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**A STUDY OF SHORT-RANGE CORRELATION EFFECTS ON NUCLEAR SPATIAL AND
MOMENTUM DISTRIBUTION**

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ABSTRACT

The relation between the nuclear spatial and momentum densities is studied by means of their integral representations in terms of uniform distributions. Examples of this approach are given for the harmonic oscillator model of ${}^4\text{He}$ and ${}^{16}\text{O}$, with and without Jastrow correlations, and for the single-particle potential model of ${}^{40}\text{Ca}$.

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1. - INTRODUCTION

The complete many-body nuclear wave function $\psi(r_1, \dots, r_A)$ determines both the nuclear density $\rho(r)$ and the nuclear momentum distribution $n(k)$. Since a single-particle wave function can be expressed either in the spatial or the momentum representation, these being Fourier transforms of each other, there is a correspondingly close relation between $\rho(r)$ and $n(k)$. This is not however always true for particular nuclear models which use approximate solutions of the many-body problem, e.g. it has been shown quite generally [1] that no Hartree-Fock calculations can reproduce simultaneously both the density and the momentum distributions. The physical reason for this is that the nucleon-nucleon interaction has short-range features that affect the wave functions at short distances and introduce short-range correlations (SRC) in the motions of the nucleons and also raise nucleons above the Fermi sea. These short-range correlations result in high-momentum components in the momentum distribution and these cannot be given by a model that includes only the long-range properties of the nucleon-nucleon interaction. It is thus possible for a model to give excellent density distributions and yet fail to give the high-momentum components of the momentum distribution. Since these are important for many interactions it is necessary to develop a simple and reliable method of calculating nuclear momentum distributions.

Several methods have been developed [2-6] to include the effects of SRC in nuclear density and momentum distributions. These give more accurate ground state wave functions, particularly at smaller distances, and are thus able to reproduce simultaneously both $\rho(r)$ and $n(k)$. However these calculations become prohibitively complicated for all but the lightest nuclei.

Recently the coherent fluctuation model (CFM) has been developed [7-9] and this gives a relation between $\rho(r)$ and $n(k)$ that enables $n(k)$ to be easily calculated for all nuclei. It has already been shown that $n(k)$ obtained from the CFM has the desired high-momentum tail and agrees with some experimental data. The CFM thus includes some of the effects of SRC. It is thus desirable to test the CFM by applying it to light nuclei for which more exact calculations are available.

While in the CFM model the relation between $\rho(r)$ and $n(k)$ is given but there is an open question about the character of SRC included in this model, in the approaches [2-6] the situation is just the opposite: the nucleon-nucleon correlations are included explicitly (by means of the Jastrow correlations factors or nuclear forces) but the link between $\rho(r)$ and $n(k)$ still remains unrevealed.

The aim of this paper is to throw light on the relation between $\rho(r)$ and $n(k)$ in different approaches and with various kinds of SRC involved in them. For this purpose in Section 2 a method is developed which allows to compare the spatial and momentum densities on the same footing, by means of weighting factors in the integral representations of these densities in terms of uniform distributions. In Section 3 this method is applied to the harmonic oscillator model of ${}^4\text{He}$ and ${}^{16}\text{O}$, with and without Jastrow correlations, and to the ${}^{40}\text{Ca}$ nucleus in the single-particle potential model [10,11]. Some comparisons are made with the CFM. The conclusions of the work are given in Section 4.

2. - INTEGRAL REPRESENTATION IN TERMS OF UNIFORM DISTRIBUTIONS

The distributions $\rho(r)$ and $n(k)$ may always be written in the form:

$$\rho(r) = \int_0^{\infty} dx w_{\rho}(x) (4/3 \pi x^3)^{-1} \theta(x-r), \quad (1)$$

$$n(k) = \int_0^{\infty} dp w_n(p) (4/3 \pi p^3)^{-1} \theta(p-k) \quad (2)$$

with the weighting functions

$$w_{\rho}(x) = -4/3 \pi x^3 dp(r)/dr \Big|_{r=x}, \quad (3)$$

$$w_n(p) = -4/3 \pi p^3 dn(k)/dk \Big|_{k=p}, \quad (4)$$

$\theta(x-r)$ and $\theta(p-k)$ being the step functions (unity for positive, zero for negative arguments) describing uniform distribution of radius x or p in variable r or k , respectively.

With the above choice of numerical factors the weighting functions are normalized in the same way as the corresponding density and momentum distributions:

$$\int_0^{\infty} dx w_{\rho}(x) = 4\pi \int_0^{\infty} dr r^2 \rho(r) \quad (5)$$

$$\int_0^{\infty} dp w_n(p) = 4\pi \int_0^{\infty} dk k^2 n(k) \quad (6)$$

However we consider it more instructive to choose for $n(k)$ another integral representation in terms of uniform distributions:

$$n(k) = 1/(2\pi)^3 \int_0^{\infty} dx w_n(x) 4/3 \pi x^3 \theta((\alpha/x)-k) \quad (7)$$

with

$$w_n(x) = -(2\pi)^3 (4/3 \pi x^5/\alpha)^{-1} dn(k)/dk \Big|_{k=\alpha/x}, \quad (8)$$

α being an arbitrary number. Then the weighting functions for the density and momentum distributions are both functions of spatial radius x . One may thus expect that comparing the weights $w_{\rho}(x)$ and $w_n(x)$ the relations between the spatial and momentum densities will become more transparent and this can provide additional information on the unique functional relation existing between these two quantities.

The normalization of $w_n(x)$ is:

$$\int_0^{\infty} dx w_n(x) = (9\pi/2\alpha^3) 4\pi \int_0^{\infty} dk k^2 n(k) \quad (9)$$

Further on, to facilitate comparison with (6), we choose $\alpha = (9\pi/2)^{1/3}$.

The weighting functions $w_\rho(x)$ and $w_n(x)$ can be calculated in any nuclear model. Generally they will be different since $w_n(x)$ peaks at

$$r_0 = (9\pi/2)^{1/3} 1/k_F, \quad (10)$$

k_F being the Fermi momentum, while $w_\rho(x)$ peaks at the value

$$R_A = (A/4)^{1/3} r_0 \quad (11)$$

which is close to the nuclear radius.

The maxima in $w_\rho(x)$ and $w_n(x)$ given by Eqs (10) and (11) reflect overall features of nuclear structure resulting from the long-range properties of the N-N interaction. Notice that the value of r_0 is roughly independent of the mass number A. When the SRC are switched on there is only a small effect on the spatial density $\rho(r)$ (at small r) hence also the shape of $w_\rho(x)$ should not be considerably changed. However, the situation can be quite different for the function $w_n(x)$ where the SRC may induce an additional structure due to the rapid change of the momentum density $n(k)$ at high values of k .

Before going to a numerical analysis of $w_\rho(x)$ and $w_n(x)$ in some nuclear models let us point out that in the CFM [7-9] a particularly simple relation between the two weighting functions has been imposed. In this model the nucleon density and momentum distribution are determined as follows:

$$\rho(r) = \int_0^\infty dx f(x) A (4/3 \pi x^3)^{-1} \theta(x-r), \quad (12)$$

$$n(k) = 4/(2\pi)^3 \int_0^\infty dx f(x) 4/3 \pi x^3 \theta(k_F(x) - k) \quad (13)$$

where $k_F(x) = (9\pi A/8)^{1/3} 1/x$ is the Fermi momentum for A nucleons uniformly distributed in a sphere of radius x .

Thus in the CFM the two weighting functions are the same:

$$w_\rho(x) = w_n(x) = A f(x) \quad (14)$$

$f(x)$ being a weighting function of uniform distributions in the density matrix.

Taking into account Eqs (13, 14) together with (3) the following functional relation between $n(k)$ and $\rho(r)$ is obtained [7,8]:

$$n(k) = (4\pi/3)^2 1/A 4/(2\pi)^3 [6 \int_0^{R(k)} dr \rho(r) r^5 - R^6(k) \rho(R)] \quad (15)$$

with

$$R(k) = (9\pi A/8)^{1/3} 1/k$$

The CFM produces a high-momentum tail (at $k > 2 \text{ fm}^{-1}$) in $n(k)$ which could mean that the model effectively accounts for some type of SRC. The very nature of this effect might be revealed by comparing Eq. (15) with momentum distributions obtained in nuclear models including such correlations explicitly. Also the study of the weighting function w_p and w_n in correlated nuclear models would be of great help in recognizing the origin of the high-momentum tail in CFM.

3. - NUMERICAL ANALYSIS OF WEIGHTING FUNCTIONS

As an illustration we calculate $w_p(x)$ and $w_n(x)$ by means of Eqs. (3) and (8) in the case of ${}^4\text{He}$ using: i) the single-particle model with harmonic oscillator wave functions and ii) the Jastrow correlation model of Bohigas and Stringari [4]. The results, shown in Figs. 1 and 2, have several noteworthy features: the uncorrelated $w_p(x)$ and $w_n(x)$ although they peak at nearly the same value $x \cong 1.9 \text{ fm}$ (which corresponds via Eq. (10) to $k_F \cong 250 \text{ MeV}/c$) do strongly deviate from each other at small values of x . The coincidence of the two maxima is rather exceptional though not accidental feature (see Eqs (10) and (11)) of the ${}^4\text{He}$ nucleus.

The difference between the two functions for $x \leq 1.0 \text{ fm}$ reflects the lack of high-momentum components in the single-particle model. In fact, in the Jastrow correlation model the situation is changed. The correlations imply mainly a change of $w_n(x)$ at small x , producing a second narrow peak at $x \cong 0.6 \text{ fm}$ which is due to the change of the slope of $n(k)$ in the region $k \cong 2.0\text{-}2.5 \text{ fm}^{-1}$. On the other hand, the change of $w_p(x)$ due to the correlations is quite small, as shown in Fig. 2. Thus the correlations lead to an overlap of $w_p(x)$ and $w_n(x)$ over a wide range of x . This could explain, at least for ${}^4\text{He}$, the presence of high-momentum components in the CFM as the simulation of short-range correlations.

The same calculations have been repeated for the ${}^{16}\text{O}$ nucleus using the result of Malecki and Picchi [2,3] and the results are shown in Fig. 3. As anticipated in Section 2 the main peak of the function $w_n(x)$ remains at the same position while that of $w_p(x)$ is shifted to greater values of x in accordance with Eq. (11). In the presence of SRC the function $w_n(x)$ acquires an additional narrow peak at $x \cong 0.6\text{-}0.7 \text{ fm}$. In the same time the "uncorrelated" broad maximum in $w_n(x)$ gets somewhat lowered. Thus although the two functions w_p and w_n no longer overlap, their values, at least for $1.5 < x < 2.5 \text{ fm}$, are closer to one another in the correlated case than in the single-particle model. This explains why the CFM contains some high-momentum components which are however weaker than those, induced by the Jastrow-type SRC, as shown in Fig. 4.

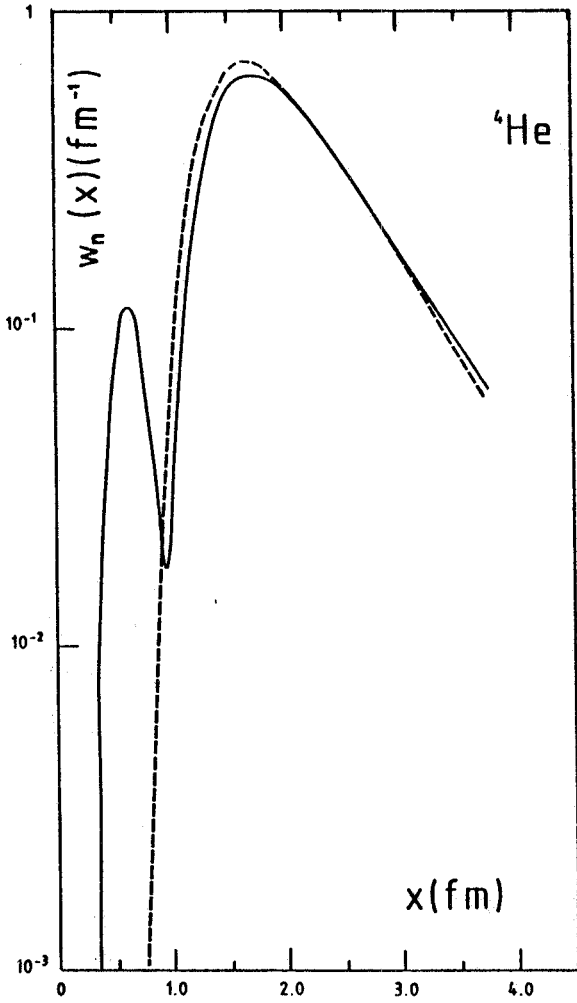


FIG. 1 - Weighting function $w_n(x)$ for ${}^4\text{He}$ in the single-particle model (dashed line) and in the Jastrow correlation model (solid line).

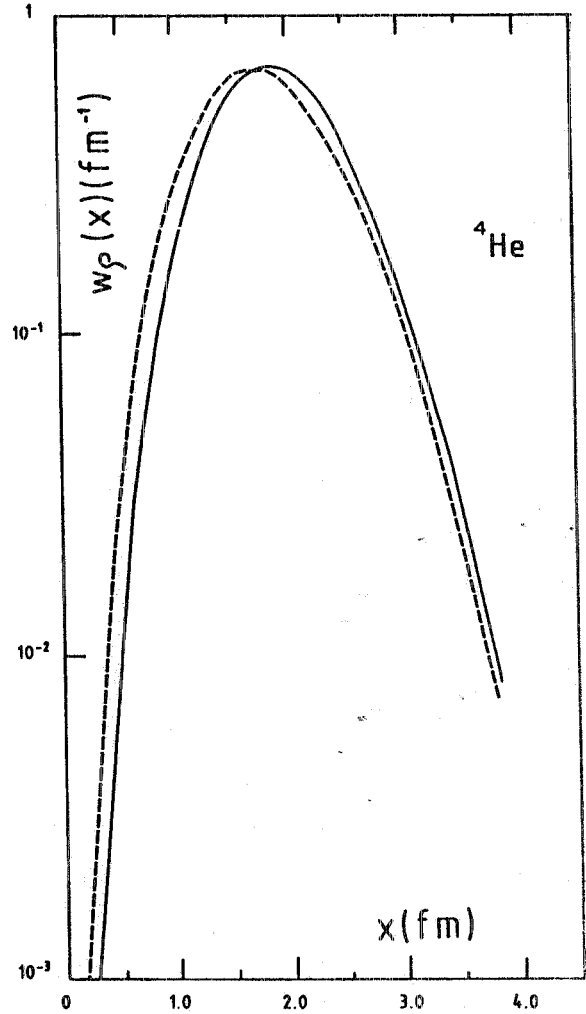


FIG. 2 - Weighting function $w_\rho(x)$ for ${}^4\text{He}$ in the single-particle model (dashed line) and in the Jastrow correlation model (solid line).

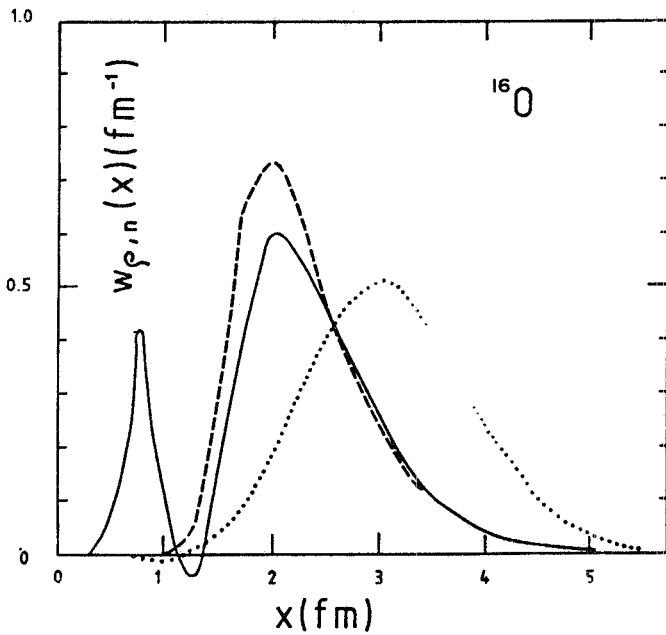


FIG. 3 - Weighting function $w_n(x)$ for ${}^{16}\text{O}$ in the single-particle model (dashed line) and in the Jastrow correlation model (solid line), and $w_\rho(x)$ in the single-particle model (dotted line).

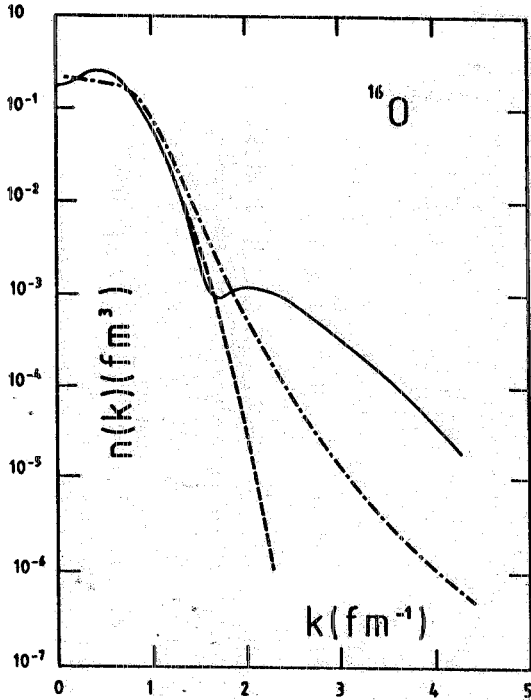


FIG. 4 - Momentum distributions in ^{16}O calculated according to i) the shell-model calculations with correlations of Malecki and Picchi [2,3] (solid line), ii) the coherent fluctuation model based on the symmetrized Fermi-type density distribution [8] (dot-dashed line) and iii) the shell-model calculations without correlations [2,3] (dashed line).

Judging from the results for ^4He and ^{16}O the two-maximum structure of $w_n(x)$ is a characteristic feature of the correlated momentum distributions. The broad bump at $x=x_0$ reflects long-range effects while the narrow peak at $x \simeq 0.7$ fm is a signature of the short-range part in the N-N interactions. Unfortunately these general properties cannot be confirmed for heavier nuclei since the correlated calculations then become prohibitively complicated.

We have also calculated the weighting functions w_ρ and w_n for the ^{40}Ca nucleus in the framework of the single-particle potential (SPP) method [10,11]. In Fig. 5 the results for these two functions calculated by means of (3) and (8) and the relations

$$\rho(r) = 1/4 \pi \sum_{nlj} (2j+1) \bar{n}_{nlj} |R_{nlj}(r)|^2, \quad (16)$$

$$n(k) = 1/4 \pi \sum_{nlj} (2j+1) \bar{n}_{nlj} |R_{nlj}(k)|^2, \quad (17)$$

$$R_{nlj}(k) = (2/\pi)^{1/2} (-i)^l \int_0^\infty dr r^2 j_l(kr) R_{nlj}(r), \quad (18)$$

are presented. The radial functions $R_{nlj}(r)$ and the occupation numbers \bar{n}_{nlj} are taken from the SPP method [10,11]. Unlike the case of the ^4He $w_\rho(x)$ and $w_n(x)$ are quite different over the whole range of x . It should be noticed that the two weighting functions for ^{40}Ca confirm the general properties expressed in Eqs (10) and (11).

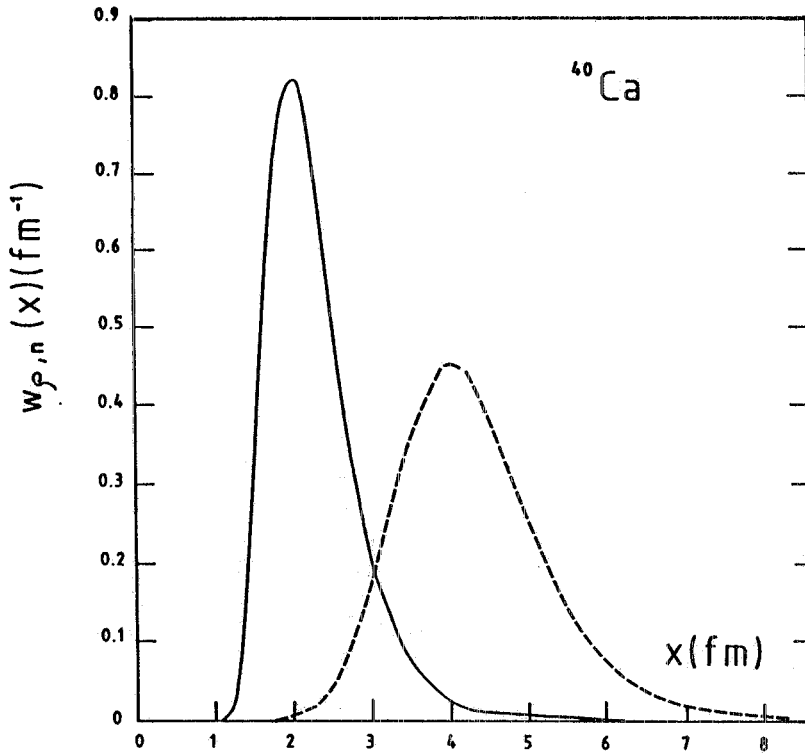


FIG. 5 - Weighting functions w_n (solid line) and w_p (dashed line) for ^{40}Ca calculated by the single-particle potential method of Malaguti et al. [10,11].

4. - CONCLUSIONS

This work shows that the relation between the nuclear density and momentum distributions can be studied by means of the weighting functions originating from the integral representation of $\rho(r)$ and $n(k)$ in terms of uniform distributions.

In particular, it shows more clearly the effect of correlations on the nucleon momentum distribution. The function $w_n(x)$ shows two peaks if the nucleons are correlated but only one if they are not correlated. In the former case, one peak is associated with the Fermi momentum and does not vary from one nucleus to another. The other peak, at a smaller value of x , is attributable to the correlations, which produce a change in slope of the nucleon momentum distribution $n(k)$. Thus the weighting function enables the feature of the momentum distribution to be connected with nuclear structure and with the correlations. The weighting function w_p peaks at a value connected with the nuclear radius and for light uncorrelated nuclei is very similar to $w_n(x)$, as in the coherent fluctuation model which is not the case for the heavier nuclei.

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