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FROM LIGHT MODULATION TO COMPLETE MODE-LOCKING IN AN ARGON-ION LASER

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We discussed in a foregoing paper(1) the Harris and Mc Duff theory(2) of the axial mode-locking when applied to unusual long cavities, such as the one we set up on the Adone storage ring of the Frascati National Laboratory for the production of the LADON beam(3).

In Ref. (1), we showed that in the presence of a given polarization P(z,t), quasi stationary forced oscillations of the electric field can be expanded in normal mode eigenfunctions

$$E(z,t) = \Sigma_n A_n(t)\, U_n(z) \tag{1}$$

where the set $U_n(z)$ obeys the usual conditions

$$\int_o^L U_n(z)\, U_m(z)\, dz = \frac{L}{2}\, \delta_{nm} \; ; \frac{d^2U_n}{dz^2} = -K_n^2 U_n(z) \tag{2}$$

where L is the cavity length and $K_n^2 = \Omega^2{}_n/c^2$ ($\Omega_n = n\,(\pi c/L)$ is the geometrical frequency of the n-th mode).

Under the normal running conditions, the polarization P(z,t) relates to the electric field E(z,t) through a generalized susceptibility χ

$$P(z,t) = \varepsilon_o \Sigma_n P_n = \varepsilon_o \Sigma_n (\chi'_n + i\,\chi''_n)\, U_n(z)\, A_n(t) \qquad (3)$$

whose quadrature component χ'' is associated with the extinction coefficient of the medium and accounts also for the gain contribution resulting from the inverted population effect.

We also showed that if a local sinusoidal modulation of the losses is introduced in the cavity, Eq. (3) generalizes into

$$P_n(z,t) = \varepsilon_o \Sigma_n \chi_n(t)\, A_n(t)\, U_n(z) \qquad (4)$$

where

$$\chi_n(t) = \chi_n + i\,[\,\chi''_n + \Delta\chi''(z)\,(1 + \cos\Omega_M t)\,] \qquad (5)$$

and Ω_M is the modulation angular frequency. The contribution due to the modulation $\Delta\chi''(z)$ concentrates in a small region of length l inside the cavity and is related to the modulated single pass losses, by the equation

$$\alpha(t) = (\omega_n l/c)\;\Delta\chi''(z)\,(1 + \cos\Omega_M t) = w\,(1 + \cos\Omega_M t) \qquad (6)$$

where w represents the mean value over one modulation period of the single pass losses.

If the solution of the Maxwell equations is choosen in terms of a set of oscillating frequencies equispaced around the central frequency of the atomic line ω_o, in the exact amount Ω_M, then the Lamb's self-consistency equations become[2]

$$(d\varphi_n/dt + n\,\Delta\omega\ 1/2\ \omega_n\,\chi''_n)\,E_n = -\alpha_c\,(c/2L)\,[\,E_{n+1}\sin(\varphi_{n+1}-\varphi_n) - E_{n-1}\sin(\varphi_n-\varphi_{n-1})\,] \qquad (7)$$

$$dE_n/dt + 1/2\ \omega_n[\,1/Q_n + \chi''_n\,]\,E_n = -\alpha_a\,(c/2L)E_n - \alpha_a\,(c/2L)\,[\,E_{n+1}\cos(\varphi_{n+1}-\varphi_n) + E_{n-1}\cos(\varphi_n-\varphi_{n-1})\,] \qquad (8)$$

where E_n, φ_n, Q_n are amplitude, phase and Q-value for the nth-mode. The parameter $\Delta\omega = \Omega_M - \pi c/L$ has been introduced to account for possible detunings between the driving frequency Ω_M and the geometrical beating of the cavity. Furthermore, since l«L and the modulator is located very closely to the terminal mirror, one has

$$\alpha_c = \alpha_a / 2 = w/2 \qquad (9)$$

When the phase-locking conditions are achieved, $\dot{\varphi}_n = 0$ and, since $E_n \simeq E_{n+1} \simeq E_{n-1} \simeq$, from Eq. (7) one has

$$\varphi_n = \varphi_{n-1} + \arcsin\left(h - \sum_{0}^{n-1}{}_k\, b_k\right); \qquad n \geq 1 \tag{10}$$

where

$$b_k = (2L / \alpha_a c)\,(K\Delta\omega + 1/2\ \omega_k\, \chi'_k)$$

Moreover, the symmetry of the problem around the central frequency ω_0 necessarily implies that $\varphi_{-n} = -\varphi_n$. The arbitrary constant h, appearing in Eq. (10) can be determined with considerations based on Eq. (2) which states that the losses seen by the n-th mode are minimal when

$$\varphi_{n\pm1} - \varphi_n = \pi \tag{11}$$

On the other hand, this condition does not satisfy Eq. (7) where the non-linear behaviour of $\chi'(\omega)$ requires a non-equispaced set of phases. Nevertheless the solution can not substantially depart from Eq. (11), at least for the modes that are in the proximity of the central frequency ω_0 where $\chi'(\omega)$ is almost completely linear. Thus with a very good approximation, we can put h=0 in Eq. (10) and consequently $\varphi_1 = -\varphi_{-1} = \pi$.

With this condition Eq. (10) says that the phase difference between adjacent modes drops from the initial value of π down to $\pm\,\pi/2$ when n is such that

$$\sum_{0}^{n-1}{}_k\, b_k = \pm\, 1.$$

Beyond this point the system of Eqs. (7) and (8) does not admit a real solution with φ_n=0 and all the modes in this region start loosing the phase correlation with the modulation frequency.

The relative phases among them are not defined anymore and the second term on the right hand side of Eq. (8) averages down to zero. This describes a situation where those modes behave as they were freely oscillating in the cavity and the losses they experience account also for the extra average loss α_a introduced by the modulator. Under the same circumstances Eq. (7) reduces to

$$\dot{\varphi}_n = -\,(n\,\Delta\omega + 1/2\ \omega_n\ \chi'_n) \tag{12}$$

which shows that for these unlocked modes the phases are linear functions of time and therefore represent pure frequency shifts. The solution of Eqs. (10) and (12), seems to require only the knowledge of χ', but, as it is very well known, the saturation of the gain strongly affects the real and imaginary parts of the susceptibility function which in Ref. (1) have been parametrized as follows:

$$\chi'(\omega_n) = cg\,(\omega_o) / (\omega_n L \sqrt{\pi}\,) \tag{13}$$

$$2\,\xi_n\,(1 - 2/3\,\xi^2_n + \ldots) / [1 + \Sigma_i\ (P(w_i)/P^i_m)\,(\gamma^2/(\gamma^2+(w_i-w_n)^2)]^{1/2}$$

$$\chi''(\omega_n) = cg(\omega_o) / (\,\omega_n L)$$

$$e^{-\xi^2_n} / \ [1 + \Sigma_i\ (P(w_i) / P^i_m)\,(\gamma^2/(\gamma^2+(w_i-w_n)^2)\,]^{1/2} \tag{14}$$

where $\xi_n = (\omega_n - \omega_o) / (\delta/2)$ and $P_n \propto |\,E_n\,|^2$ and we have assumed that $P^i_m = P^o_m\ e^{-\xi^2_i}$. The following values have been estimated with measurements done on shorter cavities:

$g(\omega_o) = 0.72$	center of band non saturated gain	
$\alpha\quad = 17.10^{-2}$	total cavity losses assumed to be the same for all the modes	
$P^o_m\ = 0.17$ W	center of band saturation parameter	(15)
$\gamma\quad = 2\text{x}10^8$ rad/s	homogeneous broadening	
$\delta\quad = 2.8\text{x}10^{10}$ rad/s	inhomogeneous broadening	

This saturation effect has the consequence that Eqs. (7) and (8) become strongly coupled and thus their solution can only be found with an iterative procedure.

To the extent that the laser parameters are given by the values (15), the only parameters left free in the calculation are the modulation depth α_a and the detuning $\Delta\omega$ whose best value is not predicted to be zero even for short cavities. In the computer code we wrote to solve Eqs. (7) and (8), the procedure is initiated starting from the electric amplitudes that fulfill the free running condition

$$dE_n/dt + 1/2\ \omega_n\,[\,(1/Q_n\,) + \chi''_n\,]\,E_n = 0 \tag{16}$$

and subsequently determining the power that can be stored in the cavity. In Ref. (1) we showed that with the values (15), in a 17.5 m long cavity, one typically has 60W, in good agreement with the experimental datum.

When converging is achieved, the solution is specified in terms of the set of E_n and φ_n that fully satisfy Eqs. (7) and (8). The sum

$$|\,E\,|^2 = |\,\Sigma_n E_n\,e^{-i\,(\omega_n t + \varphi_n(t))}\ |^2 \tag{17}$$

extended over all the oscillating modes, gives expression to the time dependence of the light intensity inside the cavity. The contribution coming from the modes with $\dot{\varphi}_n \neq 0$ generates a non modulated continuous that reproduces the level of CW-power in the free-running limit. As the external modulation depth increases, Eq. (10) is fulfilled for more and more modes and Eq. (17) starts describing the building-up of a single laser pulse bounching between the two terminal mirrors. But the length of this pulse is not related to the number N of locked modes according to the simple expression $\Delta\tau = 2L/cN$ assumed in Ref. (4), because this would be true only for the case where the phases between adjacent modes would be exactly equispaced.

As shown in Fig. 1, both the total power and that of the locked modes ($\dot{\varphi}_n=0$) have a maximum at a particular value of the detuning parameter $\Delta\omega=-6.5 \cdot 10^3$ rad/s which does not depend upon α_a. This value is related to the frequency derivative of the susceptibility function $\chi'(\omega)$ around the central frequency ω_0. Fig. 2 shows the behaviour of the total power and pulse width versus the modulation α_a at the optimal value of the detuning parameter and Fig. 3 shows how the pulse builds-up with the increasing α_a.

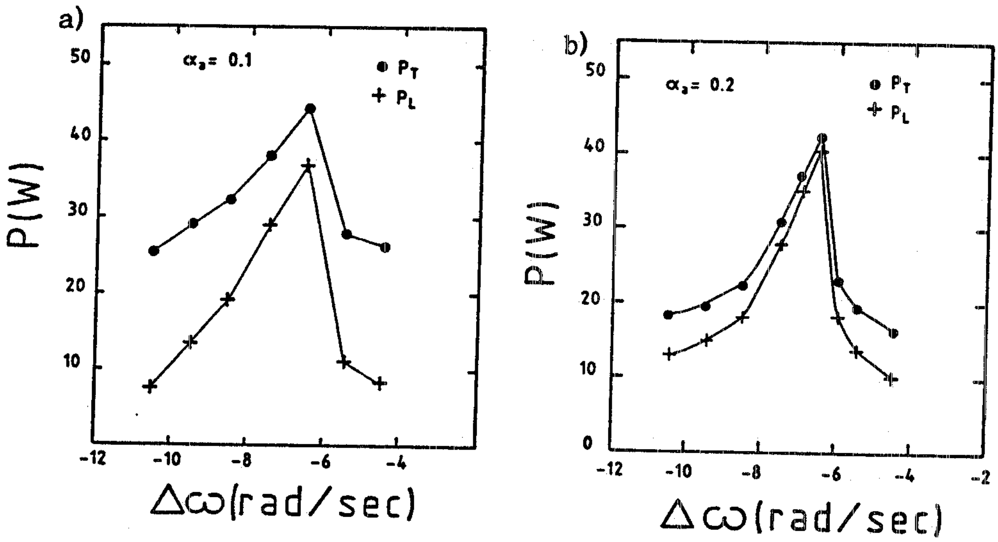

FIG. 1 - Computed behaviour of the total power (P_T) and of the locked modes power (P_L) for α_a=0.1 (Fig. 1a) and α_a=0.2 (Fig. 1b) as a function of the detuning $\Delta\omega$.

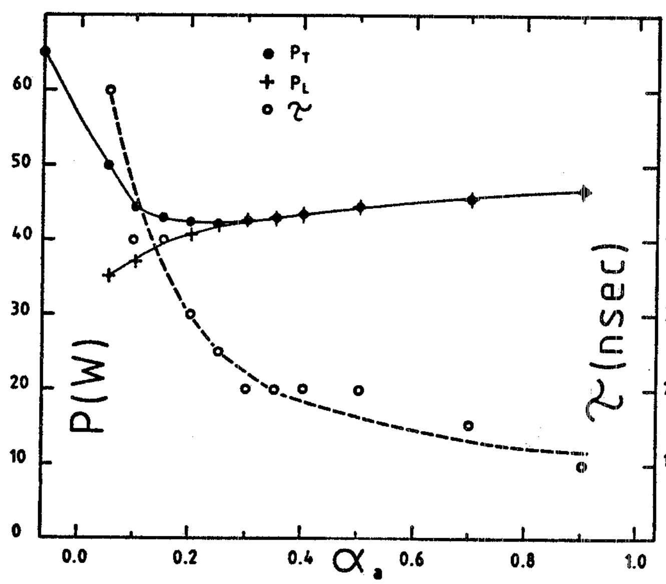

FIG. 2 - Total power (P_T), locked modes power (P_L) and FWHM duration of the pulses (τ) versus the modulation depth α_a at the optimum detuning frequency ($\Delta\omega$=–6500 rad/s).

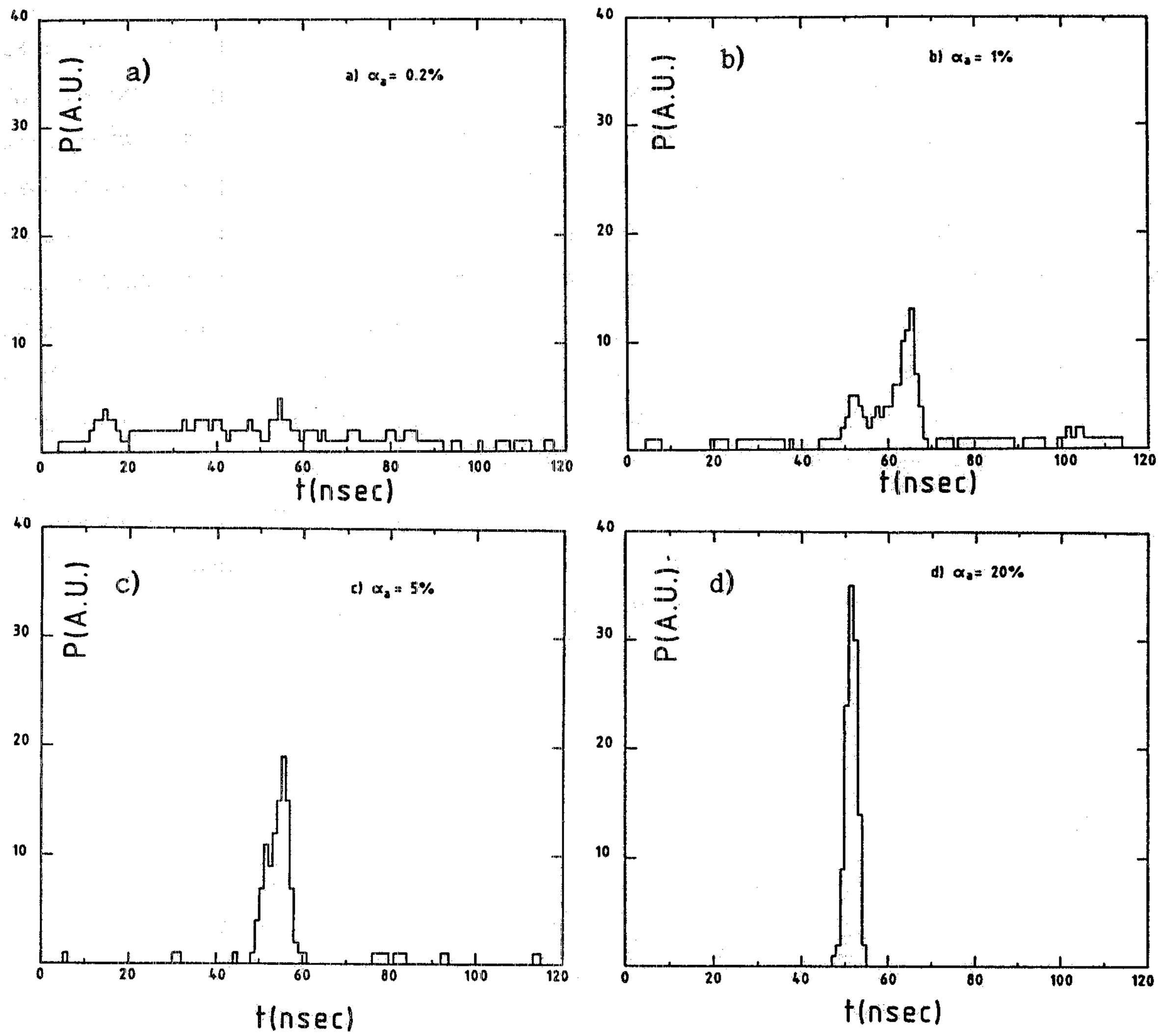

FIG. 3 - Time behaviour of the power inside the cavity at $\Delta\omega$=–6500 rad/s for four different values of α_a.

It is interesting to compare now this situation with the experimental results. Experimentally, starting from a power level of ~ 60W, one can reach a locking condition where ~25W are stored in one single pulse approximately ~15 nsec wide, with $\alpha_a \simeq$ 20% and no CW-component. This is substantially consistent with the calculation we discussed so far.

Much less consistent is the experimental behaviour of the total power that varies monotonically with α_a in conflict with Fig. 2 where no substantial dependence is shown beyond $\alpha_a \simeq$ 30%. But more serious is the practical no-dependence upon $\Delta\omega$ that we experimentally observed both for the power and the pulse width.

Part of the difficulties can definitely come from the basic assumptions that have been made in the calculation, namely that:

- only longitudinal modes are oscillating in the cavity;

- the modulation is purely sinusoidal;
- the active medium fills-up the whole cavity;
- the argument of the real and imaginary parts of the susceptibility function is assumed to be real.

We know that the transverse mode contributions is far from being negligible and we also know that the acusto-optic effect leads to a modulation which is sinusoidal only in first approximation.

Another part of the problem can arise from the length of the cavity, because when this becomes several meters long, as in our case, the homogeneous broadening of the modes starts exceeding their frequency separation and this can create complications which have not been taken into account. When the frequency beating of the cavity decreases down to few MHz, also the scattering angle of the diffused modes inside the modulator decreases and can become of the same order of the angular divergency. At this point is not clear how much of the scattered light is ejected from the cavity and consequently how much is the loss introduced by the acousto-optic deflector.

As a final conclusion of this discussion our claim is that the conventional approach(2) to the mode-locking phenomenon in very long laser cavities can describe its basic features but is not sufficient for a quantitative analysis of what is experimentally observed.

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