders in α. Usually, i. e. for non-resonant cross sections, this leads[7] to the exponentiated form $\propto \exp\{(4\alpha/\pi)\ln(2E/m)\ln(\Delta\omega/E)\}$, where m is the electron mass, 2E is the c.m. energy and $\Delta\omega$ is the resolution of the experiment. In the case of resonance production, the above factor is modified, and for a very narrow resonance-like J/ψ -the correction becomes[8] $\exp\{(4\alpha/\pi)\ln(2E/m)\ln(\Gamma/M)\}$. Physically, this is understood by saying that the width Γ provides a natural cut-off in damping the energy loss in the initial state. For the case of the Z_o boson, where neither of the preceding cases applies ($\Gamma/M \sim \Delta\omega/E$), the soft correction is[9] a complicated function of to E, M, $\Delta\omega$ and Γ raised to the power $(4\alpha/\pi)\ln(2E/m)$, of the order of 50%.

Nowaday charged particles are often detected in electromagnetic calorimeters which do not discriminate between the particle and the accompanying collinear photons. The effect of emission of hard and collinear photons becomes increasingly important at high energies and its contribution has to be included in the observable cross sections[10]. In perturbation theory, as well known, this corresponds to large logarithms associated to the mass singularities[11] of the emitting charged particles. Then introducing the angular resolution δ of the calorimeter, the final correction includes powers of logarithmic terms of the type $(\alpha/\pi)\ln\delta^2\ln(\Delta\omega/E)$ or $(\alpha/\pi)\ln\delta^2$. In Bhabha scattering, for example, the inclusion of such terms is crucial to obtain a high precision monitor of the beam luminosity[12,13].

The aim of the present review is to summarize the status of e.m. radiative corrections[14] at

LEP/SLC energies, giving in addition simple analytical prescriptions of immediate experimental applications. We consider the infrared factors to all orders in perturbation theory. All finite terms of order α are then explicitly reported for the main processes of experimental interest. We will not consider polarization effects. Some of them are discussed in Ref. (15). The plan of the paper is the following. In sect. 2 we define our notation and collect the relevant formulae for the Born terms. We also discuss how the most relevant weak corrections can be reabsorbed into the redefinition of the Z_0 mass and width and of $\sin^2\theta_W$, and into the renormalization of the fine structure constant. Real and virtual one-loop corrections are discussed in sect. 3. In particular a very compact general formula for the box diagrams is given. Differential cross section to order α are also obtained. Multiple soft photon effects are summarized in sect.4, and included into the expressions for the observable cross sections in sect. 5, for the reaction $e\bar{e} \rightarrow f\bar{f}$, with $f\bar{f} \neq e\bar{e}$. Collinear hard photon effects are discussed in sect. 6, and corrispondingly the relevant formulae for calorimetric-type experiments are then given. A complete analysis of radiative corrections to Bhabha scattering is presented in sect. 7. These results allow to use the process of $e\bar{e}$ scattering as a high precision monitor of the beam luminosity.

Hard bremsstrahlung corrections are discussed in sect. 8. The basic results for neutrino counting are summarized in sect. 9. A brief discussion of higher order weak effects is then presented in sect. 10. A general discussion of the results presented, including some numerical applications, is finally given in sect. 11

2. - BORN TERMS: NOTATION AND FORMULAE

In the SU(2) ⊗ U(1) standard electroweak model the Z_0 comples to a fermion pair as

$$\mathcal{L}_{Zf\bar{f}} = -M_Z\,(G_F/\sqrt{2})^{1/2}\,\bar{f}\,\gamma_\mu\,[\,(v_f - a_f\,\gamma_5)\,/\sqrt{2}\,]\,f\,Z_\mu \qquad (2.1)$$

where

$$v_f = 2\,(I_3^L + I_3^R)_f - 4\,Q_f \sin^2\theta_W\,,$$
$$a_f = 2\,(I_3^L - I_3^R)_f\,,$$

G_F=(1.16635 ± 0.00002) x10^{-5} GeV^{-2} is the Fermi constant, defined from the precise determination of the muon lifetime, and I_{3f}^L (I_{3f}^R) is the weak isospin of the left - (right-) handed fermion field (1/2) (1−(+) γ_5)f. In the minimal model, which includes only left-handed isospin doublets and right-handed singlets, we have, for three generations,

$$v_e = v_\mu = v_\tau = -1 + 4\sin^2\theta_W \qquad Q_f = -1$$
$$a_e = a_\mu = a_\tau = -1 \qquad (2.2)$$

$v_\nu = a_\nu = 1$ $\qquad$ $Q_f = 0$

$$v_u = v_c = v_t = 1 - (8/3)\sin^2\theta_W \qquad Q_f = 2/3 \qquad (2.2)$$
$$a_u = a_c = a_t = 1$$

$$v_d = v_s = v_b = -1 + (4/3)\sin^2\theta_W \qquad Q_f = -1/3$$
$$a_d = a_s = a_b = -1$$

In the above eqs. M_Z and $\sin^2\theta_W$ represent the physical mass of the Z_o and weak mixing angle related through

$$\sin^2\theta_W = 1 - M_W^2 / M_Z^2 \qquad (2.3)$$

where M_W is the physical W mass. At the tree level the intermediate vector boson masses are given by

$$M_W = \frac{\mu}{\sin\theta_W}; \quad M_Z = \frac{\mu}{\sin\theta_W \cos\theta_W} \qquad (2.4)$$

with

$$\mu = \left(\frac{\pi\alpha}{\sqrt{2}\, G_F}\right)^{1/2} = 37.281 \text{ GeV}.$$

Radiative corrections modify the above relations. As discussed in great detail elsewhere(16,17), the simplest and most natural choice to minimize the uncertainties in the theoretical predictions of higher order effects is to define $\sin^2\theta_W$ through the intermediate vector boson masses, namely eq. (2.3) is considered exact to next order of perturbation theory. Then the theoretical prediction of higher order corrections to physical quantities at the vector boson scale are particularly simple and contain the large logarithms associated to strong interactions effects through the renormalization of the fine structure constant only. More precisely the above eqs. (2.4) are modified as(17)

$$M_W = \frac{\mu}{\sin\theta_W}\,\frac{1}{(1-\Delta r)^{1/2}}, \quad M_Z = \frac{\mu}{\sin\theta_W \cos\theta_W}\,\frac{1}{(1-\Delta r)^{1/2}} \qquad (2.5)$$

where Δr contains the $O(\alpha)$ corrections and it is determined quite accurately. In particular, one finds

$$\Delta r = \text{Re}\, \delta_{VP}(M_Z^2) + \delta\,,$$

where Re $\delta_{VP}(M_Z^2)$ represents the largest correction, due to the photon vacuum polarization, which defines the e.m. running coupling constant at the scale $s = M_Z^2$, i.e. (see sects. 2-3)

$$\hat{e}^2(s) = \frac{e^2}{1\text{-Re}\ \delta_{VP}\ (s)} \tag{2.6}$$

The remaining quantity δ represents the genuine weak correction and it depends on $\sin^2\theta_W$, the Higgs and the top masses. This will be discussed in more detail in sect. (10). Quantitatively one finds[17] $\Delta r \simeq 0.07$, leading to an important shift in the value of the vector boson masses[18].

At the three level (Δr=0) eq.(2.1) simply reduces to

$$\mathcal{L}_{Zf\bar{f}} = \frac{-e}{2\sin(2\theta_W)} \bar{f}\gamma_\mu(v_f - a_f\gamma_5)\, f\, Z_\mu \tag{2.1'}$$

which gives, for the partial widths:

$$\Gamma(Z \to f\bar{f}) = \frac{\alpha M_Z}{12\sin^2(2\theta_W)}\ (v_f^2 + a_f^2) \tag{2.7}$$

As shown in ref. (19) the above equation also suffers large radiative corrections. They can be absorbed, however, into the vector boson mass by a suitable parametrization of the lowest order compling, as in eq. (2.1). To illustrate this point, let us consider for simplicity the decay rate $\Gamma(Z\to \nu\nu)$. To zeroth order we have, from eqs. (2.4) and (2.7),

$$\Gamma_o(Z\to \nu\bar{\nu}) = \frac{\alpha\,\mu}{24\sin^3\theta_W\cos^3\theta_W}\ .$$

Computation of one-loop corrections gives[19,20]

$$\Gamma(Z\to \nu\bar{\nu}) = \Gamma_o\,(Z\to \nu\bar{\nu})\ [1 + (3/2)\,\Delta r + \varepsilon\,(\nu\bar{\nu})]$$

where Δr is defined above, and $\varepsilon(\nu\nu)$ is the remaining weak correction. Recalling eq. (2.5), we see that the above result can be recast in the form

$$\Gamma(Z\to \nu\bar{\nu}) = \frac{M_Z^{\ 3}}{12\pi}\ \frac{G_F}{\sqrt{2}}\ [1 + \varepsilon(\nu\bar{\nu})],$$

with $\varepsilon(\nu\bar{\nu}) \sim o\ (0.1\%)$.

More generally the Z decay width can be then written as

$$\Gamma_Z \equiv \Gamma(Z\to\text{all}) \simeq \frac{M_Z^{\ 3}G_F}{24\,\pi\sqrt{2}} \{[1 + (1-4\sin^2\theta_W)^2]\, N_l + 2\, N_\nu \tag{2.8}$$

$$+ 3[1+(1-8/3\ \sin^2\theta_W)^2]N_u + 3[1+(1-4/3\ \sin^2\theta_W)^2]\, N_d\}$$

where N_l, N_ν, N_u and N_d are the numbers of charged leptons, neutrinos, charged 2/3 and −1/3 quarks respectively, with masses appreciably smaller than $M_Z/2$. Using $\sin^2\theta_W = 0.216$, Δr=0.07 and $N_l = N_\nu = N_u = N_d = 3$ eqs. (2.5) and (2.8) give $M_Z = 94$ GeV, $\Gamma(Z\to\nu\bar{\nu}) = 0.18$ GeV and $\Gamma_Z \simeq 3$ GeV, with the ratios

$$\Gamma(Z\to\nu\bar{\nu}) : \Gamma(Z\to l\bar{l}) : \Gamma(Z\to u\bar{u}) : \Gamma(Z\to d\bar{d}) \approx 2 : 1.02 : 3.54 : 4.52.$$

For the reaction

$$e^-(p_1) + e^+(p_2) \to f\,(p_3) + \bar{f}\,(p_4) \qquad\qquad (f\neq e^-)$$

we define

$$\begin{aligned} s &= (p_1+p_2)^2 = 4E^2 = W^2 \\ z &= \cos\theta = \hat{p}_1\cdot\hat{p}_3 = \hat{p}_2\cdot\hat{p}_4, \qquad a = \sin\theta/2, \qquad b = \cos\theta/2 \\ t &= (p_1-p_3)^2 = -s\,(1-z)/2 \\ u &= (p_4-p_1)^2 = -s\,(1+z)/2\,. \end{aligned} \tag{2.9}$$

It is useful to introduce a reference cross section, which we take to be, as usual,

$$\sigma_{pt} \equiv \sigma_{QED}(e\,\bar{e} \to \mu\,\bar{\mu}) = \frac{4\pi\alpha^2}{3s} \simeq \frac{87}{s(\mathrm{GeV}^2)}\,\mathrm{nb} \tag{2.10}$$

The Born graphs are indicated in Fig. 1. With the same notation we have

$$\begin{aligned} M_o^{QED}(s) &= -(e^2/s)\, Q_f\, J_\mu(s)\, J'_\mu(s), \\ M_o^{RES}(s) &= -\frac{e^2}{s-M^2_R}\,\frac{1}{4\sin^2(2\theta_W)}\,[g_V J_\mu(s) + g_A A_\mu(s)]\,[\,g_V^f J'_\mu(s) + g^f_A A'_\mu(s)] \end{aligned} \tag{2.11}$$

where $(g_V, g^f_V) \equiv (v_e, v_f)$, $(g_A, g_A^f) \equiv -(a_e, a_f)$, $M^2_R = M^2_Z - i\,M_Z\,\Gamma_Z$ and

$$\begin{aligned} J_\mu(s) &= \bar{v}\,(p_2)\,\gamma_\mu\, u\,(p_1) & A_\mu(s) &= \bar{v}\,(p_2)\,\gamma_\mu\gamma_5\, u\,(p_1) \\ J'_\mu(s) &= \bar{u}\,(p_3)\gamma_\mu\, v\,(p_4) & A'_{\bar{\mu}}(s) &= u\,(p_3)\,\gamma_\mu\gamma_5\, v\,(p_4) \end{aligned}$$

By a suitable redefinition of the effective lowest order couplings, the Born amplitudes (2.11) can be parametrized in such a way that the higher-order corrections are free, for all values of s, from the large contribution associated to the vacuum polarization. This can be done by replacing e^2 by the $\hat{e}^2(s)$, the e.m. running compling constant at the scale s (eq. 2.6), and $e^2/\sin^2\theta_W\cos^2\theta_W$ by $e^2M^2_Z/\mu^2 \sim O(M^2_ZG_F)$, as follows

$$M_o^{QED}(s) = \frac{-\hat{e}^2(s)}{s} Q_f J_\mu(s) J'_\mu(s),$$

$$M_o^{RES}(s) = -\frac{\sqrt{2}\,G_F M^2_Z}{4}\frac{1}{s-M^2_R}[g_V J_\mu(s) + g_A A_\mu(s)][g^f_V J'_\mu(s) + g^f_A A'_\mu(s)] \tag{2.12}$$

In the following it will be implicity assumed that the Born cross sections, based on the amplitudes (2.11), can be implemented according to eqs. (2.12).

Then, in absence of polarization of the initial beams, the lowest order cross sections are

$$\frac{d\sigma^{QED}}{d\Omega} = \frac{\alpha^2}{4s}(Q_f^2)(1+z^2)$$

$$\frac{d\sigma^{INT,V}}{d\Omega} = \frac{\alpha^2}{4s}(-Q_f)(1+z^2)(2\,\mathrm{Re}\,\chi(s))\,r_V$$

$$\frac{d\sigma^{INT,A}}{d\Omega} = \frac{\alpha^2}{4s}(-Q_f)(2z)(2\,\mathrm{Re}\,\chi(s))\,r_A \tag{2.13}$$

$$\frac{d\sigma^{RES}}{d\Omega} = \frac{\alpha^2}{4s}\left[(1+z^2)\frac{(v^2_f+a^2_f)}{(v^2_e+a^2_e)} + 8z\,r_V\,r_A\right]|\chi(s)|^2$$

where

$$r_V = \frac{v_e\,v_f}{v^2_e+a^2_e} \qquad r_A = \frac{a_e\,a_f}{v^2_e+a^2_e}$$

and, using eq. (2.4),

$$\chi(s) = \frac{M_Z^2}{16\,\mu^2}(v^2_e+a^2_e)\frac{s}{s-M^2_Z+i\,M_Z\,\Gamma_Z}. \tag{2.14}$$

The integrated forward-backward asymmetry, defined as,

$$A_{FB} = \frac{\int_o^1 d\cos\theta\, d\sigma - \int_{-1}^{o} d\cos\theta\, d\sigma}{\int_{-1}^{+1} d\cos\theta\, d\sigma} \equiv \frac{F-B}{F+B}$$

is given by the expression

$$A_{FB} = \frac{3\,[\,(-Q_f)\ \mathrm{Re}\,\chi(s)\ r_A + 2\, r_A\, r_V |\chi(s)|^2]}{2\,[Q_f^2 + 2\,(-Q_f)\,\mathrm{Re}\,\chi(s)\, r_v + (v^2_f + a^2_f)\,|\chi(s)|^2 / (v^2_e + a^2_e)\,]} \tag{2.15}$$

A non-vanishing effect clearly requires $a_e\, a_f \neq 0$.
At the resonance ($s \simeq M^2_Z$) the asymmetry becomes

$$A_{FB} \simeq \frac{3\,(v_e\, a_e)\,(v_f\ a_f)}{(v_e^2 + a_e^2)\,(v_f^2 + a_f^2)}, \tag{2.15'}$$

and because $\sin^2\theta_W$ is close to 1/4, for the process $e^+e^- \rightarrow \mu\bar{\mu}$ one finds $A_{FB}\ (s = M^2_f)$ $\sim 3(1\text{-}4\sin^2\theta_W)^2$, which is very small. Then a clear understanding of the radiative correction is crucial for the experimental determination of the weak parameters.

A better sensitivity to $\sin^2\theta_W$, near the Z_o pole, is provided by the left-right asymmetry $A_{L,R}$, which is defined in case of polarization of the e^- beam.

Indeed $A_{L,R}$ is given by

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}, \tag{2.16}$$

where $\sigma_{L,R}$ are the cross sections for $e^-_{L,R} + e^+ \rightarrow X$, where X can be any channel. Then, for $s \simeq M^2_Z$, one obtains

$$A_{LR} \simeq \frac{2\, v_e\, a_e}{(v_e^2 + a_e^2)} P_e, \tag{2.16'}$$

where P_e is the e^- longitudinal polarization, which is only linear in $(1-4\sin^2\theta_W)$.

For the process of Bhabha scattering, i.e.

$$e^-(p_1) + e^+(p_2) \rightarrow e^-(p_3) + e^+\ (p_4)$$

eqs. (2.11-2.13) have to be extended to include the effect of t-channel exchanges, as shown in Fig. 2.

The corresponding Born terms are

$$M_o^{QED}(t) = \frac{e^2}{t} J_\mu(t)\, J'_\mu(t)$$

$$M_o^{W}(t) = \frac{e^2}{(t-M_z^2)} \quad \frac{1}{4\sin^2(2\theta_w)} [v_e\, J_\mu(t) + a_e\, A_\mu(t)]\,[\, v_e\, J'_\mu(t) + a_e A'_\mu(t) \qquad (2.17)$$

with

$$J_\mu(t) = u(p_3)\,\gamma_\mu\, u(p_1) \qquad A_\mu(t) = u(p_3)\,\gamma_\mu\,\gamma_5\, u(p_1)$$

$$J'_\mu(t) = v(p_2)\,\gamma_\mu\, v(p_4) \qquad A'_\mu(t) = v(p_2)\gamma_\mu\,\gamma_5\, v(p_4) \qquad (2.18)$$

Then it is useful to define the various lowest-order cross section as follows

$$d\sigma_o\,[\,\gamma(s)\,,\gamma(s)] = \left(\frac{\alpha^2}{4s}\right)(1+z^2) \equiv d\sigma_o\,(1)$$

$$d\sigma_o\,[\,\gamma(s)\,,\gamma(t)] = -\left(\frac{\alpha^2}{4s}\right) 2\,(1+z)^2\,/\,(1-z) \equiv d\sigma_o\,(2)$$

$$d\sigma_o\,[\,\gamma(t)\,,\gamma(t)] = \left(\frac{\alpha^2}{4s}\right)[\,2\,/\,(1-z)^2\,][(1+z)^2+4] \equiv d\sigma_o\,(3)$$

$$d\sigma_o\,[\,\gamma(s)\,,Z(t)] = -\left(\frac{\alpha^2}{4s}\right) 2\,\chi'(t)\,(1+z)^2 \equiv d\sigma_o\,(4)$$

$$d\sigma_o\,[\,\gamma(t)\,,Z(t)] = \left(\frac{\alpha^2}{4s}\right) 2\,\chi'(t)\,[\,2\,/\,(1-z)]\,[\,(1+z)^2+4\,(r_V-r_A)] \equiv d\sigma_o\,(5)$$

$$d\sigma_o\,[\,Z(t)\,,Z(t)] = \left(\frac{\alpha^2}{4s}\right) 2\,[\chi'(t)]^2\{\,(1+z)^2\,[1+4\,r_V\,r_A]+4\,[1-4\,r_V\,r_A\,]\,\} \equiv d\sigma_o\,(6) \qquad (2.19)$$

$$d\sigma_o\,[\,Z(s)\,,\gamma(s)] = \left(\frac{\alpha^2}{4s}\right) 2\,[\,\mathrm{Re}\,\chi(s)\,]\,[(1+s)^2\,r_V+2\,z\,r_A] \equiv d\sigma_o\,(7)$$

$$d\sigma_o\,[\,Z(s)\,,\gamma(t)] = -\left(\frac{\alpha^2}{4s}\right) 2\,[\mathrm{Re}\,\chi(s)\,]\,[(1+z)^2\,/\,(1-z)] \equiv d\sigma_o\,(8)$$

$$d\sigma_0\,[\,Z(s)\,,Z(t)] = -\,(\frac{\alpha^2}{4s})\,2\,[\mathrm{Re}\,\chi(s)]\,\chi'(t)\,(1+z)^2\,[1+4\,r_V\,r_A] \equiv d\sigma_0\,(9)$$

$$d\sigma_0\,[\,Z(s)\,,Z(s)] = (\frac{\alpha^2}{4s})\,|\chi(s)\,|^2\,[\,(1+z)^2+8\,z\,r_V\,r_A] \equiv d\sigma_0\,(10)$$

where

$$\chi'(t) = \frac{1}{2}\,\frac{s}{M_Z^2-t}\,(\frac{M_Z}{4\mu})\,(v_e^2+a_e^2),$$

and we have given explicit evidence to s-and t -exchanges.

This completes the list of the formulae for the Born terms.

3. - FIRST ORDER e.m. RADIATIVE CORRECTIONS

3.1. - Soft Photon Bremsstrahlung

In this section we consider the single bremsstrahlung reaction

$$e^-(p_1) + e^+(p_2) \to f(p_3) + \bar{f}\,(p_4) + \gamma(k), \qquad (f \neq e^-)$$

corresponding to Feynman diagrams of Fig. 3.

These will be evaluated in the soft photon approximation, namely under the condition that the maximal energy $\Delta\omega$ carried by the photon, fixed by the experimental resolution, is small compared to the beam energy E. Then the effect of the photon emission on the fermion momenta can be neglected in the numerator the fermion propagators. As well known, the matrix elements then simply factorize in the form

$$M_\mu(1\gamma) = [I_\mu(k) + F_\mu(k)]\,M_0^{QED}(s) + [\,I_\mu(k)\frac{s-M_R^2}{s-2\sqrt{s}\,k-M_R^2} + F_\mu(k)\,]\,M_0^{RES}(s) \qquad (3.1)$$

where

$$I_\mu(k) = \frac{ie}{(2\pi)^{3/2}}\,[\frac{p_{1\mu}}{p_1\cdot k} - \frac{p_{2\mu}}{p_2\cdot k}]$$

$$F_\mu(k) = \frac{ieQ_f}{(2\pi)^{3/2}}\,[\frac{p_{3\mu}}{p_3\cdot k} - \frac{p_{4\mu}}{p_4\cdot k}] \qquad (3.2)$$

are the classical currents associated to the initial and final particles respectively.

We define[8,9] (*) $\Delta \equiv \Delta\omega/E$ and

$$\beta_e = \frac{2\alpha}{\pi}\left[\ln\left(\frac{s}{m_e^2}\right) - 1\right]$$

$$\beta_f = \frac{2\alpha}{\pi} Q_f^2 \left[\ln\left(\frac{s}{m_f^2}\right) - 1\right] \tag{3.3}$$

$$\beta_{int} = -Q_f \frac{4\alpha}{\pi} \ln \mathrm{tg}\ (\theta/2)$$

Then the soft bremsstrahlung cross section corresponds to isotropic emission up to a maximal photon momentum, given by the experimental energy resolution $\Delta\omega$. Regulating the infrared divergence by a small photon mass λ, the results can be expressed as multiplicative corrections to the lowest order cross sections:

$$\frac{d\sigma^{soft}(1\gamma)}{d\Omega} = \frac{d\sigma_{QED}}{d\Omega}\,\delta^{QED}(1\gamma) + \frac{d\sigma_{INT}}{d\Omega}\,\delta^{INT}(1\gamma) + \frac{d\sigma_{RES}}{d\Omega}\,\delta^{RES}(1\gamma) \tag{3.4}$$

with[8,21]

$$\delta^{QED}(1\gamma) = (\beta_e + \beta_f + 2\beta_{int}) \ln(2E/\lambda) + (\beta_e + \beta_f + 2\beta_{int}) \ln \Delta$$
$$- (\alpha/\pi)\, B(m_e^2) - (\alpha/\pi)\, Q_f^2\, B(m_f^2) - (2\alpha/\pi)\, Q_f\, F(a,b) \tag{3.5a}$$

$$\delta^{INT}(1\gamma) = (\beta_e + \beta_f + 2\beta_{int}) \ln(2E/\lambda) + (\beta_f + \beta_{int}) \ln \Delta$$
$$+ (\beta_e + \beta_{int})\, \mathrm{Re}\left\{ \frac{e^{i\delta_R(s)}}{\cos\delta_R(s)} \ln\left[\frac{\Delta(M_R^2 - s)}{M_R^2 - s + \Delta s}\right]\right\}$$
$$- (\alpha/\pi)\, B(m_e^2) - (\alpha/\pi)\, Q_f^2\, B(m_f^2) - (2\alpha/\pi)\, Q_f\, F(a,b) \tag{3.5b}$$

(*) For the effective definition of $\Delta\omega$ in terms of the maximum accollinearity angle of the final particles, see sect 6.

$$\delta^{RES}(1\gamma) = (\beta_e + \beta_f + 2\beta_{int}) \ln (2E/\lambda) + \beta_f \ln \Delta +$$

$$+ (\beta_e + 2\beta_{int}) \ln \left| \frac{\Delta(M_R^2 - s)}{M_R^2 - s + \Delta s} \right| - \beta_e \frac{(M_z^2 - s)}{M_z \Gamma_z} [\, \mathrm{arctg} \left(\frac{\Delta s + M_z^2 - s}{M_z \Gamma_z} \right)$$

$$- \mathrm{arctg} \left(\frac{M_z^2 - s}{M_z \Gamma_z} \right)] - (\alpha/\pi)\, B(m_e^2) - (\alpha/\pi)\, Q_f^2\, B(m_f^2) - (2\alpha/\pi)\, Q_f\, F(a,b). \tag{3.5c}$$

We have defined

$$B(m^2) = (1/2) \ln^2 (s/m^2) - \ln (s/m^2) + \pi^2/3 \,, \tag{3.6}$$

$$F(a,b) = 2(\ln^2 a - \ln^2 b) + Sp\,(b^2) - Sp(a^2), \tag{3.7}$$

with the Spence function

$$Sp(z) = Li_2\,(z) = -\int_0^z \frac{dx}{x} \ln (1 - x). \tag{3.8}$$

Besides the standard correction $\delta^{QED}(1\gamma)$, eqs. (3.5) explicity show how additional contributions are generated from the Z propagator. These additional factors, depending on the parameters of the Z can be casted in a rather compact form, introducing the notations[8,9]

$$\frac{s}{M_R^2 - s} \equiv \frac{s}{M_z \Gamma_z} \sin \delta_R(s)\, e^{i\,\delta_R(s)} \tag{3.9}$$

$$\mathrm{tg}\, \delta_R(s) = \frac{M_z\, \Gamma_z}{M_z^2 - s} \tag{3.10}$$

$$\delta(s, \Delta) = \mathrm{arctg} \left(\frac{\Delta s + M_z^2 - s}{M_z \Gamma_z} \right) - \mathrm{arctg}\, \frac{M_z^2 - s}{M_z \Gamma_z} \tag{3.11}$$

with the arctangent defined between $\pm\, \pi/2$. Then eqs. (3.5 b-c) can be rewritten as

$$\delta^{INT}(1\gamma) = \delta^{QED}(1\gamma) - (\beta_e + \beta_{int})\, \mathrm{Re} \left\{ \frac{e^{i\,\delta_R(s)}}{\cos \delta_R(s)} \ln \left[1 + \frac{\Delta s}{M_z \Gamma_z}\, e^{i\,\delta_R(s)} \sin \delta_R(s) \right] \right\} \tag{3.12a}$$

$$\delta^{RES}(1\gamma) = \delta^{QED}(1\gamma) - (\beta_e + 2\beta_{int}) \ln \left| 1 + \frac{\Delta s}{M_z \Gamma_z} e^{i \delta_R(s)} \sin \delta_R(s) \right|$$

$$- \beta_e \, \delta(s, \Delta) \, \mathrm{ctg} \, \delta_R(s) \; . \qquad (3.12b)$$

The last term in eq. (3.12 b) is responsible for the radiative tail of the resonance. In fact for very narrow resonances - like the J/Ψ - when $s{\sim}M^2$ and $\Delta\omega \gg \Gamma$, $\delta(s,\Delta)$ reduces to $\delta_R(s)$, the usual phase shift of the Breit - Wigner resonance.

Combining eqs. (3.4) with the virtual corrections discussed in the next section, the λ dependence drops out, as usual, and one gets a finite correction to the Born amplitudes.

When the first order corrections are large and negative one must take higher orders also into account. The exponentiation of the leading logarithmic terms will be discussed later.

3.2. - Virtual Corrections

The electromagnetic virtual corrections to the Born amplitudes arise from the vertex corrections (Fig.4), the vacuum polarization (Fig. 5) and box diagrams (Figs. 6-7). The pure QED terms are well known(22). The corrections to the Z exchange have been calculated more recently. Let us discuss them in detail.

A. Vertex corrections

The relevant diagrams are shown in Fig. 4, where, for $s \gg m_e^2, m_f^2$

$$\delta_{VC}(s) \equiv \delta_{VC}^{R}(s) + i\delta^{I}_{VC}(s) = \delta_{VC}(s, m^2_e) + Q^2_f \, \delta_{VC}(s, m^2_f), \qquad (3.13)$$

$$\delta_{VC}(s, m^2_e) = \{ - (1/2) \beta_e \ln (2E/\lambda) + (\alpha/2\pi) [(1/2) \ln^2(s/m^2_e) - \ln (s/m^2_e)]$$

$$+ (3/8) \beta_e + (\alpha/\pi) (\pi^2/3 - 1/4)\} + i (\alpha/\pi)[\pi \ln (2E/\lambda) - (3/4)\pi], \qquad (3.14)$$

and $\delta_{VC}(s, m^2_f)$ is obviously obtained from (3.14) with the substitution ($m_e \rightarrow m_f$), also in β_e. The imaginary part of the vertex corrections δ^I_{VC} does not contribute to the cross section to order α^3.

B - Vacuum polarization

The vacuum polarization part, corresponding to the diagram of Fig. 5, is given in the relativistic limit ($s \gg m^2_i$) by $M_o^{QED} \cdot \delta_{VP}(s)$, with

$$\delta_{VP}(s) \equiv \delta^R{}_{VP}(s) + i\, \delta_{VP}{}^I(s) =$$

$$= (\alpha / 3\pi) \sum_{i=l,q} Q_i^2 [\ln (s/m_i^2) - 5/3] + i\,[(-\alpha/3) \sum_{i=l,q} Q_i^2] \qquad (3.15)$$

The sums extend to all fermion-leptons and quarks-loops, and $Q_l^2=1$, $Q_i^2=4/3$ (up quarks) $Q_i^2=1/3$(down quarks). The imaginary part of the vacuum polarization, unlike the pure QED case, gives an additional contribution to the cross section, interfering with the lowest order resonant amplitude which is now complex. The hadronic vacuum polarization corresponding to light quarks is usually evaluated numerically by means of a dispersion integral over the total hadronic e^+e^- cross section[23]. Furthermore, for heavy quarks, with $s > 4m_i^2$, the first term in the r.h.s. of eq. (3.15), is modified as follows

$$[\ln (s/m_i^2) - 5/3] \rightarrow 2\sqrt{1-(4m_i^2/s)}\,(1+2\,m_i^2/s)\,\ln\,[(\sqrt{s}/2\,m_i)\,(1+\sqrt{1-(4m_i^2/s)})]$$

$$-(4m_i^2/s) - (5/3). \qquad (3.16)$$

As already discussed above, the real part of the renormalized vacuum polarization function $\delta_{VP}(s)$ can be absorbed, to all orders, into the definition of the e.m. running compling constant, namely

$$e^2(1+\delta^R{}_{VP}(s) + \ldots) = e^2/(1-\delta^R{}_{VP}(s)) \equiv \hat{e}^2(s).$$

C. Box diagrams

The two photon box diagrams are shown in Fig. 6.

The corresponding amplitude has been first evaluated by Khriplovich[24], and subsequently given by other authors[25,26]. It can be put in the very simple form[9]

$$M^{QED}_{box}(s) = (2\alpha^2/s)\, Q_f^2 \{J_\mu(s)\, J'_\mu(s)[V_1{}^\gamma + 2\pi\, i\, V_2{}^\gamma] + A_\mu(s)\, A'_\mu(s)\, [A_1{}^\gamma + 2\pi\, i\, A_2{}^\gamma]\} =$$

$$= M_o^{QED}(s)\, \{-(\alpha/2\pi) Q_f [V_1{}^\gamma + 2\pi\, i\, V_2{}^\gamma]\} + M_5^{QED}(s)\, \{-(\alpha/2\pi) Q_f [A_1{}^\gamma + 2\pi\, i\, A_2{}^\gamma]\} =$$

$$\equiv M_o^{QED}(s)\, C^{\gamma\gamma}(s,t) + M_5^{QED}(s)\, C_5^{\gamma\gamma}(s,t) \qquad (3.17)$$

with

$$M_5^{QED} = M_o^{QED}(\gamma_\mu \rightarrow \gamma_\mu\gamma_5)$$

and where

$$V_1^\gamma \equiv V_1^\gamma(s,t) = -8\ln(a/b)\ln(2E/\lambda) - z\,[\,(\ln^2 a/b^4) + (\ln^2 b/a^4)] + [(\ln a/b^2) - (\ln b/a^2)]$$

$$\equiv -8\ln(a/b)\ln(2E/\lambda) + V_{1f}^\gamma$$

$$V_2^\gamma \equiv V_2^\gamma(s,t) = 2\ln\,(a/b) - (1/2)\,z\,[\,(\ln a/b^4) + (\ln b/a^4)\,] - z/(1-z^2)$$

$$A_1^\gamma \equiv A_1^\gamma(s,t) = -z\,[(\ln^2 a/b^4) - (\ln^2 b/a^4)] + [(\ln a/b^2) + (\ln b/a^2)]$$

$$A_2^\gamma \equiv A_2^\gamma(s,t) = -(1/2)z\,[(\ln a/b^4) - (\ln b/a^4)] + 1/(1-z^2) \qquad (3.18)$$

The index f in V_{1f}^γ stands for free of infrared divergences. The interference of $M_{box}^{QED}(s)$ with the lowest order one-photon matrix element $M_o^{QED}(s)$ gives the dominant background to the forward-backward charge asymmetry originated by Z exchange.

The γ–Z box diagrams are shown in Fig. 7.

The corresponding contribution has been first evaluated(9) in the limit $s \sim M^2_Z$, extending the previous computation(8,26) for a resonance with vector coupling only, e.g. J/Ψ. This approximation corresponds to neglect terms of order $(M^2_Z/s)\ln(M^2_Z/s)$ and simplifies the calculation tremendously. In this limit one finds(9):

$$M^{\gamma Z}{}_{box}(s) = M^{RES}{}_o(s) \times \delta^{\gamma Z}{}_{box}(s) \qquad (3.19)$$

with

$$\delta^{\gamma Z}{}_{box}(s) = -(\alpha/2\pi)Q_f\,.$$

$$\cdot\,\{4\ln(b/a)\ln[\,(M^2{}_R - s)^2/(\lambda^2 s)\,] + 2[Sp(a^2) - Sp(b^2)] - 4\,(\ln^2 a - \ln^2 b)\}$$

$$(3.20)$$

Subsequently the complete calculation of the γ-Z box diagrams has been performed in ref. (27). The final result takes a rather long expression and will not be repeated here. More recently however a very compact formula has been derived in ref. (28), which holds for all energies, and in the limit $M_Z \rightarrow \lambda$ reduces to the pure QED $\gamma\gamma$ box result (3.19). With the notations of eqs. (11) it can be written as

$$M^{\gamma Z}{}_{box}(s) = \frac{2\alpha^2 Q_f}{4\sin^2(2\theta_w)} \{[f(s,t,u) - f(s,u,t)]\,[\,g_V J_\mu + g_A A_\mu]\,[\,g^f_V J'_\mu + g^f_A A'_\mu]$$

$$+ [f(s,t,u) + f(s,u,t)]\,[\,g_V A_\mu + g_A J_\mu]\,[\,g^f_V A'_\mu + g^f_A J'_\mu\,]\}\,, \qquad (3.21)$$

where (for simplicity we put $M^2{}_R = M^2$)

$$f(s,t,u) = \frac{1}{M^2-s} \{[\,\ln \frac{(ut)^{1/2}}{\lambda^2} + \ln(1-\frac{s}{M^2})^2]\ \ln(\frac{u}{t}) + [Sp(1+\frac{u}{M^2})$$

$$- Sp(1+\frac{t}{M^2})]\} + \frac{u-t-M^2}{u^2}\{\ln(1-\frac{s}{M^2})\ln(\frac{-t}{s}) + Sp(1+\frac{t}{M^2})$$

$$- Sp(1-\frac{s}{M^2})\} + \frac{1}{u}\{(\frac{M^2}{s}-1)\ln(1-\frac{s}{M^2}) + \ln(\frac{-t}{M^2})\}, \qquad (3.22)$$

and $s \equiv s+i\,\varepsilon$.

Then for $M^2 \to \lambda^2$ one recovers the $\gamma\gamma$ box diagrams results (eq. 3.17)

$$f(s,t,u) - f(s,u,t) \to -(1/s)\,[V_1{}^\gamma + 2\pi\, i\, V_2{}^\gamma\,]$$

$$f(s,t,u) + f(s,u,t) \to -(1/s)\,[A_1{}^\gamma + 2\pi\, i\, A_2{}^\gamma\,]$$

On the other hand for $s \sim M^2$ eq. (3.21) reproduces the approximate result of eq. (3.19)

$$M_{box}{}^{\gamma Z}(s) = -(\alpha/2\pi)\, Q_f\,\{4\ln(b/a)\ln[(M_R{}^2-s)^2/\lambda^2 s\,] + 2[Sp(a^2)-Sp(b^2)]$$

$$-4\,(\ln^2 a - \ln^2 b)\}\, M_o{}^{RES}(s) + o\,(s-M^2). \qquad (3.23)$$

Finally the result (3.21) can be cast in the form

$$M_{box}{}^{\gamma Z}(s) = M_o{}^{RES}(s)\,\{-(\alpha/2\pi)\,Q_f\,[4\ln(b/a)\ln[(M_R{}^2-s)^2/\lambda^2 s\,] + V^{\gamma Z}(s,t,u)]\}$$

$$+ M_5{}^{RES}(s)\,\{-(\alpha/2\pi)\,Q_f\, A^{\gamma Z}(s,t,u)\}$$

$$\equiv M_o{}^{RES}(s)\, C^{\gamma Z}(s,t,u) + Q_f\, M_5{}^{RES}(s)\, C_5{}^{\gamma Z}(s,t,u) \qquad (3.24)$$

with

$$M_5^{RES}(s) = M_o^{RES}(s)\ (\gamma_\mu \rightarrow \gamma_\mu \gamma_5,\ \gamma_\mu \gamma_5 \rightarrow \gamma_\mu)$$

and

$$V^{\gamma Z}(s,t,u) = 4 \ln\left(\frac{b}{a}\right) \ln\left(\frac{s^2}{M^4} ab\right) + 2 \left[\mathrm{Sp}\left(1+\frac{u}{M^2}\right) - \mathrm{Sp}\left(1+\frac{t}{M^2}\right)\right]$$

$$+ (M^2 - s) \left\{ (2/u) \left[\ln\left(1-\frac{s}{M^2}\right) \ln\left(-\frac{t}{s}\right) + \mathrm{Sp}\left(1+\frac{t}{M^2}\right) - \mathrm{Sp}\left(1-\frac{s}{M^2}\right) + \right.\right.$$

$$+ (1/2) \ln\left(\frac{-t}{M^2}\right) \Big] + (s - M^2) \left[(1/u^2) \left(\ln\left(1-\frac{s}{M^2}\right) \ln\left(\frac{-t}{s}\right) + \mathrm{Sp}\left(1+\frac{t}{M^2}\right)\right.\right.$$

$$\left.\left.\left. - \mathrm{Sp}\left(1-\frac{s}{M^2}\right)\right) - (1/su) \ln\left(1-\frac{s}{M^2}\right) \right] - (t \leftrightarrow u) \right\}$$

$$\equiv V_1^Z + 2\pi i\, V_2^Z \tag{3.25}$$

$$A^{\gamma Z}(s,t,u) = (M^2 - s) \left\{ (2/u)\left[\ln\left(1-\frac{s}{M^2}\right) \ln\left(\frac{-t}{s}\right) + \mathrm{Sp}\left(1+\frac{t}{M^2}\right) - \mathrm{Sp}\left(1-\frac{s}{M^2}\right) + \right.\right.$$

$$+ (1/2) \ln\left(-\frac{t}{M^2}\right)\Big] + (s - M^2) \left[(1/u^2) \left(\ln\left(1-\frac{s}{M^2}\right) \ln\left(\frac{-t}{s}\right) + \mathrm{Sp}\left(1+\frac{t}{M^2}\right) -\right.\right.$$

$$\left.\left.\left. - \mathrm{Sp}\left(1-\frac{s}{M^2}\right)\right) - (1/su) \ln\left(1-\frac{s}{M^2}\right) \right] + (t \leftrightarrow u) \right\}$$

$$\equiv A_1^Z + 2\pi i\, A_2^Z \tag{3.26}$$

This concludes the discussion of the $\gamma\gamma$ and γZ box diagrams.

3.3. - Differential Cross Sections to one Loop

The invariant matrix element $M_V(s)$ for the reaction $e^+ e^- \to f \bar{f}$, including the virtual e.m. corrections of order α discussed in the preceding sections, is given by

$$M_V(s) = M_o^{QED}(s)\{1 + \delta_{VC}(s) + \delta_{VP}(s) + C^{\gamma\gamma}(s,t)\} + M_5^{QED}(s)\, C_5^{\gamma\gamma}(s,t) +$$

$$+ M_o^{RES}(s)\{1 + \delta_{VC}(s) + C^{\gamma Z}(s,t,u)\} + M_5^{RES}(s)\, C^{\gamma Z}(s,t,u), \qquad (3.27)$$

Then by adding the real (eq.3.4) and virtual (eq. 3.27) contributions the infrared divergences are exactly cancelled and we obtain*

$$d\sigma/d\Omega = d\sigma^{QED}/d\Omega + d\sigma^{INT}/d\Omega + d\sigma^{RES}/d\Omega, \qquad (3.28)$$

where

$$d\sigma^{QED}/d\Omega \sim |\, M_o^{QED}(s)\,|^2 \{ 1 + (\beta_e + \beta_f + 2\beta_{int}) \ln \Delta + (3/4)(\beta_e + \beta_f)$$

$$+(\alpha/\pi)(1 + Q_f^2)(\pi^2/3 - 1/2) - (\alpha/\pi) Q_f [\, 2 F(a,b) + V_{1f}^{\gamma}(s,t)] + 2 \operatorname{Re} \delta_{VP}(s)\}$$

$$- (\alpha/\pi) Q_f A_1^{\gamma} \operatorname{Re}\{ M_o^{QED}(s)^* M_5^{QED}(s)\}, \qquad (3.29)$$

$$d\sigma^{INT}/d\Omega \sim 2 \operatorname{Re}\{[\, M_o^{QED}(s)\, M_o^{RES}(s)^*\,][\, 1 + (\beta_f + \beta_{int}) \ln \Delta +$$

$$+ \beta_{int} \ln\left(\frac{\Delta s}{M_R^2 - s + \Delta s}\right) + \beta_e \ln\left(\frac{\Delta (M_R^2 - s)}{M_R^2 - s + \Delta s}\right) + (3/4)(\beta_e + \beta_f)$$

$$+ \alpha/\pi\, (1 + Q_f^2)(\pi^2/3 - 1/2) - (\alpha/2\pi) Q_f (4 F(a,b) + V_{1f}^{\gamma} + V_1^{Z}) + \operatorname{Re} \delta_{vp}(s)]\}$$

$$- 2 \operatorname{Im}\{ M_o^{QED}(s)\, M_o^{RES}(s)^*\} \{ \operatorname{Im} \delta_{VP}(s) - \alpha Q_f (V_2^{\gamma} - V_2^{Z})\} +$$

$$+ 2 \operatorname{Re}\{ M_5^{QED}(s)\, M_o^{RES}(s)^*\} \{ - (\alpha/2\pi) Q_f A_1^{\gamma}\}$$

(*) As usual, the corrections proportional to Re $\delta_{vp}(s)$ can be appropriately reabsorbed in the running coupling constant, by implementing $M_o^{RES}(s)$ as in eq. (2.12).

$$- 2\,\mathrm{Im}\,\{ M_5^{QED}(s)\, M_o^{RES}(s)^* \}\, \{ - \alpha\, Q_f\, A_2^{\gamma} \}$$

$$+ 2\,\mathrm{Re}\,\{ M_o^{QED}(s)^*\, M_5^{RES}(s) \}\, \{ - (\alpha/2\pi)\, Q_f\, A_1^{z} \}$$

$$- 2\,\mathrm{Im}\,\{ M_o^{QED}(s)^*\, M_o^{RES}(s) \}\, \{ - \alpha\, Q_f\, A_2^{z} \}, \qquad (3.30)$$

and

$$d\sigma^{RES}/d\Omega \sim |\, M_o^{RES}(s)\, |^2 \{ 1 + \beta_f \ln \Delta + 2\beta_{int} \ln \left| \frac{\Delta s}{M_R^2 - s + \Delta s} \right|$$

$$+ \beta_e \ln \left| \frac{\Delta\,(M_R^2 - s)}{M_R^2 - s + \Delta s} \right| - \beta_e\, \delta\,(s, \Delta)\, \mathrm{ctg}\, \delta_R(s) +$$

$$+ (3/4)\,(\beta_e + \beta_f) + \alpha/\pi\,(1 + Q^2{}_f)\,(\pi^2/3 - 1/2) - \alpha/\pi\, Q_f\, [2\, F(a,b) + V_1^{z}] \}$$

$$+ \mathrm{Re}\,\{ M_o^{RES}(s)^*\, M_5^{RES}(s) \}\, \{ - (\alpha/\pi)\, Q_f\, A_1^{z} \}$$

$$- \mathrm{Im}\,\{ M_o^{RES}(s)^*\, M_5^{RES}(s) \}\, \{ - \alpha\, Q_f\, A_2^{z} \}. \qquad (3.31)$$

The appropriate sum over the intial and final helicities is implicity assumed. In the absence of polarization of the incoming beams this leads to

$$|\, M_o^{QED}(s)\, |^2 \sim (\alpha^2/4s)\, Q^2{}_f\, (1 + z^2)$$

$$\mathrm{Re}\,\{ M_o^{QED}(s)^*\, M_5^{QED}(s) \} \sim (\alpha^2/4s)\, Q^2{}_f \cdot 2\,z$$

$$2\,\mathrm{Re}\,\{ M_o^{QED}(s)\, M_o^{RES}(s)^* \} \sim (\alpha^2/4s)\,(- Q_f)(2\,\mathrm{Re}\,\chi\,(s))\,[(1 + z^2)\, r_V + 2z\, r_A]$$

$$2\,\mathrm{Im}\,\{ M_o^{QED}(s)\, M_o^{RES}(s)^* \} \sim - (\alpha^2/4s)\,(- Q_f)\,(2\,\mathrm{Im}\,\chi\,(s))\,[(1 + z^2)\, r_V + 2z\, r_A]$$

$$2\,\mathrm{Re}\,\{ M_5^{QED}(s)\, M_o^{RES}(s)^* \} \sim (\alpha^2/4s)\,(-Q_f)\,(2\,\mathrm{Re}\,\chi\,(s\,))\,[2\,z\,r_V + (1+z^2)\,r_A]$$

$$2\,\mathrm{Im}\,\{ M_5^{QED}(s)\;\; M_o^{RES}(s)^* \} \;\sim -\,(\alpha^2/4s)\,(-Q_f)\,(2\,\mathrm{Im}\,\chi\,(s))\,[\,2\,z\,r_V + (1+z^2)\,r_A\,]$$

$$2\,\mathrm{Re}\,\{ M_o^{QED}(s)\,^*\, M_5^{RES}(s)\} \sim (\alpha^2/4s)\,(-Q_f)(2\,\mathrm{Re}\,\chi\,(s\,))\,[2\,z\,r_V + (1+z^2)\,r_A]$$

$$2\,\mathrm{Im}\,\{ M_o^{QED}(s)^*\, M_5^{RES}(s)\} \sim (\alpha^2/4s)\,(-Q_f)\,(2\,\mathrm{Im}\,\chi\,(s\,))\,[(2\;\; z\,r_V + (1+z^2)\,r_A]$$

$$|\, M_o^{RES}(s)\,|^2 \sim (\alpha^2/4s)\;\; [(1+z^2)\frac{v^2{}_f + a^2{}_f}{v^2 + a^2} + 8\;\; z\,r_V\;\; r_A\,]\;|\,\chi\,(s)\,|^2$$

$$\mathrm{Re}\,\{ M_o^{RES}(s)\, M_5^{RES}(s)\} \;\sim (\alpha^2/4s)\;\; [2z\;\frac{v^2{}_f + a^2{}_f}{v^2 + a^2} + 4\;\;(1+z^2)\;\; r_V\;\; r_A\;|\,\chi\,(s)\,|^2$$

$$\mathrm{Im}\,\{ M_o^{RES}(s)^*\, M_5^{RES}(s)\} \sim 0, \qquad (3.32)$$

having used the notation of sect.2.

By insertion of the above results (3.32) into the eqs. (3.29-3.31) one then obtains the final expression of the soft differential cross section to order α . Some remarks ought to be made here.

The first concerns the necessity of including higher order corrections. Given the precision required[5] at LEP/SLC, where one wishes to measure M_Z and Γ_Z to an accuracy $\delta M_Z \sim \delta\Gamma_Z \sim 10$MeV, it is clear that the QED corrections have to be under control at the level of ~0.5%. From eqs. (3.29-3.31) it follows that large logarithms appear to one loop level of order $\beta_e \ln(M_Z / \Gamma_Z) \sim 37\%$ or $\beta_e \sim 11\%$. Therefore one needs to include higher order terms in the perturbative expansion. Actually it is known[7,29] how to sum up the whole series of leading and some next-to-leading logarithms. The related formalism has been extensively studied[8] in connection to the production of very narrow resonances like the J/Ψ and generalized[9] to the case of the Z_o. It will be reviewed in the next section.

The second remark concerns the emission of hard photons collinear to the charged particles, which becomes more and more important, as the energy increases. Nowadays charged particles are often detected in electromagnetic calorimeters which do not discriminate between the charged particle and the accompayning collinear photons. To include this contribution in the observed cross section, one has to introduce a small but finite angular resolution δ, in addition to the energy resolution Δ, similarly to jets analyses[10] in QCD. Then, as well known, one encounters large terms in the perturbative expansion associated to the mass sigularities[11] of the charged particles. The final result includes logarithms of order $(\alpha/\pi)\ln\delta^2\ln\Delta$ or $(\alpha/\pi)\ln\delta^2$, which are quite relevant[12,13] for the analysis of the data. A discussion of these contributions is given in sect.6.

The final remark concerns the effect of emission of hard-not collinear-photons which, whenever required by the experimental kinematical conditions have to be explicitly added. They correspond to terms of order $(\alpha/\pi)\ \ln(\Delta,\delta)$, and have been also calculated[30] in a way which is suitable for a Monte Carlo simulation of events. A short discussion will be given in sect.8.

4. - MULTIPLE SOFT PHOTON EFFECTS

In this section we will discuss the collective effect of multiple soft emission, reviewing the results of previous analyses, to which we adress the reader for a detailed derivation of the main formulae.

From the previous section, in particular from eqs. (3.5) and (3.29-3.31), it follows that the most relevant corrections come from real soft photon emission, namely

$$d\sigma \sim (\alpha/\pi)\,[\,\ln(s/m_e^2) - 1\,] \int \frac{dk}{k}\,[1+(1-k/E)^2]\,d\sigma_0\,[s\,(1-k/E)], \tag{4.1}$$

in the case of initial-state radiation, and similarly for interference and final-state radiation terms. The resummation of the full series of double logarithms and some simple logarithms, corresponding to multiple soft emission, is quite well known[7] for QED processes, where the basic cross section $d\sigma_0$ is not a rapidly varying function of the energy, or the momentum transfer. One has

$$d\sigma^{QED} = \frac{d\sigma_0}{\gamma^\beta\Gamma(1+\beta)}\ (\Delta\omega/E)^\beta + \ldots\ldots \tag{4.2}$$

where $\ln\gamma=0.5772$ is Euler's constant and $\beta=\beta_e + 2\,\beta_{int} + \beta_f$. It has been generalized in ref. (8), using the technique of coherent states[29], to discuss the radiative effects generated in presence of a narrow resonance like the J/Ψ, whose decay width is much smaller than the energy resolution of the experiment, and then in ref. (9), without any restriction on the relative size of Γ and $\Delta\omega$, just in connection of Z_0 production .We will briefly review those results.

When the reaction

$$a+b\rightarrow c+d+...$$

proceeds through the production of a resonance in the s-channel, the classical current associated to the external charged particles is modified as follows:

$$j_\mu^R(k) = \frac{W-M+(1/2)\,i\Gamma}{W-M-k+(1/2)\,i\Gamma}\, j_\mu^{(i)}(k) + j_\mu^{(f)}(k), \tag{4.3}$$

where $j_\mu^{(i)}(k)$ and $j_\mu^{(f)}(k)$ are the usual classical currents associated to the initial and final particles respectively, namely

$$j_\mu^{(i,f)}(k) = \frac{ie}{(2\pi)^{1/3}} \sum_l^{(i,f)} \varepsilon_l \frac{p_\mu^{(l)}}{(p_l \cdot k)} . \tag{4.4}$$

This modification takes into account the finiteness of the time interval between the formation of the resonance and the creation of the final state, as can be seen from the Fourier transform of (4.3). In perturbation theory this is equivalent to write the matrix element for the emission of one soft photon as

$$M_\mu^{(1\gamma)}(k) = j_\mu^{(i)} M^o(W-k) + j_\mu^{(f)} M^o(W), \tag{4.5}$$

and accounts for the shift in the c.m. energy when the radiation is emitted from the initial state.

Let us introduce the action Λ_R relative to the distribution of classical current $j_\mu^R(k)$ of eq. (4.3)

$$\Lambda_R = \frac{1}{(2\pi)^4} \int d^4k\, j_\mu^{(R)}(k)\, A^\mu(-k), \tag{4.6}$$

where $A_\mu(k)$ is quantized electromagnetic field. Then, for a pure resonant process, the matrix element

$$\overline{M}_R = \frac{1}{\sqrt{N}} < f \,|\, e^{-i\Lambda_R^+} S \,|\, i >, \tag{4.7}$$

with $N = < e^{-i\Lambda_R^+} e^{i\Lambda_R} >$, can be shown[8] to be: (i) finite and does not therefore possess an infrared divergence, (ii) separable in the infrared factors and (iii) directly comparable with the observable process section which results proportional to $|\overline{M}_R|^2$. This results extends the previous one[29], valid for QED non resonant process, where $j_\mu^{(R)}(k)$ simply reduces to $j_\mu^{(c)}(k) \equiv j_\mu^{(i)}(k)+j_\mu^{(f)}(k)$ (see eq. 4.11).

Without entering into the details of the derivation, one obtains[9] for ee $\to$ R $\to$ ff

$$d\sigma^{RES} = d\sigma_0^{RES} \{ (\Delta\omega/E)^{\beta_f} \left| \frac{2\Delta\omega}{\sqrt{s}} \frac{M_R^2 - s}{M_R^2 - s + 2\sqrt{s}\,\Delta\omega} \right|^{\beta_e} \cdot$$

$$\cdot \left| \frac{2\sqrt{s}\,\Delta\omega}{M_R^2 - s + 2\sqrt{s}\,\Delta\omega} \right|^{2\beta_{int}} [\, 1 + \frac{s - M^2}{M\Gamma} \beta_e\, \delta\,(s, \Delta\omega)] + C_F^{RES} \} \tag{4.8}$$

where $M_R^2 = M^2 - i\,M\Gamma$,

$$\delta\,(s, \Delta\omega) = \mathrm{arctg}\, \frac{2\sqrt{s}\,\Delta\omega - (s - M^2)}{M\Gamma} + \mathrm{arctg}\, \frac{s - M^2}{M\Gamma}, \tag{4.9}$$

and $C_F^{RES} \sim O(\alpha)$ accounts for the rest of the finite corrections and has to be calculated perturbatively. We have not included in eq. (4.8) and the subsequent eq. (4.13) factors of the type $\gamma^{-\beta}\,\Gamma^{-1}(1+\beta)$, as in eq. (4.2), which contribute to order β^2 only. For a discussion see the appendix of ref. (9).

First order expansion of eq.(4.8) clearly coincides with the soft term of eq. (3.31). C_F^{RES} is then determined by comparison with the remaining terms of eq. (3.31).

Let us briefly discuss this result. First, it is easily seen that the infrared factors appearing in eq. (4.8) reduce to the standard one $(\Delta\omega/E)^{\beta_e+\beta_f+2\beta_{int}}$ in the limit $\Delta\omega \ll |(M^2_R\text{-}s)/2\sqrt{s}|$, as they should.

On the other hand, in the case of a narrow resonance like the J/Ψ, for which in a typical experiment $\Delta\omega \gg |\,(M_R^2\text{-}s)/2\sqrt{s}\,|$, the $\Delta\omega$ dependence drops completely out, namely the width of the resonance provides a natural cut-off in damping the energy loss in the initial state. Furthermore, the β_{int} dependence eq. (4.8) also cancels out, giving no interference between the soft emission from the initial and final states.

Finally the term proportional to $\delta(s, \Delta\omega)$, with the arctangent defined to have values in the interval $(-\pi/2, \pi/2)$, gives the radiative tail of the resonance. In fact for narrow resonances $\delta(s,\Delta\omega)$ reduces to $\delta_R(s)$, the usual phase shift of the Breit-Wigner resonances.

We would now like to discuss the radiative effect arising from the interference of a resonant term with a pure QED term. As shown in detail in ref. (8) the procedure to follow is quite clear.

One starts with the sum of two infrared-finite matrix elements, $\overline{M}_{QED}$ and $\overline{M}_R$, as

$$\overline{M} = \overline{M}_{QED} + \overline{M}_R = \langle f | e^{-i\Lambda_c} S_{QED} | i \rangle + (1/\sqrt{N}) \langle f | e^{-i\Lambda^+_R} S_R | i \rangle \qquad (4.10)$$

where Λ_c is the action relative to the distribution of pure classical currents

$$j^c_\mu(k) = j^{(i)}_\mu(k) + j^{(f)}_\mu(k), \qquad (4.11)$$

which describes the infrared properties of a pure QED process[29]. More explicitly Λ_c is given by eq. (4.6) when $j^R_\mu(k) \rightarrow j^c_\mu(k)$. The full observed cross section is then proportional to $|\overline{M}|^2$. While $[\overline{M}_{QED}]^2 \propto d\sigma^{QED}_{corr}$ and $|\overline{M}_R|^2 \propto d\sigma^{RES}_{corr}$ the interference term comes only from that part of $\langle f | \exp(-i\Lambda_c)$ which overlaps with $\langle f | \exp(-i\Lambda^+_R)/\sqrt{N}$. Introducing, therefore, a matrix element $\overline{M}^{int}_{QED}$ as

$$\overline{M}^{int}_{QED} = (1/\sqrt{N}) \langle f | e^{-i\Lambda^+_R} S_{QED} | i \rangle, \qquad (4.12)$$

all the interference effects will then come from Re $(\overline{M}^{int}_{QED}\, \overline{M}^*_R)$, as one can see by comparison with perturbation theory.

With that in mind, one finds

$$d\sigma^{INT} = d\sigma_0^{INT} \left\{ \left(\frac{\Delta\omega}{E}\right)^{\beta_f+\beta_{int}} (1/\cos\delta_R) \cdot \right.$$

$$\left. \cdot \, \mathrm{Re}\left[e^{i\delta_R} \left(\frac{\Delta}{1+\Delta s/(M_R^2-s)}\right)^{\beta_e} \left(\frac{\Delta}{\Delta+(M_R^2-s)/s}\right)^{\beta_{int}} \right] + C_F^{INT} \right\} \qquad (4.13)$$

with $s/(M_R^2-s) \equiv (s/M\Gamma) \sin\delta_R\, e^{i\delta_R}$.

Again, taking the limit $\Delta \ll (\Delta \gg) | (M_R^2 s)/s |$ one gets the usual QED, or narrow resonance-like behaviour respectively.

As for the pure resonant case, comparing first order expansion of (4.13) with the pertubative result (3.30) one recovers the infrared terms and determines the rest of the finite correction C_f^{INT} by subtraction.

From the above formulae (4.8) and (4.13) it is clear that, in esperiments studying forward-backward asymmetries, dips and similar subtle effects, the Γ and $\Delta\omega$ dependence in the infrared factors has to be taken into account properly.

A particularly simple case is provided by the measurement of the line shape of the Z_0 in the total cross section $e^+e^- \rightarrow X$. Then one can take the limit $\Delta\omega \gg | (M_R^2-s)/2\sqrt{s} |$ in eq. (4.8) and obtains

$$d\sigma^{RES} = d\sigma_o \left[\frac{(s-M^2)^2 + M^2\Gamma^2}{M^4} \right]^{\beta_e} \left[1+ \beta_e \frac{s-M^2}{M\Gamma} \delta_R \right] + \ldots, \tag{4.14}$$

the dots indicating additional non leading terms, in exact analogy to the case[8] of production of a very narrow risonance. Physically, this is understood by saying that the width Γ provides a natural cut-off in damping the energy loss in the initial state.

The resummation of the leading and next-to-leading logarithms from initial state bremsstrahlung has been also studied[31] more recently by extending to QED the formalism used in the context of the QCD-improved parton model[32]. In this case one defines electron or position densities e(x,s) at the scale $s \simeq M^2$. The corrected cross section is then obtained by a convolution of the e^- and e^+ densities with a reduced cross section $\tilde{\sigma}$.

$$\sigma(s) = \int^1 \int^1_{x_1+x_2>2-\Delta\omega/E} dx_1\, dx_2\, \theta(x_1 x_2 s - s_o)\, e(x_1, s)\, s\,(x_2, s)\, \tilde{\sigma}(x_1 x_2 s) \tag{4.15}$$

in exact analogy to Drell-Yan processes in QCD[32,33].

The electron density e(x,s) and photon density $\gamma(x,s)$ in an electron satisfy the evolution equations[34]

$$s\left(\frac{d}{ds}\right) e(x, s) = (\alpha(s)) / (2\pi) \int_x^1 [\, e(y,s)\, P_{ee}(x/y, s) + \gamma(y,s)\, P_{e\gamma}(x/y, s)] dy/y$$

$$s\left(\frac{d}{ds}\right) \gamma(x, s) = (\alpha(s)) / (2\pi) \int_x^1 [\, e(y,s)\, P_{\gamma e}(x/y, s) + \gamma(y,s)\, P_{\gamma\gamma}(x/y, s)] dy/y \tag{4.16}$$

with

$$P_{ee}(x,s) = [(1+x^2)/(1-x)]_+ + O[\alpha(s)]$$

$$P_{e\gamma}(x,s) = [x^2+(1-x)^2] + O[\alpha(s)]$$

$$P_{\gamma e}(x,s) = [1 + (1-x)^2] / x + O[\alpha(s)]$$

$$P_{\gamma\gamma}(x,s) \simeq O[\alpha^2(s)] \tag{4.17}$$

In the above equations $\alpha(s)$ is the QED running coupling constant ($\alpha \equiv \alpha(m^2_e)$)

$$\alpha/\alpha(s) \simeq 1 - (1/3\pi)\, \alpha \sum_f Q_f{}^2 \ln (s/m^2{}_f) \tag{4.18}$$

and the sum extends over quarks and leptons with masses $m^2{}_f \lesssim s$. Clearly $e(x,m^2{}_e)= \delta(1-x)$ and $\gamma(x,m^2{}_e)=0$.

As in QCD, the solutions of eqs. (4.16) take a simple form in terms of the moments, which have to be transformed back to the x space. To order α/π one obtains a result of the form

$$\sigma(s) = \sigma_o + \delta\sigma_{LL} + \delta\sigma_F, \tag{4.19}$$

where $\delta\sigma_{LL}$ is the leading logarithmic correction

$$\delta\sigma_{LL} = (\alpha/\pi) \ln (s/m^2{}_e) \int_o^1 \sigma_o (z\, s)\, P_{ee}(z)\, dz, \tag{4.20}$$

and $\delta\sigma_F$ is the remaining connection, which has to be extracted from the complete first order calculation.

In the soft limit ($\Delta\omega/E \ll 1$), the contribution to (4.15) comes from the non-singlet part of the distributions $e(x,s)$ only, corresponding to the annihilation of the initial electrons after emission of the soft radiation.

Then the solution to eqs. (4.16) can be computed analytically if $\sigma(s)$ is a smooth function of s, and takes the form

$$\sigma(s) = \tilde{\sigma}(s)\, R\, (1 - \Delta\omega/E,\, s) \tag{4.21}$$

with[35]

$$R(x,s) = \frac{(1-x)^\eta}{\gamma^\eta \Gamma(1+\eta)}\, e^{\,3/4\,\eta} \tag{4.22}$$

and

$$\eta = \int_{m^2}^{s} \frac{2\alpha(Q^2)}{\pi} \frac{dQ^2}{Q^2} = -6 \ln\, [1 - (\alpha/3\pi) \ln (s/m^2)] \tag{4.23}$$

To leading order $\eta=\beta e$ and eq. (4.22) coincides with the previous result (4.2) a part the term exp $(3/4\eta)$, arising from the improvement of simple soft breemsstrahlung spectrum $\sim 2/x$ by $[1+(1-x)^2]/x$ [eqs.(4.17)].

A generalization of the concept of coherent state in QED to account for the emission of hard and collinear photons as well has been discussed in ref. (36). It is shown there the equivalence for the non-singlet case of the two methods of resummation discussed above - the coherent state picture and the evolution equations - leading to eq. (4.21).

It has to be emphasized that this factorized result is not valid when the cross section $\tilde{\sigma}(s)$ is a rapidly varying function of s, as in the case of resonance production with $\Gamma \lesssim \Delta\omega$. Furthermore it does not include the emission from the final state and interference effects, as in eqs (4.8) and (4.13), and therefore can only be applied to the observation of the line shape of the resonance, after taking appropriately into account the s dependence of $\tilde{\sigma}(s)$.

5. - $e^+e^- \rightarrow f\bar{f}$: FINAL FORMULAE

The discussion of the previous section on multiple soft-photon effect, together with the complete one-loop results of sect. 3, leads[9] to following form for the radiatively corrected cross section, in case of unpolarized beams,

$$d\sigma/d\Omega\,(e^+e^- \rightarrow f\,\bar{f}) = d\sigma^{QED}/d\Omega\,(C_{infra}{}^{QED} + C_F{}^{QED})$$
$$+ d\sigma^{INT}/d\Omega\,(C_{infra}{}^{INT} + C_F{}^{INT}) + d\sigma^{RES}/d\Omega\,(C_{infra}{}^{RES} + C_F{}^{RES}), \tag{5.1}$$

where the infrared factors are given by(*)

$$C_{infra}{}^{QED} = \Delta^{\beta_e+\beta_f+2\beta_{int}} ,$$

$$C_{infra}{}^{INT} = \Delta^{\beta_f+\beta_{int}} \; [\,1/\cos\delta_R].$$

$$. \,\mathrm{Re}\,\{\, e^{i\delta_R(s)} \left[\frac{\Delta}{1+\frac{\Delta s}{M\Gamma}\, e^{\,i\delta_R(s)} \sin\delta_R(s)}\right]^{\beta_e} .$$

(*) We have not included factors of the type $\gamma^{-\beta}\,\Gamma^{-1}(1+\beta)$, which contribute to order β^2 only.

$$\left[\frac{\Delta}{\Delta+\frac{M\Gamma}{s}\, e^{-i\delta_R(s)} / \sin\delta_R(s)}\right]^{\beta_{int}} \Big\} \tag{5.2}$$

$$C_{infra}^{RES} = \Delta^{\beta_f} \left|\frac{\Delta}{1+\frac{\Delta s}{M\Gamma}\, e^{i\delta_R(s)} \sin\delta_R(s)}\right|^{\beta_e} .$$

$$\left|\frac{\Delta}{\Delta+\frac{M\Gamma}{s}\, e^{-i\delta_R(s)} / \sin\delta_R(s)}\right|^{2\beta_{int}} \{1-\beta_e\, \delta(s,\Delta)\, \mathrm{ctg}\, \delta_R(s)\}$$

The finite factors $C^{(i)}{}_F$ are simply obtained from eqs. (3.29-3.31) with the help of eqs. (3.32) and include the leftover contributions of order (α/π)

$$C_F^{QED} = (3/4)\,(\beta_e+\beta_f) + (\alpha/\pi)\,(1+Q_f^2)\,(\pi^2/3-1/2) + 2\mathrm{Re}\,\delta_{VP}(s)\ , \tag{5.3a}$$

$$-\ (2\alpha/\pi)\,Q_f\,F(a,b) - (\alpha/\pi)\,Q_f\,\{V^{\gamma}{}_{1f} + [\,2z/(1+z^2)\,]\,A^{\gamma}{}_1\}\ ,$$

$$C_F^{INT} = (3/4)\,(\beta_e+\beta_f) + (\alpha/\pi)\,(1+Q_f^2)\,(\pi^2/3-1/2) + \mathrm{Re}\,\delta_{VP}(s)\ , \tag{5.3b}$$

$$-\,(\,2\alpha/\pi\,)\,Q_f\,F(a,b) + [\,\mathrm{Im}\,\chi(s)/\mathrm{Re}\,\chi(s)\,]\,\mathrm{Im}\,\delta_{VP}(s)$$

$$-\,(\,\alpha/2\pi\,)\,Q_f\,\{V^{\gamma}{}_{1f} + V^{Z}{}_1 + 2\pi\,[\,\mathrm{Im}\,\chi(s)/\mathrm{Re}\,\chi(s)\,]\,(\,V^{\gamma}{}_2 - V^{Z}{}_2\,)\,\}$$

$$-\,(\,\alpha/2\pi\,)\,Q_f\,[\,2z\,r_V + (1+z^2)\,r_A\,]\ /\,[\,2z\,r_A + (1+z^2)\,r_V\,]\ .$$

$$.\,\{\,A^{\gamma}{}_1 + A^{Z}{}_1 + 2\pi\,[\,\mathrm{Im}\,\chi(s)/\mathrm{Re}\,\chi(s)\,]\,(A^{\gamma}{}_2 - A^{Z}{}_2\,)\}\ ,$$

$$C_F^{RES} = (3/4)\,(\beta_e+\beta_f) + (\alpha/\pi)\,(1+Q_f^2)\,(\pi^2/3-1/2) - (\,2\alpha/\pi\,)\,Q_f\,F(a,b) \tag{5.3c}$$

$$-\,(\,\alpha/\pi\,)\,Q_f\,\{\,V^{Z}{}_1 + [\,4\,(1+z^2)\,r_V.r_A + 2z\,(v^2{}_f + a^2{}_f)/(v^2+a^2)\,]\ .$$

$$/\,[\,(1+z^2)\,(v^2{}_f + a^2{}_f)/(v^2+a^2) + 8\,z\,r_V\,r_A\,]\ A^{Z}{}_1\,\}\ .$$

Notice that the vacum polarization corrections due to Re $\delta_{VP}(s)$ should be removed from eqs. (5.3 a,b) when introducing the running conpling constant $\hat{e}^2(s)$ in M_o^{QED}, as already discussed in sect.(2.3).

The above results for $C_F^{(i)}$ have to be appropriately modified, as explained in next section, when hard collinear radiation is detected together with the final particles, as in calorimetric-type experiments.

The reaction $e^+e^- \rightarrow \mu^+\mu^-$ has been investigated in great detail[37]. Indeed radiative effects lead to rather large corrections to the naive expectations.

First order corrections, for example, reduce the Z peak cross-section by more than 50%, or shift the zero in the forward-backward asymmetric by about (±300) MeV, for an energy resolution of (10^{-1}-10^{-2}). It is therefore crucial that higher-order corrections are properly into account if the Z mass and width have to be measured to an accuracy of the order of 50 MeV. Numerical results are given in sect. (11).

The observation of hadronic jets at very high energies also provides a test of the standard electroweak model. In fact the production cross section of qq jets and the corresponding angular asymmetries are very sensitive to the weak vector and axial quark couplings, particularly for energies near the Z_0. Compared to the pure leptonic process, e.g. $e^+e^- \rightarrow \mu^+\mu^-$, the quark angular asymmetrics have the obvious advantage of being enhanced in statistics by a factor $R=\sigma(e^+e^- \rightarrow \text{hadrons}) / \sigma(e^+e^- \rightarrow \mu^+\mu^-)$. On the other hand the observable effects depend on the hadronization properties of the quark jets and are limited by experimental problems of particle identification. Various methods of jet analysis have been suggested[38] which are suitable for the purpose of measuring the electroweak hadronic asymmetries.

The above formulae (5.1-5.3) can be then used to describe the radiative corrections to $e^+e^- \rightarrow q\bar{q}$. In case of heavy flavours production the mass m_q, which enters in the radiative quark factor β_f, is well determined and does not lead therefore to any ambiguities. On the other hand for light quarks m_q has to be considered as an effective quark mass. Indeed the reason for that has to be traced back to the cancellation of the quark mass singularities in the physical $q\bar{q}$ cross section, after including the QCD radiation emitted from the quark legs in the definition of jets. Then, as well known, the original quark mass gets replaced by a typical hadronic average tranverse momentum $\langle p_T \rangle$. For this reason the above formulae cannot be used straightforward to large p_T events, which should be analyzed by explicit use of a qqg final state. To conclude this discussion on the quark masses we observe that in most realistic cases the possible emission of a hard collinear photon from the quark legs cannot be distinguished from the products of the hadronization of the quarks into the physically observable jets. In these cases the considerations of next section have to be applied and then β_f dependence is appropriately replaced by β_δ (see definition in next section) irrespective of the actual value of m_q.

The 2-jet cross section $d\sigma_{q\bar{q}}$ is clearly given by

$$d\sigma_{q\bar{q}}(\theta) = d\sigma_q(\theta) + d\sigma_{\bar{q}}(\theta) = d\sigma_q(\theta) + d\sigma_q(\pi-\theta) \qquad (5.4)$$

Then, for a doublet of (uu) and (dd) quark pairs we define the radiative correction factor as

$$\delta\sigma_{RC} = \frac{d\sigma_{u\bar{u}} + d\sigma_{d\bar{d}}}{d\sigma^{o}_{u\bar{u}} + d\sigma^{o}_{d\bar{d}}} - 1 \qquad (5.5)$$

In sect. (11) we will show the rapidly changing behaviour(39) of δ_{Rc} in the vicinity of M_Z.

One of the most common methods which have been envisaged to measure quark asymmetries is to look for leading particle effects in the angular distributions of inclusive produced hadrons.

Then the cross section for observing a hadron h at an angle θ, with fractional momentum $x \simeq 2E^h/\sqrt{s}$ is given by

$$d\sigma^h(\theta, x) = \sum_q [\, d\sigma_q(\theta)\, D^h{}_q(x) + d\sigma_{\bar{q}}(\theta)\, D^h{}_{\bar{q}}(x)\,]\,, \qquad (5.6)$$

where the sum goes over all quarks which are active at energy $\sqrt{s}$ and transverse momentum effects have been neglected. Correspondingly the hadronic asymmetric are defined as

$$A^h(\theta, x) = \frac{d\sigma^h(\theta, x) - d\sigma^h(\theta, x)}{d\sigma^h(\theta, x) + d\sigma^h(\theta, x)} \qquad (5.7)$$

This quantity is rather insensitive to radiative corrections(39), as shown explicitly in sect. (11).

6. - COLLINEAR HARD-PHOTON EFFECTS

The detection of charged particles in electromagnetic calorimeters does not allow to discriminate a particle from the accompanying hard collinear radiation, as remarked at the end of sect. 3. One has to include then the corresponding contribution in the observed cross section.

In perturbation theory this effect becomes increasingly important at high energies because of the large logarithms associated to the mass singularities of the emitting particle. Kinoshita, Lee and Nauenberg(11) have observed that if one sums over all degenerate states the mass singularities cancel out to all orders of pertubation theory. Then by introducing a small but finite angular resolution $\delta(\delta \ll 1)$ of the calorimeter, one is finally led to an observable cross section which contains, to leading order, logarithms of order $((\alpha/\pi) \ln \delta^2)^n$. This result is well known and rather commonly used in QCD jet analyses(10).

The formulae[37] reported below apply to a typical experiment in which the following requirements are satisfied: (i) the final state consists of a particle and an antiparticle ($f \equiv e, \mu, ...$) detected within a certain acollinearity angle J of a few degrees ($J \lesssim 5°$). The energy resolution $\Delta\omega$ then depends upon J. (see eq. 6.1). Muon pairs production is the most typical example of such a process. In the following we will take $f \equiv \mu$. (ii). An electromagnetic calorimeter of finite and small angular resolution δ is centred along the muon direction. In principle it does not discriminate between a charged particle and the accompanyng collinear photons.

Then using (i) and (ii) one would be sure that all but a fraction $\Delta \equiv \Delta\omega/E$ of the beam energy is taken by the muons and the accompanyng hard photons. For small δ and Δ, fully analytic expressions can be used, neglecting hard-photon effects of order $[(\alpha/\pi)\,\Delta,\ (\alpha/\pi)\delta]$. On the contrary, all double logarithmic terms of the form

$$(\alpha/\pi)\ \ln\,(s/m^2_e)\ \ln\,(\Delta,\ \Gamma/M)\ ,\ (\alpha/\pi)\ \ln\,\delta^2\ \ln\,\Delta,$$

or simple logs such as

$$(\alpha/\pi)\ \ln\,(s/m^2_e)\ \ ,\ \ (\alpha/\pi)\ \ln\,(\Delta,\ \Gamma/M,\ \ \ln\,\delta^2\,)$$

can be resummed to all orders, using known results on the exponentiation of the infrared and mass singularities.

For a given acollinearity angle J, the maximum energy taken by undetected soft photons, which defines the energy resolution $\Delta\omega$, is given by

$$k_{max} \equiv \Delta\omega = \frac{\sqrt{s}}{1+\cos J}\{-(1-\cos J\ +2\,[\,(1-\cos J)\,[\,1/2\ -(m^2_\mu/s)\,(\,1+\cos J)]\,]^{1/2}\,\} \tag{6.1}$$

Then for $J = 1°, 3°$, and $5°$, one obtains $\Delta \equiv \Delta\omega/E=(1.7)\%$, $(5.1)\%$ and $(8.3)\%$, respectively.

To first order in α, the contribution from collinear hard radiation ($k \geq \Delta\omega$) from the final particles, when detected within a small cone of half opening angle δ, consists of the following correction factor[10,12] to each term in the r.h.s. of eqs. (3.29), (3.30) and (3.31) : (j = QED, INT, RES)

$$\delta_j^{coll} = d\sigma_j^o\ (4\alpha/\pi)\,[\,(\,\ln\,(E/\Delta\omega) - 3/4)\ \ \ln\,(E\delta/m_\mu) -1/2\ \ln\,(E/\Delta\omega) + 1/2\,(9/4\ -\pi^2/3)] \tag{6.2}$$

Then in agreement with the Kinoshita-Lee-Nauenberg theorem on the mass singularities, the

m_μ-dependence disappears after adding Eq. (6.2) to Eqs. (3.29 - 3.31) and the overall correction factor to the Born cross-sections can be simply obtained from Eqs. (3.29 - 3.31) by the substitution

$$\beta_\mu (\ln \Delta + 3/4) \rightarrow (2\alpha/\pi) [\ln (4/\delta^2) (\ln \Delta + 3/4) + (3/2 - \pi^2/3)] \tag{6.3}$$

Notice that the contribution of the muon loop to the vacuum polarization factor Re $\delta_{VP}(s)$, in eqs. (3.29 - 3.30), is kept unchanged.

The previous result can be generalized to all orders using the known results on the exponentiation of soft and collinear divergences[40], as follows. The factor [exp ($\beta_\mu$ln Δ)], present in each term often radiatively corrected cross sections (4.2), (4.8) and (4.13) is replaced by [exp ($\beta_\delta$ln Δ)], where $\beta_\delta=(2\alpha/\pi)\ln(4/\delta^2)$. In addition the finite factors $C^{(i)}{}_F$, (i=QED, INT, RES), which account for the rest of corrections in the eqs. (5.3) are modified as

$$C^{(i)}{}_F \longrightarrow C^{(i)}{}_F\ (\beta_\mu \rightarrow \beta_\delta) + (2\alpha/\pi)\ (3/2 - \pi^2/3) \tag{6.4}$$

Numerical results are given in sect. (11).

7 - RADIATIVE CORRECTIONS TO BHABHA SCATTERING

The presence of t-channel exchanges in Bhabha scattering makes radiative corrections to this process more complicated than in the pure s-channel case, as, for example, $e^+e^- \rightarrow \mu^+\mu^-$.

The reaction $e^+e^- \rightarrow e^+e^-$, on the other hand, is particularly interesting for its large cross section and could provide a high precision monitor of the beam luminosity. Detailed studies of electro-weak radiative corrections to this process, which have been performed earlier[41-43,12], are all incomplete in some respects.

Indeed the calculation of electro-weak first order corrections, performed in ref. (41), does not extend to the energy range around the Z_0, because of the lack of finite width effects. Those were included in ref. (42), together with the complete treatment of soft photon effects, resummed to all orders. The analytical expressions for the box diagrams in the s and t channels however, were only given in the limit $s \sim M^2$, the left-over terms being of order (α/π). An attempt to improve these results has been made in ref. (43). Finally a treatment of collinear hard photon effects, quite relevant in calorimetric-type experiments, has been given in ref. (12).

Very recently a final and complete description of QED radiative effects for Bhabha scattering has been given in ref. (13), including exact analytical expressions for all one-loop diagrams, and soft and collinear hard photon effects resummed to all orders. Therefore those results include all double logarithmic terms of the form $(\alpha/\pi) \ln (s/m^2) \ln (\Delta, \Gamma/M)$, $(\alpha/\pi) \ln \delta^2 \ln \Delta$, simple logs as

$(\alpha/\pi)\ln(s/m^2)$, $(\alpha/\pi)\ln(\Delta, \Gamma/M, \delta^2)$ resumed to all orders, and all finite terms of orders (α/π). Only hard photon effects of order $(\alpha/\pi)(\Delta,\delta)$, have been neglected and have to be taken into account explicitely, whenever necessary. We summarize here the analysis of ref. (13).

Our considerations apply to a typical experiment in which the requirements of the previous section are satisfied, namely:

(i) The electron-positron pair should be detected back-to-back within a certain acollinearity angle J of a few degrees ($J \lesssim 5°$).

(ii) An electromagnetic calorimeter of finite and small angular resolution δ is centred along the electron and positron directions.

The relevant virtual graphs are shown in Fig. (8). We have:

$$M(s,t) = M_o^{QED}(s)\,[1+\delta^{QED}(s)] - M_o^{QED}(t)\,[1+\delta^{QED}(t)] + M_o^{RES}(s)\,[1+2\delta_V(s)] -$$
$$- M_o^{W}(t)\,[1+2\delta_V(t)] + M_{box}^{QED}(s) - M_{box}^{QED}(t) + M_{box}^{RES}(s) - M_{box}^{W}(t), \tag{7.1}$$

where, as in sects. 2, 3 for the Born terms and s-channel exchanges,

$$M_o^{QED}(s) = (e^2/s)\,J_\mu(s)\,J'_\mu(s), \quad M_o^{QED}(t) = (e^2/t)\,J_\mu(t)\,J'_\mu(t),$$

$$M_o^{RES}(s) = [\,e^2/(s-M_R^2)\,]\,[\,f_V J_\mu(s) + f_A A_\mu(s)\,]\,[\,f_V J'_\mu(s) + f_A A'_\mu(s)\,],$$

$$M_o^{W}(t) = [\,e^2/(t-M^2)\,]\,[\,f_V J_\mu(t) + f_A A_\mu(t)\,]\,[\,f_V J'_\mu(t) + f_A A'_\mu(t)\,],$$

$$M_{box}^{QED}(s) = (2\alpha^2/s)\,\{J_\mu(s)\,J'_\mu(s)\,[\,V_1^\gamma(s) + 2\pi\, iV_2^\gamma(s)\,] + A_\mu(s)\,A'_\mu(s)\,.$$
$$.\,[\,A_1^\gamma(s) + 2\pi\, iA_2^\gamma(s)\,]\,\},$$

$$M_{box}^{QED}(t) = (2\alpha^2/t)\,\{J_\mu(t)\,J'_\mu(t)\,[\,V_1^\gamma(t) + 2\pi\, iV_2^\gamma(t)\,] + A_\mu(t)\,A'_\mu(t)\,.$$
$$.\,[\,A_1^\gamma(t) + 2\pi\, iA_2^\gamma(t)\,]\,\},$$

$$M_{box}^{RES}(s) = (\alpha/2\pi)\,\{\,M_o^{RES}(s)\,[V_1^Z(s) + 2\pi\, iV_2^Z(s)] +$$
$$+ M_5^{RES}(s)\,[V_1^Z(s) + 2\pi\, iV_2^Z(s)]\,\},$$

$$M_{box}^{W}(t) = (\alpha/2\pi)\,\{\,M_o^{W}(t)\,[V_1^Z(t) + 2\pi\, iV_2^Z(t)] + M_5^{W}(t)\,[A_1^Z(t) + 2\pi\, iA_2^Z(t)]\,\}, \tag{7.2}$$

with

$$J_\mu(s) = \bar{v}\,(k_2)\,\gamma_\mu u(k_1),\quad J'_\mu(s) = \bar{u}\,(q_1)\,\gamma_\mu v\,(q_2),\quad A_\mu(s) = \bar{v}\,(k_2)\,\gamma_\mu\,\gamma_5 u(k_1),$$

$$A'_\mu(s) = \bar{u}\,(q_1)\,\gamma_\mu\,\gamma_5\,v\,(q_2),\quad J_\mu(t) = \bar{u}\,(q_1)\,\gamma_\mu u\,(k_1),\quad J'_\mu(t) = \bar{v}\,(k_2)\,\gamma_\mu v\,(q_2),$$

$$A_\mu(t) = \bar{u}\,(q_1)\,\gamma_\mu\,\gamma_5\,u\,(k_1),\quad A'_\mu(t) = \bar{v}\,(k_2)\,\gamma_\mu\,\gamma_5 v\,(q_2), \tag{7.3}$$

and

$$f_V = (4\sin^2\theta_w - 1)\,/\,4\sin\theta_w\cos\theta_w,\quad f_A = -\,1/4\ \sin\theta_w\ \cos\theta_w\,,$$

θ_w being the weak mixing angle. Moreover the following notations are used:

$$s = (k_1+k_2)^2 = 4E^2,\quad t = (k_1-q_1)^2 = -\,(s/2)\,(1-\cos\theta),\quad u = (k_1-q_2)^2 =$$

$$= (-\,s/2)\,(1+\cos\theta),\quad z = \cos\theta,\quad a = \sin\theta/2,\quad b = \cos\theta/2\,.$$

The matrix elements $M_5^{RES}(s)$ and $M_5^{W}(t)$ are defined as $M_5^{RES,W} = M_0^{RES,W}\,(\gamma_\mu \rightarrow \gamma_\mu\gamma_5,\ \gamma_\mu\gamma_5 \rightarrow \gamma_\mu)$.

The radiative factors in eq. (7.1) are defined as follows (see Fig. 8):

$$\delta^{QED}(x) = 2\delta_v(x) + \delta_{vp}\,(x),\quad (x = s, t\,), \tag{7.4}$$

with the vertex and vacuum polarization parts given by (see sect. 3.2)

$$\delta_v\,(s) \equiv \delta_v^R\,(s) + i\delta_v^I\,(s) = \{-\,(1/2)\,\beta_e \ln\,(2E/\lambda) + (\alpha/2\pi)\,[(1/2)\ln^2(s/m^2) - \ln\,(s/m^2)] +$$

$$+\,(3/8)\,\beta_e + (\alpha/\pi)\,(\pi^2/3 - 1/4)\,\} + i\,(\alpha/\pi)\,[\,\pi \ln\,(2E/\lambda) - (3/4)\pi\,],$$

$$\delta_v\,(t) = -\,(1/2)\,\beta_e \ln\,(2E/\lambda) - (2\,\alpha/\pi)\ln a \ln\,(2E/\lambda) + (\alpha/2\pi)\,[\,(1/2) - \ln^2(s/m^2) -$$

$$-\ln\,(s/m^2)] + (3/8)\,\beta_e + (\alpha/\pi)\,[(3/2)\ \ln a - \ln^2 a\,] + (\alpha/\pi)\,(\pi^2/12\ - 1/4\,)\,, \tag{7.5}$$

and

$$\delta_{VP}(s) \equiv \delta_{VP}^{R}(s) + i\,\delta_{VP}^{I}(s) = (\alpha/3\pi) \sum_{i=l,q} Q^2_i\,[\,\ln(s/m_i^2) - 5/3\,] - i\,(\alpha/3) \sum_{i=l,q} Q^2_i\,),$$

$$\delta_{VP}(t) = (\alpha/3\pi) \sum_{i=l,q} Q^2_i\,[\,\ln(-t/m_i^2) - 5/3\,], \qquad (7.6)$$

with

$$Q_l^2 = 1,\quad Q_q^2 = 4/3\ (\text{up}),\quad Q_q^2 = 1/3\ (\text{down}).$$

The $\gamma\gamma$ and γZ box diagrams contributions $M^{QED}{}_{box}(s)$ and $M^{RES}{}_{box}(s)$ have been discussed in sect. 3. The expressions for the t-channel box diagrams $M^{QED}{}_{box}(t)$ and $M^{W}{}_{box}(t)$ can be easily obtained by applying the crossing relation $s \leftrightarrow t$ in eqs. (3.17) and (3.24), with $M^2{}_R \rightarrow M^2$ and $t \rightarrow t + i\varepsilon$, when necessary. One recovers then the $\gamma\gamma$ results (42)

$$V_1{}^{\gamma}(t) = 8 \ln b \ln(2E/\lambda) + 8 \ln a \ln b + (1/4)\pi^2 (1-b^4) + [\,(1-b^4)/b^4] \ln^2 a +$$

$$+ (1-b^4) \ln^2(a/b) + (a^2/b^2) \ln a + a^2 \ln(a/b) = 8 \ln b \ln(2E/\lambda) + V^{\gamma}{}_{1f}(t),$$

$$A_1{}^{\gamma}(t) = -(1/4)\pi^2(1-b^4) + [(1-b^4)/b^4] \ln^2 a - (1-b^4) \ln^2(a/b) + (a^2/b^2) \ln a - a^2 \ln(a/b),$$

$$V_2{}^{\gamma}(t) = 2 \ln(2E/\lambda) + (1/2)\,\{4 \ln a - [\,(1-b^4)/b^4\,] \ln a + a^2/2b^2\} \equiv 2 \ln(2E/\lambda) + V_{2f}{}^{\gamma}(t),$$

$$A_2{}^{\gamma}(t) = -(1/2)\,\{\,[\,(1-b^4)/b^4\,] \ln a + a^2/2b^2\,\}. \qquad (7.7)$$

and, similarly to eqs (3.24 - 3.26)

$$V_1{}^{Z}(t) + 2\pi i\, V_2{}^{Z}(t) = (M^2 - t)\,[\,f(t,s,u) - f(t,u,s)] \equiv$$

$$\equiv 4 \ln(\sqrt{s}/\lambda)\,(2 \ln b + i\pi) + V_{1f}{}^{Z}(t) + 2\pi i\, V_{2f}{}^{Z}(t)$$

$$A_1{}^{Z}(t) + 2\pi i\, A_2{}^{Z}(t) = (M^2 - t)\,[\,f(t,s,u) + f(t,u,s)]. \qquad (7.8)$$

It is clear that the exact espression for the γZ box diagrams given above, allows for the complete evaluation of all (α/π) e.m. contributions to the process under consideration. A similar attempt has been made in ref. (43). Although the box contributions have been put in closed form in terms of two functions, analogous to the non-infrared parts of V_i and A_i, their analytical expression are rather cumbersome and include further simple and double logarithms of ($s-M^2$) in addition of the pure infrared terms. This concludes the discussion of the virtual contributions to one-loop corrections.

The analysis of the real photon emission contributions follows closely that sect. 3.1 as far as soft effects are concerned.

Then in terms of the Born cross sections $\delta\sigma_0(i)$ defined in sect. 2 (eqs. 2.19), the bremsstrahlung terms read as

$$d\sigma(1\gamma) = \delta^{QED}(1\gamma) = \sum_{i=1}^{6} d\sigma_0(i) + \delta^{INT}(1\gamma) \sum_{i=7}^{9} \delta\sigma_0(i) + \delta^{RES}(1\gamma)\, d\sigma_0(10), \qquad (7.9)$$

with, using eqs (3.5),

$$\delta^{QED}(1\gamma) = (2\beta_e + 2\beta_{int}) \ln(2E/\lambda) + (2\beta_e + 2\beta_{int}) \ln\Delta -$$

$$- (2\alpha/\pi)\, B(m^2) + (2\alpha/\pi)\, F(a,b), \qquad (7.10a)$$

$$\delta^{int}(1\gamma) = (2\beta_e + 2\beta_{int}) \ln(2E/\lambda) + (\beta_{int} + \beta_e) \ln\Delta + \mathrm{Re}\,(\{\exp[i\delta_R(s)] / \cos\delta_R(s)\}.$$

$$\cdot (\beta_{int} + \beta_e) \ln\{\Delta\, [1 + (\Delta s/M\Gamma) \exp[i\,\delta_R(s)] \sin\delta_R(s)]^{-1}\}) -$$

$$- (2\alpha/\pi)\, B(m^2) + (2\alpha/\pi)\, F(a,b). \qquad (7.10b)$$

$$\delta^{RES}(1\gamma) = (2\beta_e + 2\beta_{int}) \ln(2E/\lambda) + \beta_e \ln\Delta - \beta_e \delta(s, \Delta\omega) \cot\delta_R(s) +$$

$$+ (\beta_e + \beta_{int}) \ln|\Delta\, \{[1 + (\Delta s/M\Gamma) \exp[i\,\delta_R(s)] \sin\delta_R(s)\}^{-1}| -$$

$$- (2\alpha/\pi)\, B(m^2) + (2\alpha/\pi)\, F(a,b), \qquad (7.10c)$$

In experiments where the electrons are observed as a single particle track, then one obtains the final corrected cross section by simply adding the virtual and real corrections from eqs. (7.1) and (7.9) respectively, exponentiating the soft part as usual. When however collinear hard radiation

($k > \Delta\omega$) from the final particles is also detected, as in calorimetric-type experiments, one has to include further corrections, as explicitly indicated in sect. 6. We will first consider the former case. Then we obtain

$$d\sigma_{tot}\,(e^+e^- \to e^+e^-) = \sum_{i=1}^{10} d\sigma_0\,(i)\,(\,C_{infra}^{(i)} + C_F^{(i)}\,), \tag{7.11}$$

where, as in sect.4.,

$$C_{infra}^{(i)} = (\Delta)^{(2\beta_e+2\beta_{int})} \quad (i=1,\ldots,6), \tag{7.12a}$$

$$C_{infra}^{(i)} = (\Delta)^{(2\beta_e+2\beta_{int})}\,[\,1/\cos\delta_R\,(s)]\,\mathrm{Re}\,(\exp[\,i\,\delta_R\,(s)]\,\{\,\Delta\,\{1 + (\Delta s/M\Gamma)\cdot$$

$$\cdot\exp[i\,\delta_R(s)]\,\sin\delta_R(s)\,\}^{-1}\}^{\beta_e}\,\{\Delta\{\Delta + (M\Gamma/s)\,\exp\,[-\,i\,\delta_R(s)]\,/\,\sin\delta_R(s)\}^{-1}\}^{\beta_{int}})$$

$$(i = 7, 8, 9), \tag{7.12b}$$

$$C_{infra}^{(10)} = \Delta^{\beta_e}\,|\,\Delta\,\{1 + (\Delta s/M\Gamma)\,\exp\,[\,i\,\delta_R(s)\,]\,\sin\delta_R(s)\,\}^{-1}\,|^{\beta_e}\,|\Delta\,\{\,\Delta + (M\Gamma/s)\cdot$$

$$\cdot\exp[\,i\,\delta_R(s)]\,/\,\sin\delta_R(s)\}^{-1}\,|^{2\beta_{int}}[\,1-\beta_e\,\delta(s,\,\Delta\omega)\,\cot\delta_R(s)\,]\,, \tag{7.12c}$$

and the finite factors $C_F^{(i)(*)}$ include the leftover contributions of order (α/π):

$$C_F^{(1)} = (3/2)\,\beta_e + (2\alpha/\pi)\,(\pi^2/3\ - 1/2) + (2\alpha/\pi)F(a,b) + 2\delta_{VP}^R(s) + (\alpha/\pi)\,\{V_{1f}^{\gamma}(s) +$$

$$+ [2z\,/\,(1+z^2)]\,A_1^{\gamma}(s)\}, \tag{7.13a}$$

$$C_F^{(2)} = 3/2\,\beta_e + (2\alpha/\pi)\,(\,\pi^2/12 - 1/2) + (2\alpha/\pi)\,[\,(3/2)\ln a - \ln^2 a] + (2\alpha/\pi)F(a,b) +$$

$$+\,\delta_{VP}^R(s) + \delta_{VP}\,(t\,) + (\alpha/2\pi)\,[\,V_{1f}^{\gamma}(s) + A_1^{\gamma}(s) + V_{1f}^{\gamma}(t) + A_1^{\gamma}(t)\,], \tag{7.13b}$$

(*) As usual, the vacuum polarization corrections due to $\delta_{VP}^R(s)$ can be resummed by introducing the running coupling constant $\hat{e}^2(s) = e^2\,/\,[1-\delta_{VP}^R(s)]$ in $M_0^{QED}(s)$.

$$C_F^{(3)} = (3/2)\,\beta_e - (2\alpha/\pi)\,(\pi^2/6 + 1/2) + (4\alpha/\pi)\,[(3/2)\ln a - \ln^2 a)] + (2\alpha/\pi)F(a,b) +$$

$$+ 2\delta_{vp}(t) + (\alpha/\pi)\,\{ V_{lf}^{\gamma}(t) + [\,(b^4 - 1)/(b^4+1)]\,A_1^{\gamma}(t)\,]\}\,, \qquad (7.13c)$$

$$C_F^{(4)} = (3/2)\,\beta_e + (2\alpha/\pi)\,(\pi^2/12 - 1/2) + (2\alpha/\pi)\,[\,(3/2)\ln a - \ln^2 a] + (2\alpha/\pi)F(a,b) +$$

$$+ \delta_{vp}^{R}(s) + (\alpha/2\pi)\,[\,V_{lf}^{\gamma}(s) + A_1^{\gamma}(s) + V_{lf}^{Z}(t) + A_1^{Z}(t)\,], \qquad (7.13d)$$

$$C_F^{(5)} = (3/2)\,\beta_e - (2\alpha/\pi)\,(\pi^2/6 + 1/2) + (4\alpha/\pi)\,[\,(3/2)\ln a - \ln^2 a] + (2\alpha/\pi)\,F(a,b) +$$

$$+ \delta_{vp}(t) + (\alpha/2\pi)\,[\,V_{lf}^{\gamma}(t) + V_{lf}^{Z}(t)] + (\alpha/2\pi)\,\{[(f_V^2 + f_A^2)\,b^4 - (f_V^2 - f_A^2)\,]\,/$$

$$/\,[(f_V^2 + f_A^2)\,b^4 + (f_V^2 - f_A^2)\,]\}\;[\,A_1^{\gamma}(t) + A_1^{Z}(t)\,]\,, \qquad (7.13e)$$

$$C_F^{(6)} = (3/2)\,\beta_e - (2\alpha/\pi)\,(\pi^2/6 + 1/2) + (4\alpha/\pi)\,[\,(3/2)\ln a - \ln^2 a] + (2\alpha/\pi)\,F(a,b) +$$

$$+ (\alpha/\pi)\,V_{lf}^{Z}(t) + (\alpha/\pi)\,A_1^{Z}(t)\,\{\,b^4\,[\,(f_V^2 + f_A^2)^2 + 4\,f_V^2\,f_A^2] - (f_V^2 - f_A^2)^2\,\}\,/$$

$$/\,\{\,b^4\,[\,(f_V^2 + f_A^2)^2 + 4\,f_V^2\,f_A^2] + (f_V^2 - f_A^2)^2\,\}\,, \qquad (7.13f)$$

$$C_F^{(7)} = (3/2)\,\beta_e + (2\alpha/\pi)\,(\pi^2/3 - 1/2) + (2\alpha/\pi)F(a,b) + \delta_{VP}^{R}(s) +$$

$$+ [\,I'(s)/R'(s)]\,\delta_{VP}^{I}(s) +$$

$$+ (\alpha/2\pi)\,\{\,V_{lf}^{\gamma}(s) + V_{lf}^{Z}(s) + 2\pi\,[\,I'(s)/R'(s)]\,[\,V_2^{\gamma}(s) - V_2^{Z}(s)]\} +$$

$$+ (\alpha/2\pi)\,\{[\,f_A^2(1+z^2) + f_V^2\,2z\,]\,/\,[\,f_V^2\,(1+z^2) + f_A^2\,2z\,]\}\,\{\,A_1^{\gamma}(s) + A_1^{Z}(s) +$$

$$+ 2\pi\,[\,I'(s)/R'(s)]\,[\,A_2^{\gamma}(s) - A_2^{Z}(s)]\}\,, \qquad (7.13g)$$

$$C_F^{(8)} = (3/2)\,\beta_e + (2\alpha/\pi)\,(\pi^2/12 - 1/2) + (2\alpha/\pi)\,[\,(3/2)\,\ln a - \ln^2 a] + (2\alpha/\pi)\,F(a,b) +$$

$$+\ \delta_{VP}(t) + (\alpha/2\pi)\,[\,V_{1f}^{\gamma}(t) + V_{1f}^{Z}(s) + A_1^{\gamma}(t) + A_1^{Z}(s)\,] + \alpha\,[\,I'(s)\,/\,R'(s)] \cdot$$

$$\cdot\,[\,V_{2f}^{\gamma}(t) - V_2^{Z}(s) + A_2^{\gamma}(t) - A_2^{Z}(s) + 3/2\,], \qquad (7.13h)$$

$$C_F^{(9)} = (3/2)\,\beta_e + (2\alpha/\pi)\,(\pi^2/12 - 1/2) + (2\alpha/\pi)\,[\,(3/2)\ln a - \ln^2 a] + (2\alpha/\pi)F(a,b) +$$

$$+ (\alpha/2\pi\,[\,V_{1f}^{Z}(s) + V_{1f}^{Z}(t) + A_1^{Z}(s) + A_1^{Z}(t)\,] + \alpha\,[\,I'(s)\,/\,R'(s)] \cdot$$

$$\cdot\,[\,V_{2f}^{Z}(t) - V_2^{Z}(s) + A_2^{Z}(t) - A_2^{Z}(s) + 3/2\,], \qquad (7.13i)$$

$$C_F^{(10)} = (3/2)\,\beta_e + (2\alpha/\pi)\,(\pi^2/3 - 1/2) + (2\alpha/\pi)\,F(a,b) + (\alpha/\pi)\,\{\,V_{1f}^{Z}(s) +$$

$$+ [4f_V^2\, f_A^2(1+z^2) + (f_V^2 + f_A^2)^2\, 2z]\,/\,[(f_V^2 + f_A^2)^2(1+z^2) + 8\, f_V^2 f_A^2 z]\, A_1^{Z}(s)\,\},$$

$$(7.13j)$$

with

$$R'(s) + iI'(s) \equiv s/(s - M_R^2).$$

We consider now the case of calorimetric-type measurements, where collinear hard radiation from the final particles is detected within a small cone of half opening angle δ ($\delta \ll 1$). Then, as discussed in sect. 6, one has to add the correction factor (6.2) to each term in the r.h.s. of eq. (7.11), taken to first order in α:

$$\delta^{coll}(i) = d\sigma_o(i)\,(4\alpha/\pi)\,[\,(\ln(E/\Delta\omega) - (3/4))\,\ln(E\delta/m) - (1/2)\,\ln(E/\Delta\omega) +$$

$$+ (1/2)\,(9/4 - \pi^2/3)\,]\,. \qquad (7.14)$$

Then after adding eq.(7.14) to eq. (7.11) the overall correction factor to the Born cross-sections can be simply obtained from eq. (7.11), to first order in α, by the substitution

$$\beta_e\,(\ln\Delta + 3/4) \rightarrow (2\alpha/\pi)\,[\,\ln(4/\delta^2)\,\,(\ln\Delta + 3/4) + (3/2 - \pi^2/3)\,]. \qquad (7.15)$$

From the known results on the exponentiation of soft and collinear divergences, one then obtains the final result,

$$d\sigma_{tot}\,(e^+e^- \to e^+e^-) = \sum_{i=1}^{10} d\sigma_o\,(i)\,[\,\tilde{C}^{(i)}_{infra} + \tilde{C}^{(i)}_F] \qquad (7.16)$$

where

$$\tilde{C}^{(i)}_{infra} = C^{(i)}_{infra} \cdot \Delta^{\beta_\delta - \beta_e} \quad ,$$

$$\tilde{C}^{(i)}_F = C^{(i)}_F + (3/4)\ (\beta_\delta - \beta_e) + (2\alpha/\pi)\,(3/2 - \pi^2/3)$$

with

$$\beta_\delta = (4\alpha/\pi)\,\ln(2/\delta).$$

So far large-angle hard bremsstrahlung effects have not been considered. As long as the electron-positron pair is detected back-to-back with good collinearity, the accuracy of the formulae given above is of order (α/π) (Δ,δ). Hard photon effects have to be taken into account otherwise (see next section).

To conclude this section, the complete analysis presented above, which includes the exact contributions of one-loop diagrams and the whole series of double and simple logarithms in exponentiated from, allows one to use the process of Bhabha scattering as high precision monitor of luminosity at LEP/SLC anergies.The accuracy achieved so far is of order $\alpha^2 \ln^2 (s/m^2)$~1%. A complete calculation of radiative corrections up two loops would further reduce the theoretical uncertainty to much a higher level.

8. - HARD BREMSSTRAHLUNG CORRECTIONS

In the previous sections we have considered radiative corrections to various e^+e^- processes due to virtual and soft-photon effects or the emission of hard bremsstrahlung collinear to the final charged paticles. As we have stressed earlier, the level of accuracy of our analytical formulae is of order $(\alpha/\pi)\Delta$, $(\alpha/\pi)\delta$ with Δ and δ are the energy and angular resolutions. This is certainly sufficient as long as the final (e^+e^-, $\mu^+\mu^-$, ...) pair is detected to a good collinearity and, if it is the case, the electromagnetic shower cones are small. The results given above include the largest contributions from double and single logarithms to first and higher orders in α, the latter being often larger than simple $o(\alpha/\pi)$ terms.

When however the above criteria are not satisfied, namely the kinematical domain allowed in the experiment is a full three body phase space, then an additional term has to be added to the previous formulae, corresponding to hard photon bremsstrahlung. This effects is normally included in a way which is suitable for a Monte Carlo simulation of events. This also allows to account for all features of experimental detection.

It has to be stressed however that the numerical treatments usually available(44) are based on first order cross sections only, and should not be used as a full description of e.m. radiative effects, due to the relevance of higher-order terms. Furthermore numerical integration of the $o(\alpha)$ cross section in the infrared and collinear regions requires a careful analysis and is computer time consuming, while simple analytic expressions can be used alternatively. Therefore the numerical study of hard photon effects has to be appropriately performed in the large angle regions, and if it is the case, the corresponding term added to those discussed in the previous sections.

The hard bremsstrahlung contribution can be obtained on the basis of the general result(45) on the factorization properties of the cross sections in gange theories. For example, in the process $e^+e^-\rightarrow\mu^+\mu^-\gamma$, one obtains(30).

$$\frac{d\sigma^B}{d\Omega_\mu\, d\Omega_\gamma\, dk} = \frac{\alpha^3}{2\pi^2 s}\, \frac{|q_+|\, k}{2E - k + k\cos\theta_\gamma}\, X \qquad (8.1)$$

or

$$\frac{d\sigma^B}{d\Omega_\mu\, dq_+^o\, dq_-^o\, d\phi_\gamma} = \frac{\alpha^3}{2\pi^2 s}\, X, \qquad (8.2)$$

where

$$X = \frac{-m_e^2}{2s'^2}\left[A(s')\left(\frac{t^2}{k_-^2} + \frac{t'^2}{k_+^2}\right) + B(s')\left(\frac{u^2}{k_-^2} + \frac{u'^2}{k_+^2}\right)\right]$$

$$+ \frac{1}{4\, s'\, k_+ k_-}\left[A(s')\ (t^2 + t'^2) + B(s')\,(u^2 + u'^2)\right]$$

$$- \frac{m_\mu^2}{2s^2}\left[A(s)\left(\frac{t^2}{k'^2_-} + \frac{t'^2}{k'^2_+}\right) + B(s)\left(\frac{u'^2}{k'^2_-} + \frac{u^2}{k'^2_+}\right)\right] +$$

$$+\frac{1}{4\,s\,k'_+\,k'_-}\,[\,A\,(s)\,\,(t^2+t'^2)+B\,(s)\,(\,u^2+u'^2\,)]$$

$$+\frac{1}{4\,ss'}\,\left[\frac{u}{k'_-\,k_+}+\frac{u'}{k_-\,k'_+}-\frac{t}{k_+\,k'_+}-\frac{t'}{k_-\,k'_-}\right]\,[\,C\,(s,s')\,(t^2+t'^2)+$$

$$+\,D\,(s,s')\,(\,u^2+u'^2\,)\,]+\frac{(\,s-s')M_z\Gamma_z}{2k_-\,k_+\,k'_-\,k'_+}\,\varepsilon_{\mu\nu\rho\sigma}\,p^{\mu}{}_{+}\,p^{\nu}{}_{-}\,q^{\rho}{}_{+}\,q^{\sigma}{}_{-}\,F\,(s,s')\,(u^2-u'^2),\tag{8.13}$$

where ε is the antisymmetric tensor with $\varepsilon_{0123}=1$,

$$s=(\,p_++p_-\,)^2,\qquad t=(\,p_+-q_+\,)^2,\qquad u=(\,p_+-q_-\,)^2,$$
$$s'=(q_++q_-)^2,\qquad t'=(p_--q_-\,)^2,\qquad u'=(\,p_--q_+\,)^2,$$
$$k_\pm=p_\pm\,.\,k,\qquad k'_\pm=q_\pm\,.\,k,$$

and

$$A\,(s)\;=1+\frac{2s(s-M^2{}_z)\,(C^2{}_V-C^2{}_A)}{|\,Z(s)|^2}+\frac{s^2\,(C^2{}_V-C^2{}_A)^2}{|\,Z(s)|^2}\,,$$

$$B\,(s)\;=1+\frac{2s(s-M^2{}_z)\,(C^2{}_V+C^2{}_A)}{|\,Z(s)|^2}+\frac{s^2\,[(C^2{}_V+C^2{}_A)^2+4C^2{}_V\,C^2{}_A)]}{|\,Z(s)|^2}$$

$$C\,(s,s')\;=1+\left[\frac{s(s-M^2{}_z)}{|\,Z(s)|^2}+\frac{s'(s'-M^2{}_z)}{|\,Z(s')|^2}\right](C^2{}_V-C^2{}_A)$$

$$+ \frac{ss'[(s - M^2_z)(s' - M^2_z) + M^2_z \Gamma^2_z] (C^2_V - C^2_A)^2}{| Z(s)|^2 \, | Z(s')|^2} ,$$

$$D(s,s') = 1 + \left[\frac{s(s-M^2_z)}{| Z(s)|^2} + \frac{s'(s'-M^2_z)}{| Z(s')|^2} \right] (C^2_V + C^2_A)$$

$$+ \frac{ss'[s - M^2_z)(s' - M^2_z) + M^2_z \Gamma^2_z] [(C^2_V + C^2_A)^2 + 4C^2_V C^2_A]}{| Z(s)|^2 \, | Z(s')|^2} ,$$

$$F(s,s') = \frac{C_V C_A}{ss'} \left[\frac{s}{| Z(s)|^2} - \frac{s'}{| Z(s')|^2} \right] + \frac{2(s' - s) C_V C_A (C^2_V + C^2_A)}{| Z(s)|^2 \, | Z(s')|^2} . \qquad (8.4)$$

In eq. (8.4) the notations $C_A = -(2\sin 2\theta_W)^{-1}$, $C_V = -C_A(4\sin^2\theta_W - 1)$ and $Z(s) = s - M^2_z + iM_z\Gamma_z$ have been used.

In the bremsstrahlung expression the terms corresponding to initial-state radiation (1st, 2nd term), to final-state radiation (3rd, 4th term) and to their interference are clearly separated. The expression is a generalization of the soft bremsstrahlung expression (3.4).

An analytic, but rather cumbersome analysis of hard photon effects has been alternatively performed in ref. (46).

More recently, new techniques have been developed(47,48) to calculate the multidifferential bremsstrahlung cross sections in gauge theories in terms of helicity amplitudes, with the explicit introduction of polarization vectors for the radiated gauge particles.

This method has been then used(49) to obtain an event generator which can take into account all possible experimental constraints. We dont't give the details here, which can be found in the appropriate bibliography.

9. - NEUTRINO COUNTING

A precision measurement of the Z total decay width is crucial for determining the number of neutrino generations. Indeed, for each $\nu\nu$ pair we have, from eq.(2.8), $\Gamma(Z \rightarrow \nu\nu) \sim .18$ GeV with $\Gamma(Z \to all)$ ~3 GeV. Therefore the detection of an additional neutrino, beyond the three standard generations, requires the measurement of Γ_Z with an accuracy of better than 2%. The suggestion to determine the number of neutrino generations using the reaction $e\bar{e} \to \gamma Z \to \gamma\nu\bar{\nu}$ has been also widely discussed(50,51). The electron-positron colliding beam facility is to be operated at a center-of-mass energy slight above the Z mass, and a photon is observed with the appropriate energy with no additional particles detected.

The reaction $e\bar{e} \to \gamma\nu\bar{\nu}$ proceeds through Z production and W exchange as well. The differential cross section is given by

$$\frac{d\sigma}{dx\,dy} = G^2_F\,\alpha\,s\,\frac{(1-x)\,[(1-(x/2)^2 + x^2\,y^2/4]}{6\pi^2\,x\,(1-y^2)}\;\cdot$$

$$\cdot\left\{\,2 + \frac{N_V\,(g_V{}^2 + g_A{}^2)\,/\,4 + (g_V + g_A)[1 - s(1-x)/M^2_Z]}{[1 - s(1-x)/M^2_Z]^2 + \Gamma^2_Z\,/\,M^2_Z}\right\} \qquad (9.1)$$

where x is the photon energy in units of the beam energy (x=k/E),y is the cosine of the photon angle with respect to the incident beam direction, N_V is the number of low mass neutrinos and, as usual, $g_A = -1$, $g_V = -1 + 4\sin^2\theta_W$.

Initially it has been proposed(51) to carry out the experiment at a c.m. energy of about 15 GeV above the Z mass , with an effective cross section of about 2.5 x 10^{-2} nb. A potentially serious source of background is beam-beam bremsstrahlung. Indeed the total cross section for $e\bar{e} \to e\bar{e}\gamma$ is many orders of magnitude larger than for $e\bar{e} \to \nu\bar{\nu}\gamma$. Due to the strong peaking in the forward direction however, appropriate cuts in the photon angles can reduce sensibly the effective cross section of the background, down to a level which is comparable with the expected signal.

More recently the possibility has been discussed(52-54) to carry out the experiment at c.m. energies around the Z. Indeed present detectors allow a reasonable accuracy in the measurement of low energy photons, and one could therefore exploit the high statistics which can reached be on the resonance.

The expected background - radiative Bhabha scattering - represents however the main problem, and has to be calculated quite reliably. In fact, a recent study(53) suggesting a reasonable control of the situation to an accuracy level of (4-5) standard deviations, has been questioned in subsequent papers(12,54), which show that the background is comparable or even larger than the expected signal so that very accurate subtractions have to be performed in actual experiments. In particular it is essential that the minimum photon emission angle θ_{min} be reasonably large (θ_{min} > 20°) in order thet the signal could dominate the background, therefore allowing the neutrino counting experiment to be feasable. For a detailed discussion of this problem we address the reader to refs. (12,54).

10. - HIGHER ORDER WEAK EFFECTS

So far we have considered purely e.m. corrections to the Born amplitudes for the various process. The only higher order weak effects taken into account are those which can be reabsorbed into a redefinition of the basic parameters (M_Z, Γ_Z, $\sin^2\theta_W$) and the renormalization of the fine structure constant. Other purely weak effects, which are peculiar of the standard model, are contained in the full one-loop amplitude through the exchange of the heavy bosons of the theory. An additional dependence on unknown parameters, such as the Higgs boson mass m_H, the top quark mass m_t, etc., is then introduced into the various quantities which could provide a genuine test of the theory.

A complete discussion of the various effects is beyond the aim of the present paper and can be found in the literature[55,56,57]. We only present here a brief survey of the main results concerning the role played by the corrections to the forward-backward and left-right asymmetries. At the resonance ($s \simeq M^2_Z$) the lowest order expression for $e\bar{e} \to \mu\bar{\mu}$ is (see eq. 2.15')

$$A_{FB} \simeq 3 \frac{v^2 a^2}{(v^2 + a^2)^2} \simeq 3\,(1 - 4 \sin^2 \theta_w)^2, \qquad (10.1)$$

which is very small and quite sensitive to higher order effects. Indeed, using eqs.(2.3 - 2.5) to express $\sin^2\theta_W$ in terms of M_Z, the accurately known quantities α and G_F and the radiative correction Δr, one obtains[55]

$$\sin^2 \theta_w = (1/2) \{ 1 - [1 - 4\mu^2 / (M^2_Z (1-\Delta r))]^{1/2} \} \qquad (10.2)$$

Then, it follows, for M_Z=94 GeV

$$A_{FB}(s= M^2_Z) = 0.13 \quad , \quad \Delta r=0 \;\; (\sin^2\theta_W = 0.195) \qquad (10.3)$$

and

$$A_{FB}(s= M^2_Z) = 0.055 \quad , \quad \Delta r=0.07 \;\; (\sin^2\theta_W = 0.216) \qquad (10.3')$$

This shows the relevance of the effects of the standard model prediction Δr=0.07 on the forward-backward asymmetry. On the other hand it clearly reinforces the importance of an accurate understanding of purely e.m. effects.

Furthermore, the left over weak corrections appearing in the full one-loop amplitude affect of an additional ~10% the result (10.3'). In fact, as shown in ref. (56), one obtains for M_Z=94 GeV, 10GeV $\leq m_H \leq$ 1000 GeV and 30 GeV $\leq m_t \leq$ 90 GeV

$$0.0619 \leq A_{FB}\ (s{=}M^2{}_Z) \leq 0.0532 \qquad (10.4)$$

The reason for such a sizeable effect can be traced to the fact that the zeroth order expression for A_{FB} is suppressed by the very small factor $v^2 \sim (1 - 4\sin^2\theta_w)^2$ (eq. 10.1), whilst the radiative corrections contain contributions of $O(\alpha \sin^2\theta_w/\pi)$ is not inhibited by v^2.

A similar but smaller effect is found for the left-right asymmetry. At $s \simeq M^2{}_Z$ the lowest order expression is (eq. 2.16')

$$A_{LR} \simeq 2\ (va/(v^2+a^2))\ P_e \simeq 2\,(1-4\sin^2\theta_w)^2\,P_e \qquad (10.5)$$

which, with the help of eq.(10.2) and for P_e=1, leads to

$$A_{LR}(s{=}M^2{}_Z) = 0.42 \quad , \quad \Delta r = 0$$

$$A_{LR}(s{=}M^2{}_Z) = 0.27 \quad , \quad \Delta r = 0.07 \qquad (10.6)$$

in analogy to eqs(10.3-3')

The effect of the leftover one-loop weak corrections on A_{LR} is less important than in the case of A_{FB}, due to the only linear dependence on v of the zeroth-order term. Indeed, for the same range of values assumed above for m_H and m_Z one finds

$$0.26 \leq A_{LR}(s{=}M^2{}_Z) \leq 0.28 \qquad (10.7)$$

From the above discussion it follows that an accurate test of the weak corrections and of the related dependence on m_H, the number of families, etc., or the detection of possible effects of new physics beyond the standard model require a precision of $\lesssim 1\%$ and consequently an even better control of pure e.m. effects.

11. - DISCUSSION AND NUMERICAL RESULTS

In the preceding sessions we have presented a detailed study of higher-order electromagnetic effects in the reaction $e\bar{e} \to Z,\ \gamma \to f\bar{f}$, in the framework of the standard electro-weak model, for c.m. energies around the Z mass, and unpolarized electon positron beams. Infrared factors have been considered to all orders, together with complete first order corrections.

Collinear hard-photon corrections have also been discussed, and have to be explicitly included for the case of calorimetric-type experiments. The only higher-order weak effects taken into account are those which can be reabsorbed into a redefinition of the basic parameters (M_Z, Γ_Z, $\sin^2\theta_W$) and the renormalization of the fine structure constant.

These results are given by simple analytical expression in terms of the basic parameters and the experimental resolutions, and are of immediate phenomenological application. As already emphasized earlier, the description of e.m. radiative effects by first order corrections only, as often done in numerical analyses for experimental applications, is quite inadequate and unreliable, due to the relevance of higher-order effects.

In order to explicitly show the importance of the radiative effects we have presented, we show, in figs. (9-12), as an example, some numerical results for the differential and cross sections, and the forward-backward asymmetry.

We consider six leptons and six quarks. The hadronic vacuum polarization is usually evaluated using a dispersion integral over the low energy cross section $\sigma(e\bar{e}\to had)$ and the QCD predictions at higher energies. A convenient parametrization, is given[55] by

$$\delta^R_{VP}(s) \equiv \mathrm{Re}\,\Pi_{\gamma\gamma}(s) = \mathrm{Re}\,\Pi_{\gamma\gamma}\,(93\ \mathrm{GeV})^2 + (20/9)\ \ (\alpha/\pi)\ \ln\,[(\,s\,/\,(93\ \mathrm{GeV})^2] \qquad (11.1)$$

with

$$\mathrm{Re}\,\Pi_{\gamma\gamma}\,(93\ \mathrm{GeV})^2 = (6 \pm 0.04 \pm 0.05)\ \mathrm{x}\ 10^{-2}$$

where the first error reflects uncertaintiy from the dispersive integral in evaluating the light-quark contributions, and the second error is the uncertainty due to the top-quark mass (20 GeV $\leq m_t \leq$ 60 GeV).

Then in figs. (9) we show the differential cross section ($d\sigma/d\Omega$) for the reaction $e\,\bar{e}\to\mu\,\bar{\mu}$ at various energies, with $\Delta=10^{-1}$ and no use of collinear-photon corrections. We have assumed for simplicity in eqs. (2.5) $\sin^2\theta_W = 1/4$, $\Delta r = 0$ which gives $M_Z \simeq 86$ GeV, $\Gamma_Z \simeq 2.2$ and $\Gamma(Z\to e\bar{e})$ $\simeq 70$ MeV. The comparison is made with the Born cross section called "naive".

In figs. (10) for the same reaction we plot the cross section integrated on the scattering angle θ, for a calorimetric-type experiment with an angular resolution $\delta \simeq 1°$and $\Delta=10^{-1}$ (fig. 10a), $\Delta=10^{-2}$ (fig. 10b). The electroweak parameters are $\sin^2\theta_W = 0.23$, $M_Z \simeq 92$ GeV and $\Gamma = 2.9$ GeV. The dashed curves show the Born cross sections and the dot-dashed ones the first order corrections. The comparison with the full curves corresponding to the all-orders correction, shows the relevance of the effect of higher-orders.

In fig. (11) the $\mu\bar{\mu}$ forward-backward asymmetry is also shown, for the same choice of the parameters and for $\sqrt{s}\sim M_Z$. Notice the shift of the zero, which strongly depends on the value of Δ. From these results, and from the discussion on the smallness of the weak effects in sect. 10, it clearly follows that a precise determination of the standard model weak parameters, and even of the value of Γ_Z is quite sensitive to the energy resolution of the experiments.

The energy dependence of the $\mu\mu$ forward-backward asymmetry is plotted in fig. (12). In absence of photon detection ($\Delta \rightarrow 1$), namely including radiative corrections with bremsstrahlung up to the maximum energy of the electrons, the asymmetry behaves as shown by the solid line in fig. (12).

Similar results are obtained for the integrated forward-backward asymmetry for $q\bar{q}$ production. One of the most common methods which have been envisaged to measure quark asymmetries is to look for leading particle effects in the angular distributions of inclusive produced hadrons. The effect of radiative corrections in this case does not change appreciably the naive expectations[39].

So far we have disregarded, in all our considerations, the energy spread of the colliding beam machine. Although, in principle, this problem does not deserve a particular care because the spread is known and can be taken into account quite precisely, we will briefly discuss it here for the case of formation of a very narrow resonance. This problem was considered[8,58] in conjunction with the analysis of the J/Ψ, and although the physical content of the result is rather transparent, it is still treated incorrectly in the literature[58].

For a finite machine resolution G(W'-W), assumed having a Gaussian form, i.e.

$$G(W'-W) = \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-(W'-W)^2/2\sigma^2} \tag{11.2}$$

with σ the machine dispersion, such that $(\Delta W)_{FWHM} = 2.3548\,\sigma$, the experimentally observed cross section is given by

$$\tilde{\sigma}(W) = \int G(W'-W)\, \sigma(W')\, dW' \tag{11.3}$$

with $\sigma(W')$ the resonant cross section, radiatively corrected.

For a narrow resonance R decaying into hadrons, for example, one has from sect. 5, in the limit ($\Delta\omega \gg \Gamma$),

$$\sigma(W') \simeq \frac{12\pi}{W'^2}\, \frac{\Gamma(R\rightarrow ee)\,\Gamma(R\rightarrow had)}{\Gamma^2(R\rightarrow all)}\, \sin^2\delta_R(W')\left[\frac{\Gamma}{W'\,\sin\delta_R(W')}\right]^{\beta_e} \cdot$$

$$\cdot\,(1-\beta_e\,\delta_R\,\mathrm{ctg}\,\delta_R)\,(1+C_F^{RES}) \tag{11.4}$$

Then, inserting (11.2) and (11.4) in eq. (11.3) one obtains[8] for the observed cross section at the peak

$$\tilde{\sigma}(M) \simeq 6\,\pi^2 \frac{\Gamma_{ee}\,\Gamma_{had}}{\sqrt{2\pi}\,\sigma\, M^2\,\Gamma} \left(\frac{\Gamma}{M}\right)^{\beta_e} \exp\left(\frac{\Gamma}{2\sqrt{2}\,\sigma}\right)^2 \left\{ \mathrm{ercf}\left(\frac{\Gamma}{2\sqrt{2}\,\sigma}\right) + \right.$$

$$\left. + (1/2)\,\beta_e\, E_1(\Gamma^2/8\sigma^2) \right\} (1+C_F^{RES}) \qquad (11.5)$$

For resonances whose total width is smaller than the machine resolution, a simple expansion of eq. (11.5) in powers of $(\Gamma/2\sqrt{2}\,\sigma)$ leads[8] to

$$\tilde{\sigma}(M) \simeq \frac{6\pi^2\,\Gamma_{ee}\,\Gamma_{had}}{\sqrt{2\pi}\,\sigma\, M^2\,\Gamma} (\Gamma/M)^{\beta_e} \left(1+\frac{\Gamma^2}{8\sigma^2}\right)\left\{ 1-\frac{\Gamma}{\sqrt{2\pi}\,\sigma} + \beta_e \left[\ln\frac{2\sqrt{2}\sigma}{\Gamma} - \frac{\gamma}{2}\right]\right\} .$$

$$\cdot\ (1+C_F^{RES}) \qquad (11.6)$$

where $\gamma = 0.5772$ is Euler's constant. Therefore the main radiative factor is $(\Gamma/M)^{\beta_e}$, as physically clear, and not $(2\sqrt{2}\,\sigma/M)^{\beta_e}$, as often reported in the literature[58]. On the other hand, taking the opposite limit $(2\sqrt{2}\,\sigma \ll 1)$ for a resonance whose width is large compared with the energy resolution, e.g. resonances like the ρ, one finds the well known result

$$\tilde{\sigma}(M) \simeq (2/\pi\,\Gamma)\,(\Gamma/M)^{\beta_e}\ \frac{6\pi^2\,\Gamma_{ee}\,\Gamma_{had}}{M^2\,\Gamma}\ (1+C_F^{RES})\,. \qquad (11.7)$$

To conclude, we summarize here the main points of the paper. Precision tests of the standard model demand a very careful treatment of e.m. radiative corrections, well beyond the one loop level. In addition to complete $O(\alpha)$ formulae for variuos processes of interest at LEP/SLC energies, we have presented a detailed treatment of higher order effects, which sums up the full series of double leading logarithms associated to multiple soft and collinear emission, as well as some classes of single logarithms. A full account of e.m. effects below the (1%) level would require complete calculation to two-loops accuracy. Attempts along this direction have been recently under taken[59].

We would like to thank M. Consoli, G. Pancheri and Y.N. Srivastava for many illuminating discussions.

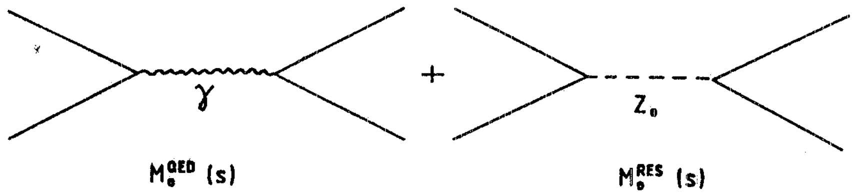


FIG. 1 - Born diagrams with γ and Z_0 in the s-channel.

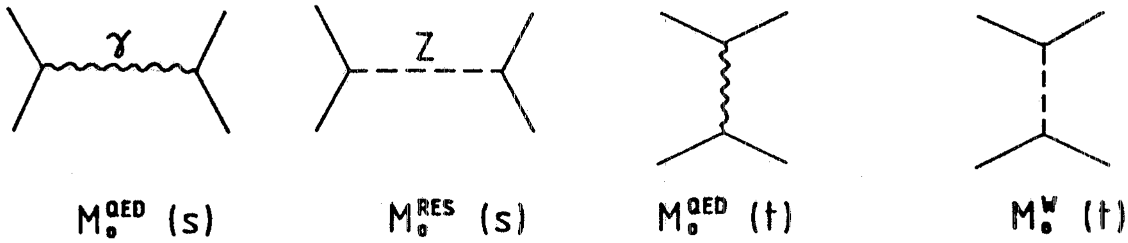


FIG. 2 - Born diagrams for the Bhabha scattering.

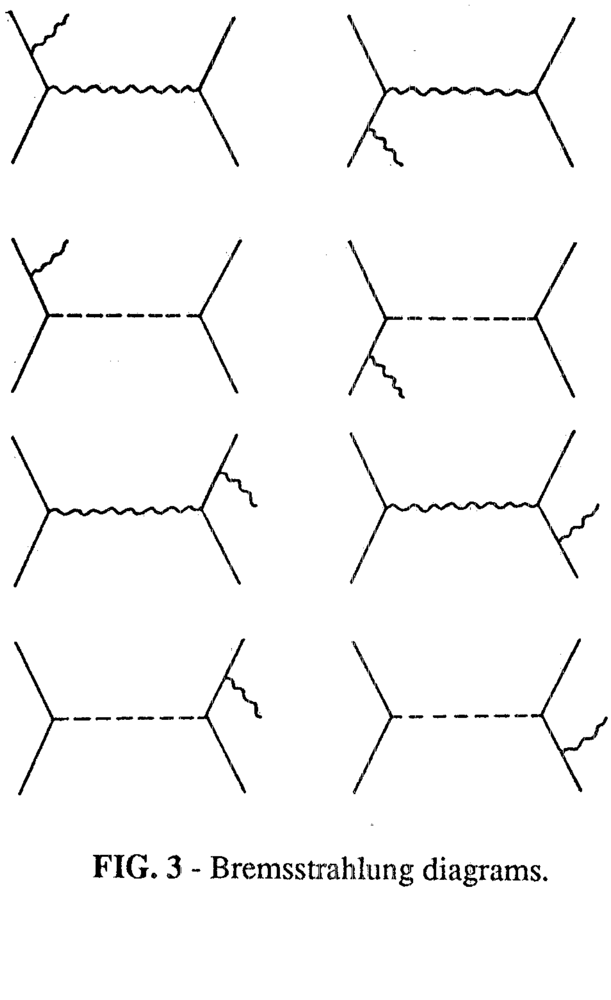


FIG. 3 - Bremsstrahlung diagrams.

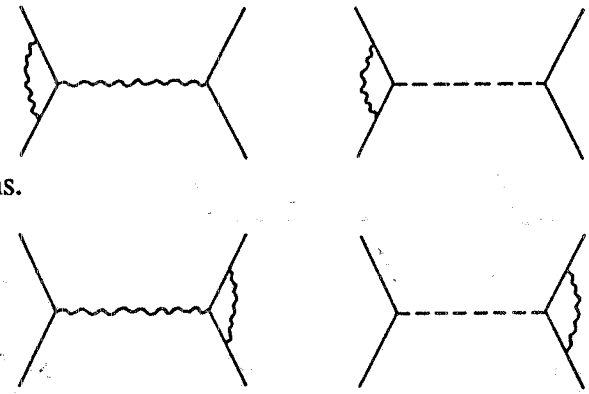


FIG. 4 - The vertex correction diagrams.

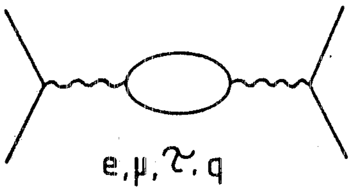
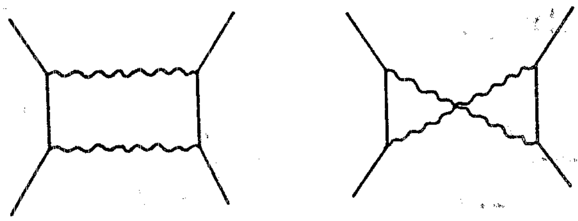


FIG. 5 - The vacuum polarization diagrams.

FIG. 6 - The pure QED box diagrams.



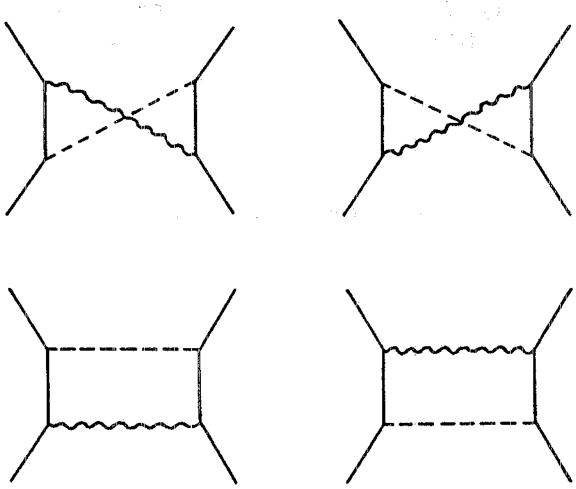


FIG. 7 - The γ, Z_0 box diagrams.

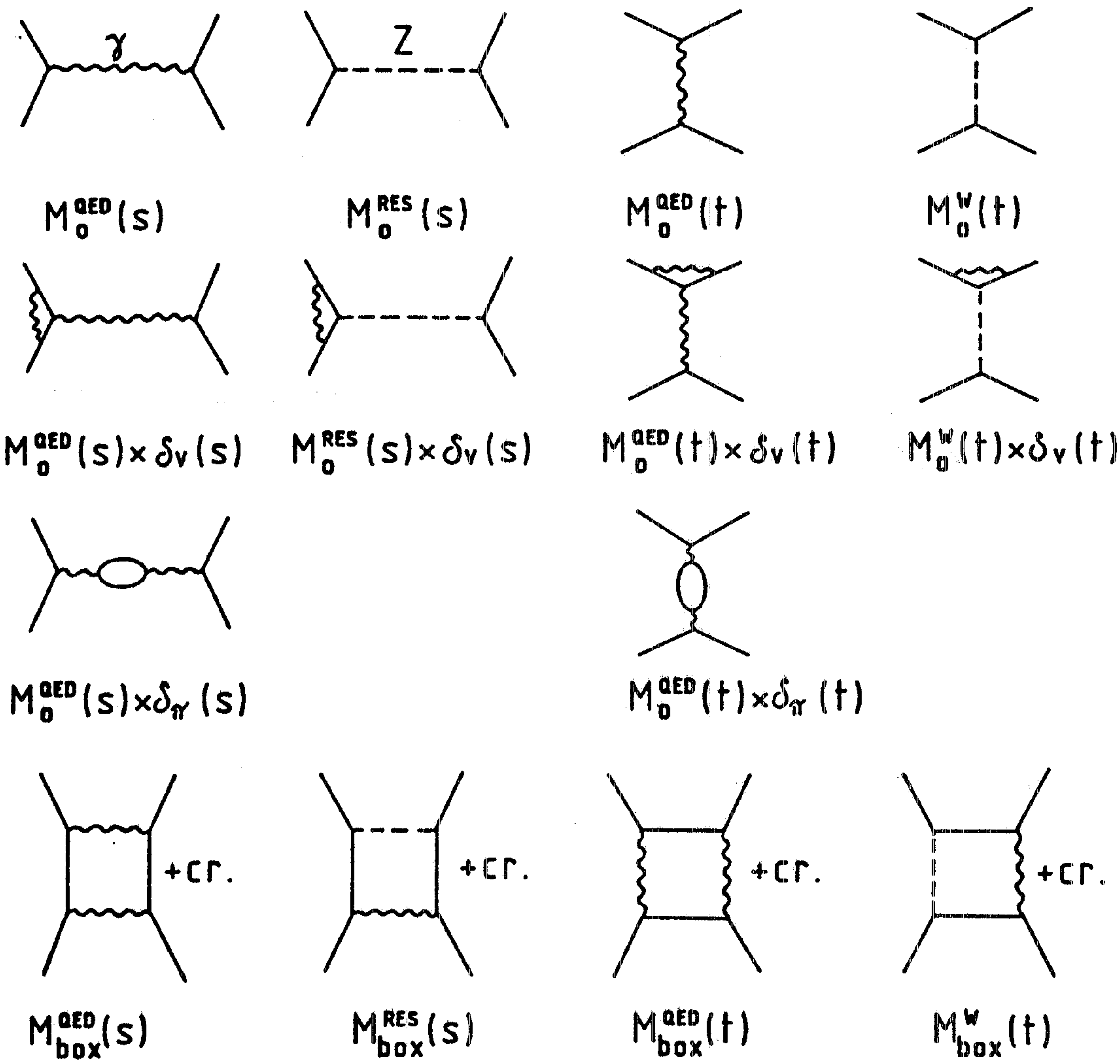


FIG. 8 - Virtual graphs for Bhabha scattering in the s and t channels.

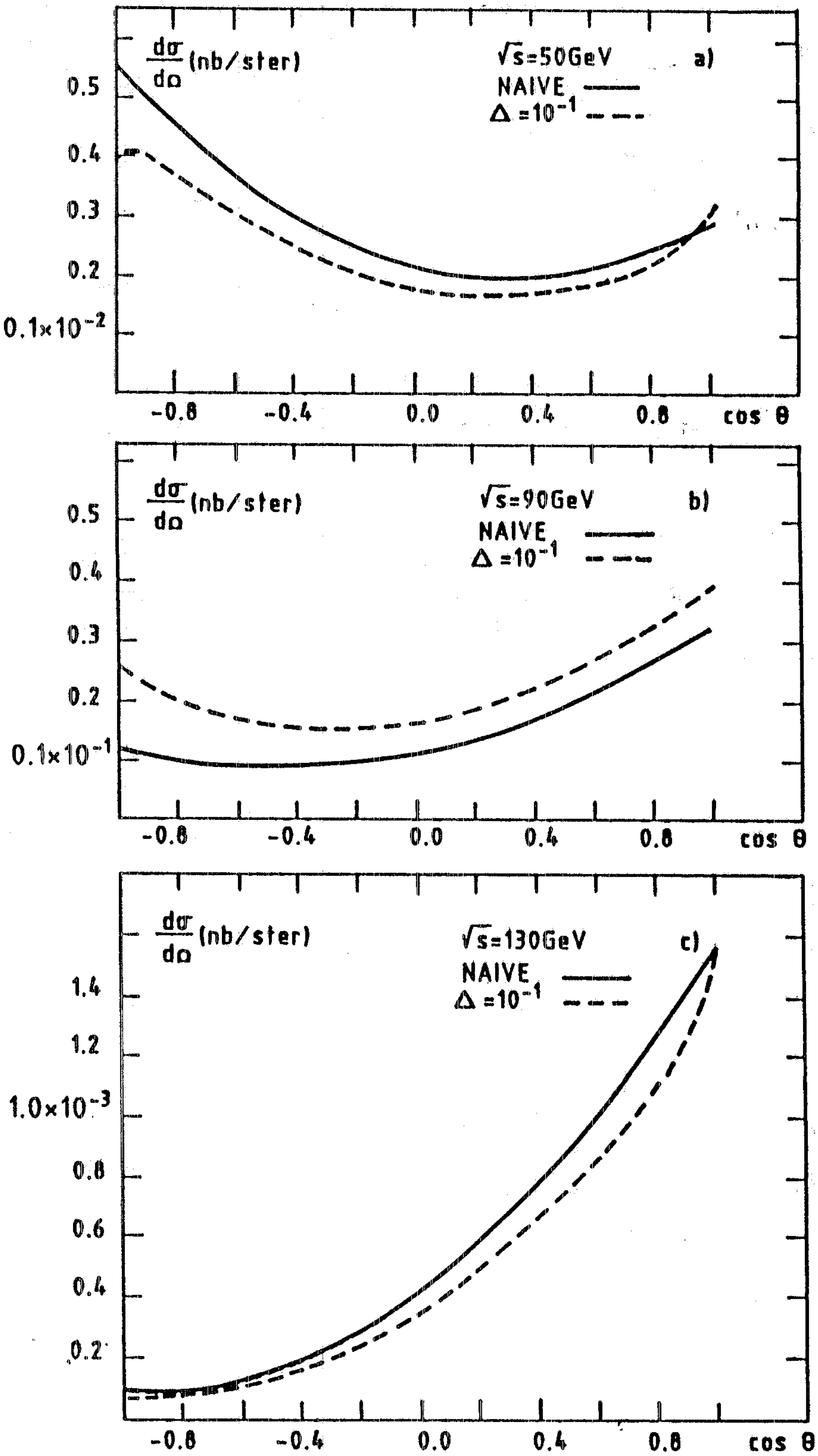


FIG. 9 - Differential cross section dσ/dΩ versus cosθ, for various energies, with and without radiative corrections (naive).

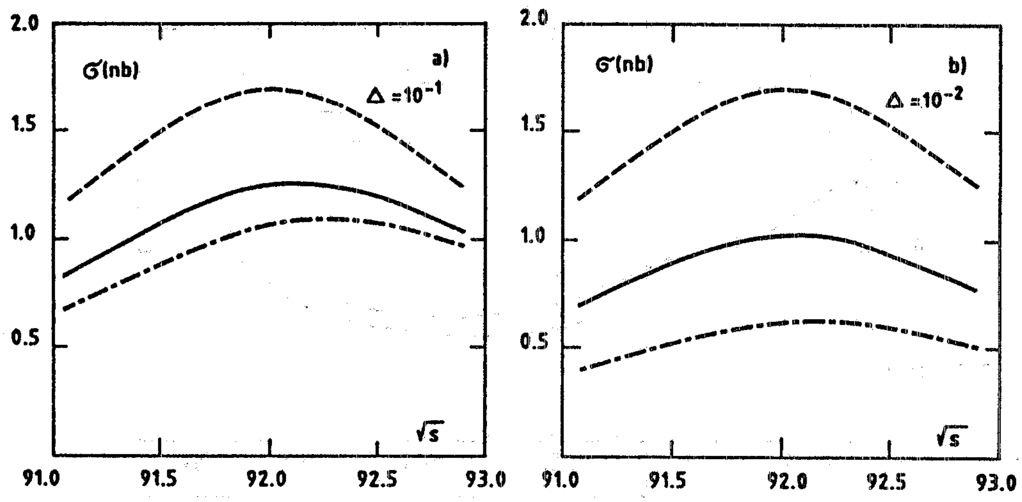


FIG. 10 - a) Cross section integrated on the scattering angle θ, for $\Delta=10^{-1}$ and $\delta \simeq 1°$. The electroweak parameters are M=92 GeV, Γ=2.9 GeV and $\sin^2\theta_W$=0.23. Dashed curve: Born cross section; dot-dashed curve: first order correction; full curve: all orders correction. b) Same as a) for $\Delta=10^{-2}$.

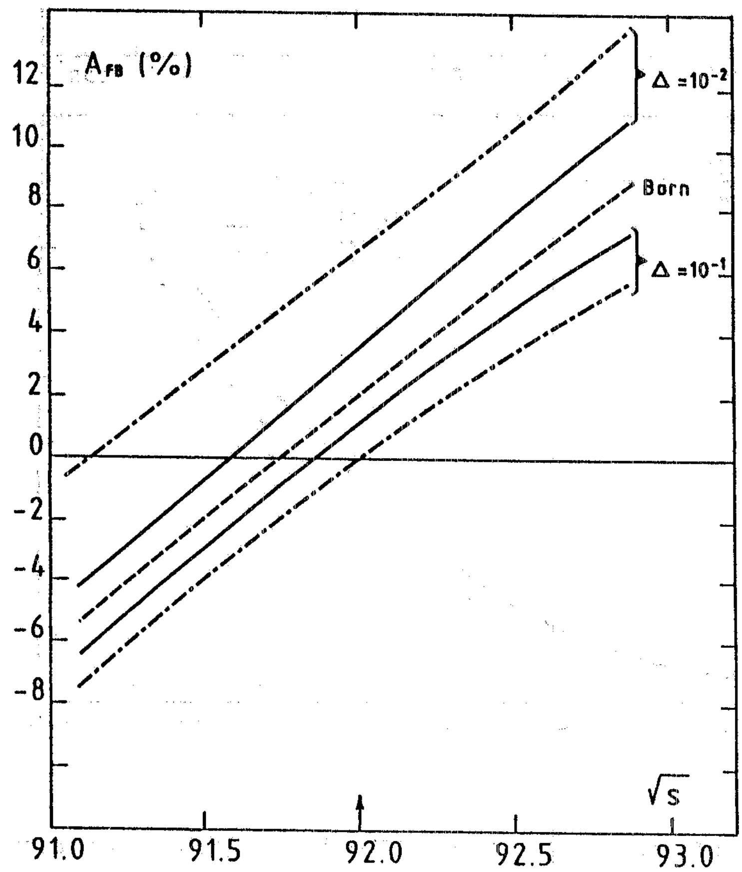


FIG. 11 - Integrated μ forward-backward asymmetry for $\sqrt{s}$~M. The notation is the same of Fig. 10.

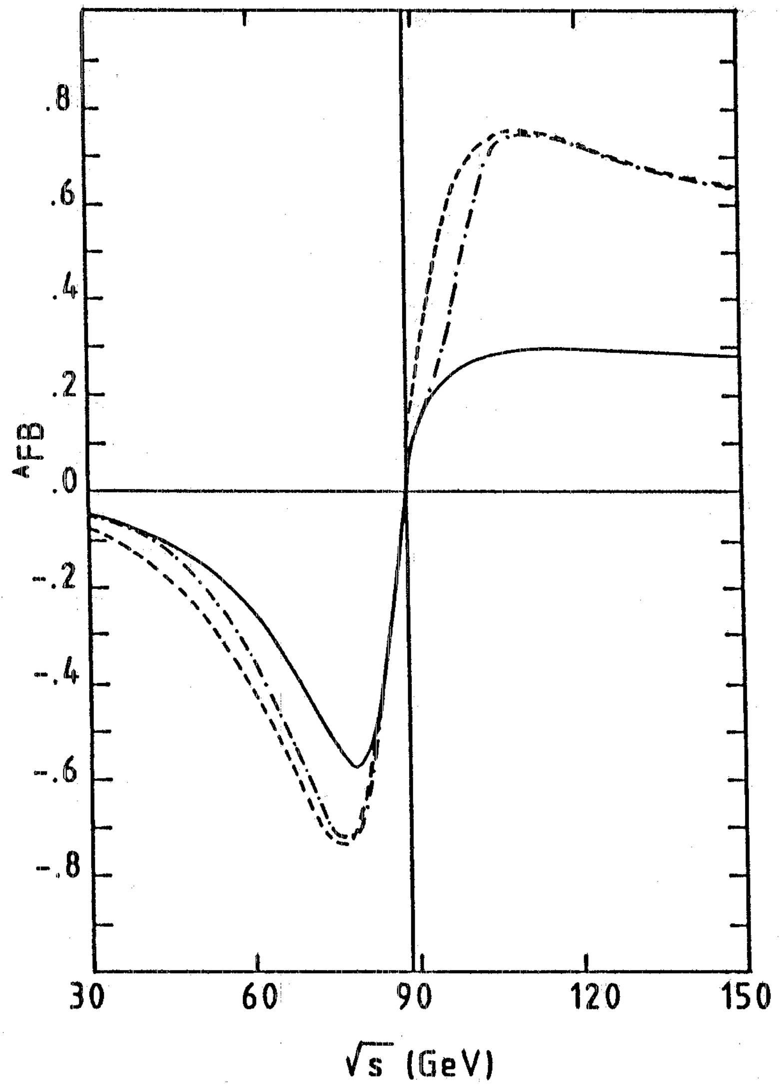


FIG. 12 - The integrated forward-backward asymmetry A_{FB} as a function of the c.m. $\sqrt{s}$. The three cases are the lowest order cross section (dashed line) and the cross section including radiative corrections with bremsstrhalung up to the maximum energy (solid line) and up to 0.2E (dotted line).

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