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ANALYTICITY CONSTRAINTS AND ABSENCE OF MULTIPLE DIPS IN ELASTIC pp AND $p\bar{p}$ SCATTERING

E. Etim and A. Małecki *
INFN - Laboratori Nazionali di Frascati, P.O.Box 13, 00044 Frascati (Italy)

ABSTRACT

Elastic pp and $p\bar{p}$ differential cross sections have just one, not multiple dips. The filling up of the diffraction zero by the real part of the scattering amplitude which gives rise to this dip structure is shown to be related, by analyticity, to the asymptotic behaviour of the amplitude in the momentum transfer, q^2. The relationship is similar to that of q^2-duality, with the Glauber model giving the low q^2 description of the scattering amplitude in its analyticity domain, $q^2 \to 0$, while the Chou-Yang model describes it in the asymptotic region, $q^2 \to \infty$. This combination fits the data better, at all energies and for all momentum transfers.

It was pointed out in a recent note[(1)] that the Glauber model[(2)], if properly applied, predicts one, not multiple dips in the pp and $p\bar{p}$ elastic differential cross sections at high energies. The essential idea is that the Glauber model involves an approximation in which the ground state wave function alone is used. If this drastic truncation of the overall wave function expansion is compensated by proportionately increasing the strength of the coupling to the ground state, one is led to a Bjorken-type limit: the density of the ground state wave function is vanishingly small but the coupling to it tends to infinity so that their product remains finite. In this limit, the Glauber model reproduces correctly the

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* Permanent address: Institute of Nuclear Physics, 31-342 Krakow, Poland.

one dip structure observed experimentally in the cross sections[3]. Only the definition, but not this simple interpretation of the diffractive limit, was given in ref. (1).

The approach of ref. (1), is however, only partially successful:

(1) It does not fit the data at high momentum transfers.

(2) It gives the scattering amplitude an ad hoc real part in order to fill up diffraction zeros.

These two facts are intimately related: the asymptotic q^2 behaviour of the scattering amplitude reflects on its local analyticity properties and vice versa. But up to now, so much attention has been paid to shadow scattering and the corresponding imaginary part of the amplitude that this important analyticity relationship has been overlooked. We interprete the various attempts to apply QCD and quark-parton model concepts[4-6] to pp and $p\bar{p}$ diffraction scattering as bearing, indirectly, on this point. From these concepts, there is some suggestion as to the possible form of the relationship between the asymptotic q^2 behaviour of the amplitude and its local analyticity properties which are responsible for diffraction zeroes and dips in the differential cross section. It is based on ideas motivated by q^2-duality[7,8], which has been quite succesful elsewhere[9].

Here is the proposal. The Glauber model gives the dominant contribution to the imaginary part of the scattering amplitude and the Chou-Yang model[10] the dominant contribution to its real part, for all q^2. The one dominates the complete amplitude in its analyticity domain, $q^2 \rightarrow 0$, and corresponds to a hadronic description in a q^2-duality type picture. The other dominates in the asymptotic region, $q^2 \rightarrow \infty$, and corresponds to a generalised Born term of a quark-parton model description, in the same duality picture. The idea is that if this models were separately and independently unitarised they would provide equivalent and dual descriptions of the scattering process. An alternative, but approximate, analytic procedure is to combine and interprete them, as proposed here, as the predominantly imaginary and real parts, respectively, of one and the same amplitude.

The result of this combination is shown in Figs. 1 and 2 for $\sqrt{s}$=53 and 546 GeV. The dashed curve is the prediction of the Glauber model alone while the dotted curve that of the Chou-Yang model alone, with a dipole form factor* . The full curves are their sum. Our notations are[1]

$$d\sigma_{pp}(s,t) / |dt| = \pi |T(s,t)|^2 \tag{1a}$$

$$T(s,t) = i \int_0^\infty db\, b\, J_0(b\sqrt{|t|}) T(s,b) \tag{1b}$$

$$T(s,b) = T_G(s,b) - i\, T_{CY}(s,b) \tag{1c}$$

$$T_G(s,b) = \eta_{pp}(s,b) + |2\lambda|^2 (1 - \eta(s,b))(1 - e^{-h_G(s,b)}) \tag{1d}$$

$$T_{CY}(s,b) = 1 - e^{-h_{CY}(s,b)} \tag{1e}$$

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* To check the reliability of our approach, we have also used a purely empirical fit to the proton electromagnetic form factor[11] in the Chou-Yang amplitude, and obtained practically the same results as those in Figs. 1 and 2.

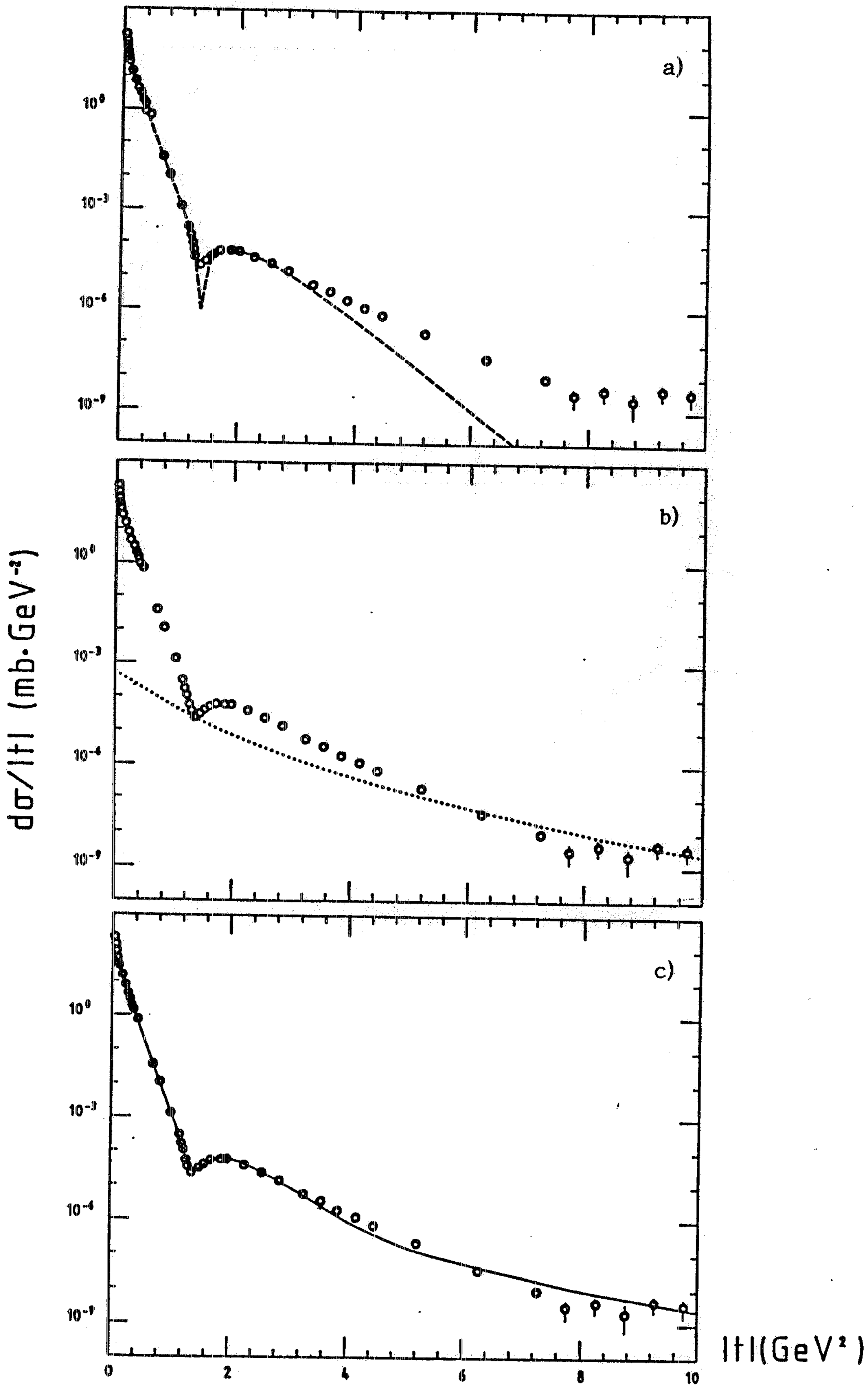

FIG. 1 - Plot of the elastic differential cross section dσ(s,t)/dt against the momentum transfer t, for CM energy √s=53 GeV. Data are from ref. (3c). The dashed curve is the prediction of the Glauber model alone; the dotted curve the prediction of the Chou-Yang model alone. The full curve is their sum as described in the text.

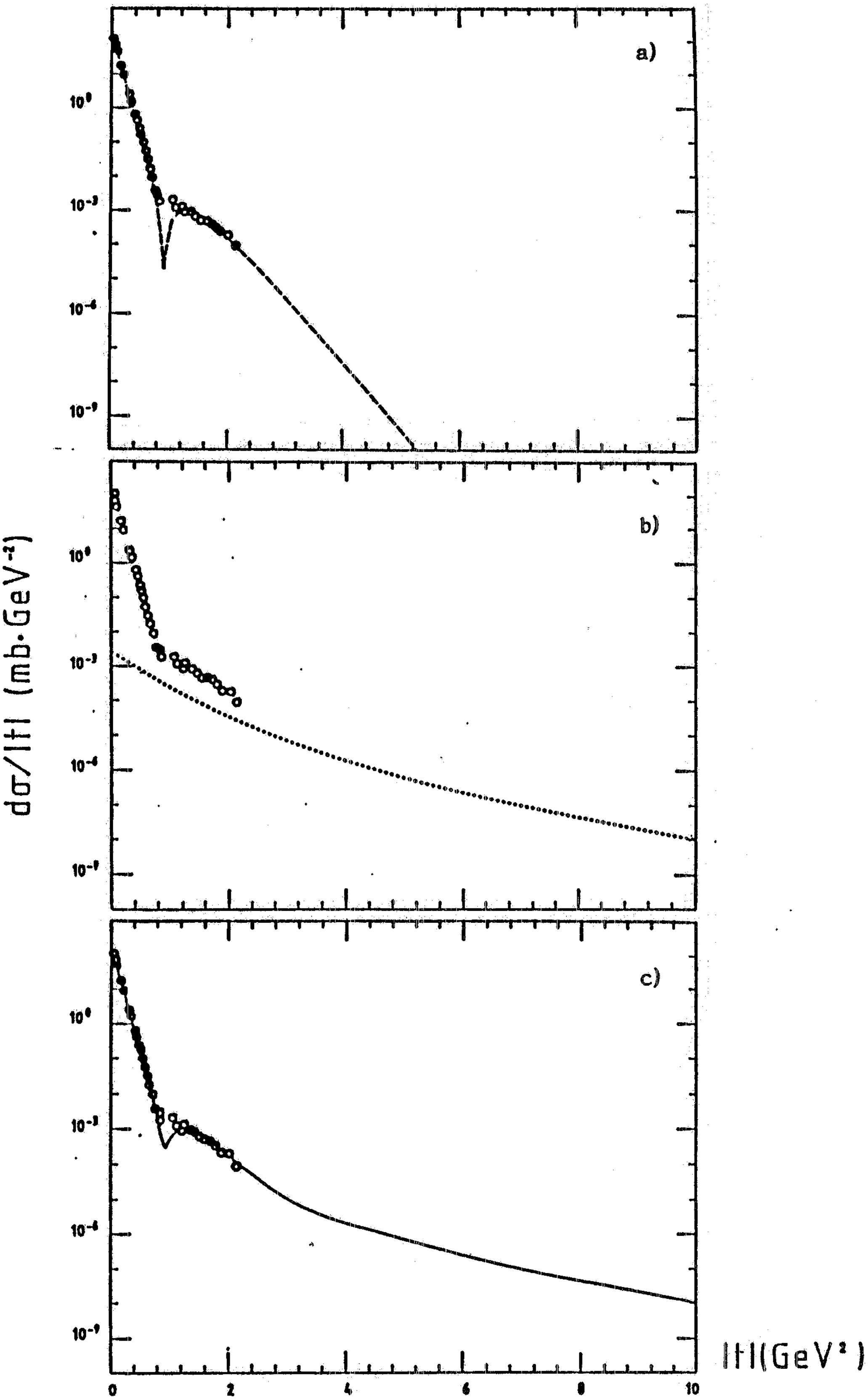

FIG. 2 - Same as in Fig. 1 for $p\bar{p}$ scattering at $\sqrt{s}$=546-630 GeV. The data are from Ref. (3d).

where

$$\eta_{pp}(s,b) = \frac{\sigma_{pp}(s)}{4\pi\ a_p(s)}\ e^{-b^2/2a_p(s)} \tag{2a}$$

$$h_G\ (s,b) = \frac{\sigma_q(s)}{4\pi\ R(s)}\ e^{-b^2/2R(s)} \tag{2b}$$

$$h_{CY}\ (s,b) = \frac{\sigma_q(s)}{4\ \pi}\ \frac{\mu^2}{48}\ (\mu b)^3\ K_3\ (\mu b) \tag{2c}$$

where $K_3(z)$ is the modified Bessel function. $\sigma_{pp}(s)$ is the pp total cross section at CM energy $\sqrt{s}$ and $a_p(s)$ the corresponding slope. $\sigma_q(s)$ is a constant multiple of the total cross section of the scattering of the proton off constituents; R(s) is the corresponding slope. μ is the mass scale in the dipole form factor $(1+ | t | / \mu^2)^{-2}$. λ is the coupling strength mentioned in the introduction and $| t | = q^2$ is the squared momentum transfer.

The theory leading to Eq. (1d) is described in Ref. (1). Eq. (1e) is obtained by dropping the term $\eta_{pp}(s,b)$ in Eq. (1d) since, by assumption, the Chou-Yang amplitude $T_{CY}(s,b)$ is contributed entirely by Born terms of scattering from constituents. The Glauber amplitude $T_G(s,b)$ on the other hand, contains this leading term which represents shadow scattering from the proton as a black disk. It dominates the diffraction peak for $t \to 0$. The values of the parameters used in the fits are shown in Table I.

TABLE I - Values of the parameters used in the fits in Figs. 1 and 2. G stands for Glauber amplitude and CY for the Chou-Yang amplitude.

$\sqrt{s}$ (GeV)	σ_{pp} (mb)	a_p $(fm)^2$	$\mid\lambda\mid^2$	R $(fm)^2$	μ (GeV)	σ_q (mb) G	σ_q (mb) C-Y
53	39.3	0.49	41.0	0.17	1.648	0.034	0.110
546	53.7	0.66	41.2	0.24	1.633	0.107	0.245

The remarkable property of this combined amplitude is that the continuation of its asymptotic real part into the low q^2 region fills up the diffraction zero of the imaginary part. There is thus no need for ad hoc complexifications of the amplitude through energy dependent parameters, such as $\sigma_{pp}(s) \to \sigma_{pp}(s)\ (1 - i\ \alpha_p)$ and $\sigma_q(s) \to \sigma_q\ (s)\ (1 - i\ \alpha_q)$. This is the analyticity constraint. It shows how the local and asymptotic behaviours of the scattering amplitude are related in a q^2-duality type manner. It is argued that this kind of analytic correlation is, theoretically, to be expected. Figs. 1 and 2 shows that experiments also support it, for all CM energies and over the entire range of available momentum transfers. The real part of the amplitude is therefore not everywhere negligible with respect to the imaginary part. The dip in the differential cross section, marks the transition from the low q^2 region, where the imaginary part is dominant, to the high q^2 region, where the real part takes over.

Lastly, some comments, on the values of the fit parameters in Table I. The most striking facts are:

(i) The mass scale in the dipole from factor is of the order of two proton masses, not of a proton mass, as in the fits to the proton electromagnetic form factor.

(ii) The cross section $\sigma_q(s)$ in the Chou-Yang amplitude $T_{CY}(s,b)$ is of the same order as that in the Glauber amplitude $T_G(s,b)$ and both much smaller than the cross section of about 50 mb used in fits of the Chou-Yang model to the ISR data in Ref. (12). Small values of σ_q are consistent with scattering from point-like constituents.

(iii) The product $|\lambda|^2 \sigma_q(s)$ is approximately constant over the energy range $\sqrt{s} = 53 - 600$ GeV. This is consistent with taking the Bjorken type limit $|\lambda|^2 \to \infty$, $\sigma_q(s) \to 0$ and $|\lambda|^2 \sigma_q(s)$ finite.

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