F. Palumbo:

THE SCISSORS MODE

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ABSTRACT

A dipole magnetic resonance has been predicted in deformed nuclei by a two-rotor model, as a state in which the nucleus rotates while the proton neutron symmetry axes stay a fixed angle in a scissors-like configuration.

This resonance has been also predicted in the VPM and in the IBA, and has been experimentally confirmed by high resolution electron scattering in three regions of the periodic table, i.e. the deformed rare earth nuclei, the $f_{7/2}$-shell nuclei ^{46,48}Ti and the actinides.

I will report the theoretical description in terms of the two-rotor model and the RPA, and I will give a survey of the experimental status.

1. - INTRODUCTION

As it is well known the E1 giant resonance is a collettive mode existing in all nuclei. It has a simple semiclassical description in terms of translational oscillations of protons against neutrons[1].

This two-fluid picture suggests the existence of another collective mode which can occur only in deformed nuclei. Protons and neutrons can be assumed to form two separate rigid bodies of ellipsoidal shape, free to rotate independently around a common axis with opposite velocities. In such a two-rotor model (TRM) the restoring force generated by the displacement of protons against neutrons gives rise classically either to relative rotational oscillations or to a configuration in which the nucleus rotates while the proton-neutron symmetry axes stay at a fixed angle in a scissors-like configuration (Fig. 1). This latter state is strongly excited by M1-radiation through the coupling to the convection current[2].

The existence of this magnetic resonance, which has been predicted also in the VPM[3] and in the IBA[4], has been proved for the first time by a high resolution electron scattering experiment[5] in ^{156}Gd, and it is by now confirmed in three regions of the periodic table, i.e. the deformed rare earth nuclei[6], the $f_{7/2}$-shell nuclei[7] ^{46,48}Ti and the actinidies[8].

While there is little doubt that the magnetic resonances found experimentally correspond to the scissors mode, a detailed understanding of the related phenomenology is still lacking. The present situation is in fact more complex than that of the E1 mode.

Unlike the E1 state, the M1 state is bound, so that instead of reproducing a broad pick the theoretical models have to reproduce the pattern of fragmentation, which is remarkably modest. Although the

investigation of the excitation spectrum at higher energy has not so far been completed, it seems that with a few exceptions the M1 strength is concentrated in few fragments of much smaller intensity than the main level and very close to it[9]. In the few exceptions the M1 state rather than fragmented, is splitted into two fragments of comparable strengh. This has suggested[10] that also in these cases one cannot talk of fragmentation in the ordinary sense and that the observed splitting might be related to a triaxial deformation. Now there exists no systematic theoretical description of this pattern of fragmentation, although the spectrum of ^{156}Gd is fairly reproduced in a HFB calculation[11].
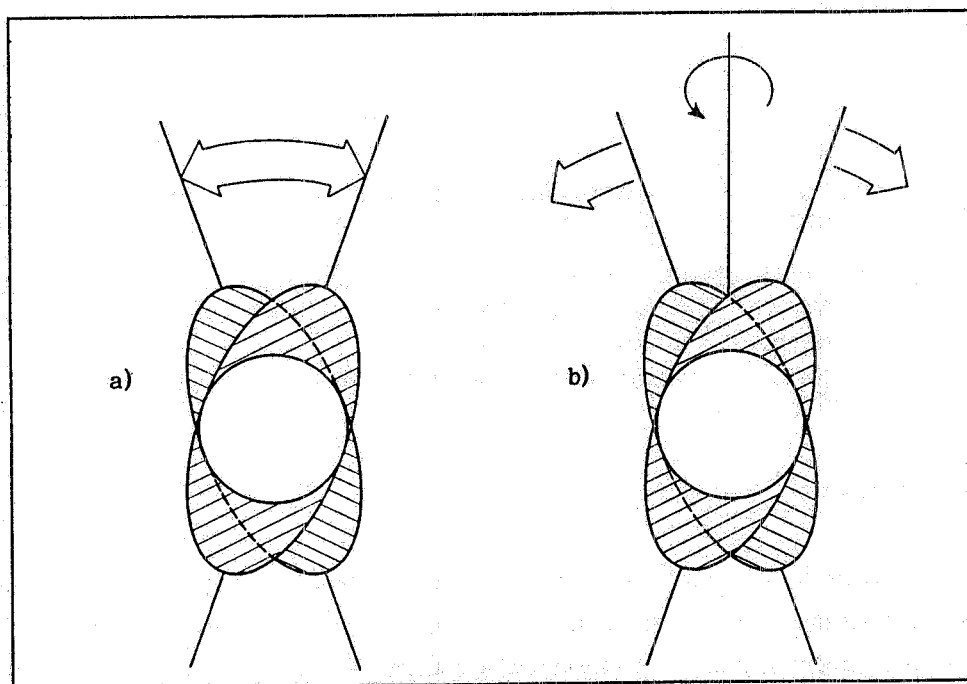

FIG. 1 - Classical motions in the TRM: (a) rotation around the ζ
axis; (b) rotational oscillation around the ξ axis.

Other theoretical features specific to the M1-mode which complicate its description are superconductivity, deformation and strong coupling of the scissors mode with quadrupole oscillations[3,12].

The mentioned reasons of complexity are, however, also the reasons why the M1-mode is the potential source of a lot of information about nuclear structure.

In these lectures I will report the theoretical description in terms of the TRM and the RPA, and I will give a survey of the experimental status. I will not discuss the important subject of the IBA approach, apart from reporting that it yields the same eigenstate equation for the M1-mode as the TRM, and for mentioning its success in some quantitative predictions.

I will be detailed in the description of the semiclassical TRM in spite of the existence of sophisticated calculations, because it is very transparent and leads to an understanding of the physics of the scissors-mode which is then conceptually simple to describe in terms of microscopic models.

2. - THE TWO ROTOR MODEL

If relative translational motion is neglected, the classical Hamiltonian of the TRM is

$$H = \frac{1}{2J_p} I^{(p)^2} + \frac{1}{2J_n} I^{(n)^2} + V, \tag{2.1}$$

where $I^{(p)}$, $I^{(n)}$, J_p and J_n are the angular momenta and moments of inertia of protons and neutrons, while V is the potential energy.

It is now convenient to introduce the total angular momentum I

$$I = I^{(p)} + I^{(n)},$$

$$S = I^{(p)} - I^{(n)}, \tag{2.2}$$

and rewrite H as

$$H = \frac{1}{2J}(I^2 + S^2) + \frac{J_n - J_p}{4J_n J_p} I \cdot S + V, \tag{2.3}$$

where

$$J = \frac{4J_n J_p}{J_p + J_n} . \tag{2.4}$$

We assume the potential to depend on the angle 2θ between the symmetry axes $\zeta^{(p)}$, $\zeta^{(n)}$ of the proton and neutron ellipsoids

$$\cos(2\theta) = \zeta^{(p)} \cdot \zeta^{(n)} \tag{2.5}$$

It is therefore natural to introduce this variable along with a set of other variables necessary to identify $\zeta^{(p)}$, $\zeta^{(n)}$. These variables may be the Euler angles α, β, γ of the intrinsic frame defined by

$$\xi = \frac{\zeta^{(p)} \times \zeta^{(n)}}{\sin(2\theta)} ,$$

$$\eta = \frac{\zeta^{(p)} - \zeta^{(n)}}{2\sin\theta} , \tag{2.6}$$

$$\zeta = \frac{\zeta^{(p)} + \zeta^{(n)}}{2\cos\theta} .$$

The correspondence $\{\zeta^{(p)}, \zeta^{(n)}\} = \{\alpha, \beta, \gamma, \theta\}$ is one to one and regular for $0 < \theta < \pi/2$.

The variables $(\zeta^{(p)}, \zeta^{(n)}) = \{\alpha, \beta, \gamma, \theta\}$ are not sufficient to describe the configurations of the classical system. However, they describe uniquely the quantized system owing to the constraints

$$I^{(p)}_{\zeta^{(p)}} = I^{(n)}_{\zeta^{(n)}} = 0, \tag{2.7}$$

appropriate to rigid bodies with axial symmetry. These constraints are automatically satisfied if we take wave functions depending on $\zeta^{(p)}$, $\zeta^{(n)}$ only.

We quantize by replacing I and S by their Cartesian operator realizations and then perform the change of variable $(\zeta^{(p)}, \zeta^{(n)}) \rightarrow (\alpha, \beta, \gamma, \theta)$. This change of variables is a unitary transformation provided the scalar product in the new variables is defined by

$$< \psi | \psi' > = \int_0^{2\pi} d\alpha \int_0^\pi d\beta \sin\beta \int_0^{2\pi} d\gamma \int_0^{\pi/2} d(2\theta) \sin(2\theta) \, \psi^*(\alpha\beta\gamma\theta) \, \psi'(\alpha\beta\gamma\theta). \tag{2.8}$$

The properties of this transformation are given in the second of the refs. (2), where it is shown that the transformed S operator and Hamiltonian are

$$S_\xi = i \, \partial/\partial\theta, \ S_\eta = -\cot\theta I_\zeta, \ S_\zeta = -\tan\theta I_\eta, \tag{2.9}$$

$$H = 1/(2J) \, I^2 + H_I,$$

$$H_I = 1/(2J) \left[\cot^2\theta \, I_\zeta^2 + \tan^2\theta \, I_\eta^2 - \partial^2/\partial\theta^2 - 2\cot(2\theta) \, \partial/\partial\theta \right] + (J_n - J_p)/(4J_n J_p) \tag{2.10}$$

$$\left[-\tan\theta \, I_\zeta I_\eta - \cot\theta \, I_\eta I_\zeta + iI_\xi \, \partial/\partial\theta \right] + V.$$

We assume an harmonic approximation for the potential which, due to the geometry of the system must have the form

$$V(\theta) = \begin{cases} 1/2 \, C\theta^2, & 0 \leq \theta \leq \pi/4, \\ \\ 1/2 \, C \left[\pi/2 - \theta \right]^2, & \pi/4 \leq \theta \leq \pi/2. \end{cases} \tag{2.11}$$

3. - THE EIGENVALUE PROBLEM

The general expression for the eigenfunctions is

$$\psi_{IM\sigma} = \left[(2I+1)/(8\pi^2) \right]^{1/2} \sum_K D^I_{MK}(\alpha\beta\gamma)\Phi_{IK\sigma}(\theta), \tag{3.1}$$

where σ stands for all necessary quantum numbers.

These eigenfunctions must satisfy the constraint

$$R_\xi^{(p)}(\pi) \, \psi_{IM\sigma} = R_\xi^{(n)}(\pi) \, \psi_{IM\sigma} = \psi_{IM\sigma}, \tag{3.2}$$

owing to the fact that configurations of the system differing by a rotation of protons or neutrons of π around the ξ axis are indistinguishable. The symmetry of the system imposes two sets of relations.

The first one is

$$\Phi_{IK\sigma}(\theta) = (-1)^I \Phi_{I-K\sigma}(\theta). \tag{3.3}$$

Using this relation we can rewrite the eigenfunctions as

$$\Psi_{IM\sigma} = [(2I+1)/(16\pi^2)]^{1/2} \sum_{K\geq0} 1/\sqrt{1+\delta_{K0}} \left[D^I_{MK} + (-)^I D^I_{M-K} \right] \Phi_{IK\sigma}(\theta). \tag{3.4}$$

The second set of constraints relates the values of the Φ's in the regions $0\leq\theta\leq\pi/4$ and $\pi/4\leq\theta\leq\pi/2$

$$\Phi_{00\sigma}(\pi/2-\theta) = \Phi_{00\sigma}(\theta),$$
$$\Phi_{1K\sigma}(\pi/2-\theta) = -\Phi_{1K\sigma}(\theta),$$
$$\Phi_{22\sigma}(\pi/2-\theta) = 1/2\,\Phi_{22\sigma}(\theta) - 1/2\,\sqrt{3}/2\,\Phi_{20\sigma}(\theta), \tag{3.5}$$
$$\Phi_{21\sigma}(\pi/2-\theta) = \Phi_{21\sigma}(\theta),$$
$$\Phi_{20\sigma}(\pi/2-\theta) = -\sqrt{3}/2\,\Phi_{22\sigma}(\theta) - 1/2\,\Phi_{20\sigma}(\theta).$$

It is therefore sufficient to solve the eigenvalue problem for $0 \leq \theta \leq \pi/4$. The solution of the eigenvalue problem is simplified by the following transformation, which eliminates the first term linear in the θ derivative from the Hamiltonian (2.10),

$$(U\phi_{IK\sigma})(\theta) \stackrel{def}{=} \sqrt{\sin(2\theta)}\Phi_{IK\sigma}(\theta) = \varphi_{IK\sigma}(\theta), \tag{3.6}$$

$$H' = UH_IU^{-1} = 1/(2J)\,\{\cot^2\theta\,I^2_\varsigma + \tan^2\theta\,I^2_\eta - \partial^2/\partial\theta^2 - [2+\cot^2\theta]\}$$
$$+ (J_n-J_p)/(4J_nJ_p)[1/2\,(\cot\theta + \tan\theta)(I_\varsigma I_\eta + I_\eta I_\varsigma) - iI_\xi\,\partial/\partial\theta] + V(\theta). \tag{3.7}$$

Consistently with the harmonic approximation for V we expand H' in powers of θ up to second order

$$H' = 1/(2J)\,[1/\theta^2\,(I^2_\varsigma-1/4)-\partial^2/\partial\theta^2-2] + 1/2\,C\theta^2+1/(2J)\,\theta^2 I^2_\eta + (J_n-J_p)/(4J_nJ_p)$$
$$[1/2(1/\theta+\theta)(I_\varsigma I_\eta+I_\eta I_\varsigma)-iI_\xi\,\partial/\partial\theta]\,,\ 0\leq\theta\leq\pi/4. \tag{3.8}$$

Let us introduce the definitions

$$\omega = \sqrt{C/J},\ \theta_0 = (JC)^{-1/4},\ x = \theta/\theta_0. \tag{3.9}$$

Omitting the constant term $-1/J$, H' can be rewritten

$$H' = 1/2\,\omega\left[-\partial^2/\partial x^2 + 1/x^2\,(I^2_\varsigma-1/4)+x^2)\right]+ 1/2\,\omega\theta^4_0\,x^2\,I^2_\eta + \omega\,\theta_0\,(J_n-J_p)/(J_n+J_p)$$
$$\left[1/2\,(x + \theta^2_0\,(1/x))\,(I_\varsigma I_\eta+I_\eta I_\varsigma) - iI_\xi\,\partial/\partial x\right]. \tag{3.10}$$

According to the estimates made in Section (6) for a heavy nucleus $\theta_0 \simeq 0.05$, so that $(J_n - J_p)$ / $(J_n + J_p) \theta_0$ is of order of 1%, and we can neglect the last term in the Hamiltonian

$$H' = 1/2\,\omega\,\left[-\partial^2/\partial x^2 + 1/x^2\,(I^2_\varsigma - 1/4) + x^2)\right], 0 \le x \le \pi/(4\,\theta_0). \qquad (3.11)$$

This Hamiltonian does not contain any coupling between states with different K. In the region $\pi/4 \le \theta \le \pi/2$ states with different K are instead coupled, because in that region, writing $[\pi/2 - \theta]/\theta_0 = y$, we have

$$H' = 1/2\,\omega\,\left[-\partial^2/\partial y^2 + 1/y^2\,(I^2_\eta - 1/4) + y^2)\right]. \qquad (3.12)$$

Constraints (3.5) can be shown to be in agreement with the above approximate expression of the Hamiltonian. In this approximation the nucleus still has axial symmetry.

Note, however, that the terms we have neglected in H', while small with respect to the intrinsic excitation energies, are comparable to the rotational energy, so that deviation from pure rotational spectrum might be expected.

The eigenfunctions of H' are

$$\varphi_{IKn}(\theta) = \varphi_{Kn}(\theta) = \left[n!/((n+K+1)\theta_0)\right]^{1/2}(\theta/\theta_0)^{K+1/2}$$

$$\times e^{-1/2(\theta/\theta_0)^2}\,L^K_n(\theta^2/\theta^2_0), \qquad (3.13)$$

with eigenvalues

$$\varepsilon_{Kn} = \omega\,(2n + K + 1). \qquad (3.14)$$

The total eigenfunctions are

$$\psi_{IMKn}(\theta) = \left[(2I+1)/(16\pi^2\,(1+\delta_{ko}))\right]^{1/2}[D^I_{MK} + (-1)^I D^I_{M-K}]\,\varphi_{K\sigma}/\sqrt{2\theta} \qquad (3.15)$$

We remark that states with n=1, K=0, correspond to the classical vibrations shown in Fig. 1a, while states with n=0, K=1, correspond to the classical rotations shown in Fig. 1b.

4. - ELECTROMAGNETIC TRANSITION PROBABILITIES AND FORM FACTORS

The electromagnetic strengths are evaluated to leading order in the deformation parameter δ. To first order only the B(M1) and B(E2) are nonvanishing, while the B(M3) turns out to be of order $|\delta|^3$. The strengths of higher multipoles are much smaller than the single particle value and will not be reported.

While the M1-transition is the signature of the mode, the study of the E2- and M3-transitions is important to establish whether a rotational band exists as predicted by the model.

A - Magnetic Transitions

The general expression for the magnetic transition operator is

$$M(M\lambda,\mu) = -1/(\lambda+1) \int dr \; (j(r) \cdot (r \times \nabla)(r^{\lambda}Y_{\lambda\mu})$$

$$= -1/(\lambda+1) \int dr \; \rho_p(r) v_p(r) \cdot (r \times \nabla)(r^{\lambda}Y_{\lambda\mu}) =$$

$$= -1/(m(\lambda+1)) \int dr \; \rho_p(r) l_p \cdot \nabla(r^{\lambda}Y_{\lambda\mu}) ,$$

(4.1)

where m is the proton mass, l the angular momentum and ρ_p the charge density normalized to the total number of protons.

The transition operator can be conveniently rewritten by using the classical relations

$$l = m \; r(r \cdot \Omega) - m \; r^2 \Omega$$

$$\Omega_p = 1/J_p \, I_p .$$

(4.2)

After some straightforward algebra we obtain

$$M(M\lambda,\mu) = -1/(2(\lambda+1)) \; S_x/J_p \int dr \; \rho_p(r)(r^2\partial_x - x \; r \cdot \nabla) \; (r^{\lambda}Y_{\lambda\mu}).$$

(4.3)

For $\lambda = 1$ we get

$$M(M1,\mu) = \delta_{\mu, \pm 1} \sqrt{3/(32\pi)} \; S_x \; \mu_N$$

(4.4)

and the corresponding strength is

$$B(M1)\uparrow = 3/(16\pi) \; J\omega \; \mu^2_N .$$

(4.5)

For $\lambda > 1$ the integral in Eq. (4.3) vanishes unless the density deformation is taken into account. We therefore use the full expression of the density

$$\rho_p(r) = \rho_p[\; r(1- \beta \; Y_{20}) \;] ,$$

(4.6)

where

$$\beta = \sqrt{4\pi/5} \; 2/3 \; \delta$$

(4.7)

and δ is the deformation parameter

$$\delta = (R_3 - R_1)/R, \qquad R = 1/3 \; (2 \; R_1 + R_3)$$

(4.8)

For $\lambda = 3$ we get (third of refs.2)

$$M(M3,\mu) = \delta_{\mu \pm 1} \sqrt{7/(3\pi)} \; 1/5 \; | \; \delta \; | \; mZ <r^4> S_x \; (1/J_p) \; \mu_N,$$

(4.9)

and the corresponding strength is

$$B(M3)\uparrow = 14/(75\pi) \, \delta^2 \, m^2 \, A^2 <r^4>^2 \, \omega/J \, \mu_N^2 \, fm^4. \tag{4.10}$$

For the numerical estimates we will assume

$$<r^n> = 3/(3+n) \, R^n. \tag{4.11}$$

B - Electric Strength

The E2 operator is

$$M(E2, \mu) = e \int dr \, \rho_p \, (R_\theta^{-1} \, r) \, r^2 \, Y_{\lambda\mu}$$

$$= e \int dr \, \rho_p \, (r) \, r^2 \, Y_{2\mu} \, (R_\theta \, r) = e \sum_\nu Q_{2\nu}^{(p)} \, <2\nu \, | \exp(-i\theta \, I_\xi)| \, 2\mu> \tag{4.12}$$

where

$$Q_{2\mu}^{(p)} = \int dr \, \rho_p(r) \, r^2 \, Y_{2\mu} = Q_{2o} \, \delta_{\mu 0} = \sqrt{5/(16\pi)} \, Q_o \, \delta_{\mu 0} \tag{4.13}$$

for an ellipsoidal shape. To first order in θ and $\pi/2 - \theta$, Eq. (4.12) becomes

$$M(E2, \mu) = M_0(E2, \mu) + M_\theta(E2, \mu), \tag{4.14}$$

where

$$M_0(E2, \mu) \cong e \, Q_{20}^{(p)} \, [\, \delta_{\mu 0} \, S(\theta - \pi/4) + <20 \, | \exp(-i \, \pi/4 \, I_\xi)| \, 2\mu> S \, (\pi/4 - \theta)] \tag{4.15}$$

$$M_\theta(E2, \mu) = -i \, e \, Q_{20}^{(p)} \sqrt{3/2} \, \delta_{\mu 1} \, [\theta \, S \, (\theta - \pi/4) + (\pi/2 - \theta) \, S \, (\pi/4 - \theta)] \tag{4.16}$$

and

$$S(x) = \begin{array}{l} 1, x < 0 \\ \\ 0, x > 0. \end{array}$$

Using Eqs. (3.5) and (3.13) we get

$$< I = 2, K = 1, n = o|| \, M \, (E2, K=1) \, || \, I = K = n = 0 > \cong$$

$$\cong - \, i \, e \, Q_{20}^{(p)} \, 2\sqrt{3} \int_0^{\pi/4} d\theta \, \varphi_{210} \, (\theta) \, \theta \, \varphi_{ooo}(\theta) \cong - \, i \sqrt{3} \, Q_{20}^{(p)}\theta_0 \, e.$$

The E2-transition probability results

$$B(E2)\uparrow = 3 \, e^2 \, \theta_0^2 \, \left(Q_{20}^{(p)}\right)^2 = 15/(16\pi) \, e^2 \, 1/(J \, \omega) \, \left(Q_0^{(p)}\right)^2 \tag{4.17}$$

For the numerical estimates we will assume

$$Q_0^{(p)} = 4/3 \, Z \, | \, \delta \, | < r^2 > .$$
(4.18)

C - Magnetic Form Factors

The general expression of the magnetic operator for electron scattering is

$$T^{(m)} \, (M\lambda,q) = - \, i \, / \sqrt{\lambda \, (\lambda+1)} \int dr \, j_p \, (r) \cdot r \times \nabla \, (j_\lambda(qr)Y_{\lambda\mu})$$

$$= - \, i \, /(2 \sqrt{\lambda \, (\lambda+1)} \, S_x \, /J_p \int dr \, \rho_p \, (r) \, (j_\lambda \, (qr) \, /r^\lambda \, [(y^2+z^2)\partial_x$$
(4.19)

$$- \, x \, y \, \partial_y - x \, z \, \partial_z \,] \, (\, r^\lambda \, Y_{\lambda\mu} \,).$$

For $\lambda = 1$ the only nonvanishing matrix element is

$$< I = K = 1, n=0 \, || \, T_{1\mu}^{(m)} \, (q) \, || \, I = K = n=0 > =$$

$$= - \sqrt{\pi/3} \, \sqrt{\omega \, /J} \, \int_0^\infty d \, r \, r^3 \, \rho(r) \, j_1 \, (qr).$$
(4.20)

In the above equation $\rho(r)$ is the spherical density normalized to the total number of nucleons and we have checked that the first order correction in δ is of the order of 1%.

For $\lambda = 3$ the only nonvanishing matrix element is

$$< I = 3, K = 1, n=0 \, || \, T_{1\mu}^{(m)} \, (q) \, || \, I = K = n=0 > =$$
(4.21)

$$= 4/15 \sqrt{2\pi/7} \, | \, \delta \, | \sqrt{\omega \, /J} \, q \int d \, r \, r^4 \, \rho(r) \, j_2 \, (qr).$$

5. - SPLITTING OF THE M1 STATE AND TRIAXIAL DEFORMATION

If the nucleus has axial symmetry the relative rotation of protons against neutrons can occur only around one of the axes orthogonal to the symmetry axis, but if the nucleus has triaxial symmetry, rotations around each of the three axes become obviously possible.

Let us assume a small deviation from axial symmetry and let us consider relative rotations around the axes orthogonal to the axis of approximate symmetry. In such a case to first order in the deformation parameter we can retain the results of the TRM, but we must take into account that the restoring constant C is now different for rotations around the ξ- or η- axes, so that we have two different frequences

$$\omega_\xi = \sqrt{C_\xi/J_\xi} \cong \sqrt{C/J} \, |R_\zeta - R_\eta|/R$$

$$\omega_\eta = \sqrt{C_\eta/J_\eta} \cong \sqrt{C/J} \, |R_\zeta - R_\xi|/R$$
(5.1)

In the above formula $J = 1/2 \, (J_\xi + J_\eta)$.

Expressing R_i ($i = \xi, \eta, \zeta$) in terms of the deformation parameters β and γ

$$R_K = R + \sqrt{5/4\pi} \; \beta \; R \cos (\gamma - 2\pi k/3), \qquad (5.2)$$

we get

$$(\omega_2 - \omega_1) / (\omega_2 + \omega_1) = 1/\sqrt{3} \; \mathrm{tg} \; \gamma. \qquad (5.3)$$

Since according to Eq. (4.5) the B(M1) is proportional to the excitation energy

$$B_2(M1)\uparrow / B_1(M1)\uparrow = (1 + 1/\sqrt{3} \; \mathrm{tg} \; \gamma) / (1 - 1/\sqrt{3} \; \mathrm{tg} \; \gamma). \qquad (5.4)$$

Note that the above equation provides a check of the hypotesis of triaxiality, relating the strengths of the fragments to the energy splitting. Once the triaxial shape is established Eqs. (5.3), (5.4) allow a very precise measurement of the γ-parameter.

6. - NUMERICAL ESTIMATES

The TRM has been developed in analogy to the Goldhaber and Teller model for the E1 giant resonance. The analogy has been followed also in the evaluation of the parameters ω and θ_0.

Untill now we have only assumed the rotors to be rigid, and this is the only assumption intrinsic to the model.

Let us now assume the rotors to have a constant density and the same deformation for protons and neutrons. In such a case the moments of inertia are

$$J_p = 2/5 \; Z \; m \; R^2 (1 + 0.31 \; \beta)$$
$$\qquad (6.1)$$
$$J_n = 2/5 \; N \; m \; R^2 (1 + 0.31 \; \beta).$$

In order to evaluate C we observe that when θ becomes larger than a critical value θ_c, $1/2 \; \rho \Delta \upsilon \; (\theta)$ neutron-proton pairs do not interact any longer causing an increase of the nuclear potential energy

$$\Delta V_N = 1/2 \; \rho \Delta \upsilon \; (\theta) \; V_0. \qquad (6.2)$$

Here ρ is the nuclear density, V_0 is the neutron-proton interaction potential and $1/2 \; \Delta \upsilon$ the volume variation of the nucleus due to the neutron-proton relative rotation

$$\Delta \upsilon \; (\theta) \simeq 16/3 \; \theta \; | \; R^2_3 - R^2_1 \; | / \; R^2_1 \; R^3_3. \qquad (6.3)$$

The value of θ_c has been determined by requiring that for $\theta = \theta_c$ the relative displacement of neutrons w.r. to protons be equal to the range of the interaction r_0

$$r_0 \simeq 4/(3\pi) \; \theta_c \; | \; R^2_3 - R^2_1 \; | / \; R^2_1 \; R_3. \qquad (6.4)$$

At $\theta=\theta_c$ the restoring force given by ΔV_N must be equal to that given by V

$$C = 32/(9\pi) \ \rho \ V_0/r_0 \ | R^2_3 - R^2_1 |^2 (R_3/R_1)^4 \qquad (6.5)$$

To lowest order in the deformation parameter

$$\delta = (R_3 - R_1) / R \qquad (6.6)$$

we obtain

$$C = 128/(9\pi) \ V_0/r_0 \ \delta^2 \ \rho \ R^4 . \qquad (6.7)$$

Assuming the usual parametrization for ρ_0

$$\rho_0 = A /(4\pi/3 \ R^3) \quad , \quad R = 1.2 \ A^{1/3} \ \text{fm} \qquad (6.8)$$

and

$$V_0 = 40 \ \text{Mev} \ ; \quad r_0 = 2 \ \text{fm}, \qquad (6.9)$$

we obtain

$$\omega = 42 \ | \ \delta \ | \ A^{-1/6} \ \text{MeV}; \qquad \theta_0^2 = 1.7 \ A^{-3/2} | \ \delta \ |^{-1} \qquad (6.10)$$

For $A=156$, $\delta = 0.25$, $\omega=4.7$ MeV, $| \ \theta_0^2 \ | = 3 \cdot 10^{-3}$, $B(M1)\!\uparrow = 17\mu^2_N$,

$$B(E2)\!\uparrow = 289 \ e^2 \ \text{fm}^4 , \quad B(M3)\!\uparrow \cong 0.4 \ \mu^2_N \ b^2. \qquad (6.11)$$

These values are to be compared to the orbital single-particle estimates[13,19]

$$B(M1)\!\uparrow \ \underset{\sim}{\sim} \ 0.1 \ \mu^2_N$$

$$\qquad (6.12)$$

$$B(E2)\!\uparrow \ \underset{\sim}{\sim} \ 270 \ e^2 \ \text{fm}^4$$

It should be noted that the above estimates for ω and θ_0 are very crude. Let me mention some effects which should appreciably alter them.

The first one is common to the E1 mode, and is related to the fact that the volume $\Delta v \ (\theta)$ lies on the surface of the nucleus. The average density ρ_0 should therefore be replaced by the surface density. This effect, which is taken into account when the restoring force is determined by comparison with the symmetry energy[14] would lower C and therefore energy and M1 strength. To have an idea of its importance we can compare with the E1-mode where, when it is neglected, one still gets a qualitative agreement with experiment with the same values of the parameters given by Eqs.(6.9).

The other features which make the above estimate inadequate are specific of deformed nuclei.

The first one is related to the geometry. To see where the geometry makes a substantial difference with respect to the E1 mode let us rewrite Eq. (6.4) by introducing the deformation parameter

$$r_0 \simeq 8/(3\pi) \, \theta_c \mid \delta \mid R \,. \tag{6.13}$$

For ^{156}Gd we get $\theta_c \sim \pi/2$, which exceeds the maximum value of $\theta = \pi/4$, (there exists no analogous limitation in the translation displacement of the E1 mode) not to mention that all our formulae have been derived by retaining the dominant terms in the expansion for small θ. The estimate (6.7) however, could be considered the extrapolation of a result valid for a very large nucleus, and in fact a similar result has been obtained by a different method[12].

The second reason of inadequacy of the estimates (6.10) is that, due to superfluidity, the moment of inertia is half the rigid body value. Such a reduction can be accounted for in a two-fluid model[15], in which the moment of inertia is the rigid body moment evaluated for the normal fluid component of the nucleus. This is the nuclear region external to the largest sphere enclosed in the nucleus, the latter being the rotationally invariant superfluid components. (This picture is similar to that resulting in the IBA, where only the bosons describing Cooper pairs of valence nucleons are dynamical). Let me emphasize that it is perfectly consistent to introduce in this way superfluidity into the TRM, whose only assumption is that of rigid motion of protons against neutrons.

The last point to be mentioned in connection with the numerical estimates is related to the possibility that protons and neutrons have a different shape, as suggested by the fact that the pairing strength for protons is larger than for neutrons[16]. Assuming R_1 to be the same for protons and neutrons, and $R_3^{(p)} < R_3^{(n)}$, we would obviously get a restoring force constant smaller than for $R_3^{(p)} = R_3^{(n)}$ (Fig. 2). Let me mention however, that there is experimental evidence[17] that the protons are more deformed than the neutrons in ^{154}Sm.
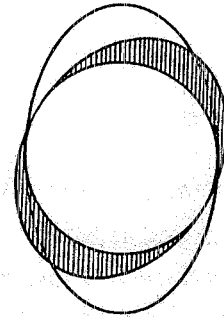

FIG. 2 - Different proton-neutron deformation

While the relevance of all the above effects is under investigation[18], it is perhaps worth-while to empasize that the procedure adopted to evaluate the parameters ω and θ_0 is not intrinsic to the TRM.

7. - THE SCHEMATIC RPA

The relation between TRM and RPA can be easily established in the framework of the unified theory of collective motion.

To this purpose it is necessary to observe that a rotation of protons against neutrons around one of the intrinsic axes, for instance the x_1-axis, induces a density variation of the type

$$\delta\rho \propto f(r)r^2 \; 1/(i\sqrt{2}) \; (Y_{21}+Y_{2,-1}) \; \tau_3\theta. \tag{7.1}$$

One would then be tempted to follow the prescriptions of the unified theory to conclude that for axially symmetric nuclei the TRM is equivalent to a (quasi-) degenerate RPA eigenvalue problem with a separable interaction of the form

$$V(1, 2) = \chi \, F^*(1) \, F(2) \, , \tag{7.2}$$

where the field F has the expression

$$F = \tau_3 \, r^2 \, Y_{21}. \tag{7.3}$$

This however would be correct only if the particle-hole (p-h) states excited by F were all (quasi-) degenerate, which is not the case.

Let us in fact assume that the nucleons move in an anisotropic harmonic oscillator potential of frequency $\omega_\perp$ and ω_3. These obey the usual volume conserving condition $\omega_3 \, \omega^2_\perp = \omega^3_0$, where ω_0 is the frequency for zero deformation.

The field F can be decomposed into the sum of two terms[3]

$$F = F_0 + F_2 = F_{01} + i \, F_{02} + F_{21} + i \, F_{22} \tag{7.4}$$

where

$$F_{01} = - \sqrt{15/(8\pi)} \; 1/(2m\sqrt{\omega_\perp \, \omega_3}) \; (a_3 \, a^+_1 + a^+_3 \, a_1)$$

$$\tag{7.5}$$

$$F_{21} = - \sqrt{15/(8\pi)} \; 1/(2m\sqrt{\omega_\perp \, \omega_3}) \; (a_3 \, a_1 + a^+_1 \, a^+_3)$$

with analogous expressions for F_{02} and F_{22}. Here the a^+ and a are creation and annihilation operators for oscillator quanta in different directions.

Now the F_0 terms excite p-h states of energy $\epsilon_0 = |\omega_\perp - \omega_3| \cong |\delta| \, \omega_0$, which are the ones entering into the M1-TRM state. The F_2 terms however excite p-h states of energy $\epsilon_2 = \omega_\perp + \omega_3 \cong 2\omega_0$, which enter into a state describing isovector quadrupole oscillations. In other words the scissors-mode is coupled to surface vibrations[12].

Such a coupling can be accounted for in the framework of a two levels RPA schematic model, whose eigenvalue equation is

$$\varepsilon_0/(\omega^2 - \varepsilon_0^2) \sum_{ph \in \varepsilon_0} |<0|F|ph>|^2 + \varepsilon_2/(\omega^2 - \varepsilon_2^2) \sum_{ph \in \varepsilon_2} |<0|F|ph>|^2 = 1/(2\chi) \quad (7.6)$$

The lowest eigenvalue is

$$\omega = |\delta| \omega_0 \sqrt{(1+2b)/(1+b)} \quad (7.7)$$

where

$$b = \chi / C_0 \quad (7.8)$$

and

$$\chi = \pi/A \ V_1/<r^4>, \ C_0 = 8/5 \ \pi \ m \ \omega_0^2/(A<r^2>), \quad (7.9)$$

V_1 being the symmetry potential.

The M1 strenght is

$$B(M1)\uparrow = 3/(16\pi) \ J\omega \ (g_p - g_n)^2 \ \mu_N^2. \quad (7.10)$$

It should be noted that the above expression coincides with that of the TRM, Eq. (4.5).
If the coupling with the quadrupole oscillation is neglected one gets

$$\omega_{degenerate} = |\delta| \omega_0 \sqrt{1+b}, \quad (7.11)$$

which is the value which can be directly compared to the TRM one.
The E2 strength is [3,19]

$$B(E2)\uparrow = 1/(1+b)^2 \ B(E2)\uparrow|_{degenerate}, \quad (7.12)$$

where

$$B(E2)\uparrow|_{degenerate} = 5/(32\pi) \ \delta^2 A <r^2>/(m\omega). \quad (7.13)$$

While the coupling with the quadrupole oscillations lowers the value of the energy but does not alter the expression of the M1 strength, it drastically reduces the $B(E2)\uparrow$ because, as we will see, $b \sim 2$.
The pairing has the following renormalizing effect[20]

$$\omega \rightarrow (E/\varepsilon) \omega$$

$$B(M1) \rightarrow (U_h V_p - U_p V_h)^2 \ B(M1) \sim (U_h^2 - V_h^2) \ B(M1) \sim (\varepsilon/E)^2 \ B(M1), \quad (7.14)$$

where E is the two quasi-particle (qp) energy, V_h, V_p (U_h, U_p) the occupation (vacancy) probability amplitudes respectively for the single-hole and-particle states entering into the p-h state of energy $|\delta| \omega_0$.

The ratio ε/E can be estimated by imposing that the moment of inertia be half the rigid body value

$$1/2 = J/J_{rig} \simeq \varepsilon/E \ (U_h^2 - V_h^2)^2 \simeq (\varepsilon/E)^3 \qquad (7.15)$$

This gives $\varepsilon/E \simeq 0.79$. A furter 10% reduction in the M1 strength comes from the neutron excess[20]. For numerical estimates we assume

$$V_1 = 130 \ MeV$$
$$< r^2 > = 3/5 \ R^2 \qquad (7.16)$$
$$< r^4 > = 3/7 \ R^4$$
$$\omega_0 = 41 \ A^{-1/3} \ MeV$$

which yields $b \cong 2$.

For ^{156}Gd we get $\omega_{degenerate} = 3.4$ MeV, $\omega = 2.6$ MeV. Such a reduction, due to the coupling with the quadrupole mode, cannot obviously be accounted for in the semiclassical TRM. Taking into account superconductivity and neutron excess we get the general formulae

$$\omega \simeq 66 \ |\delta| A^{-1/3} \ MeV \qquad (7.17)$$
$$B(M1)\uparrow \simeq 0.024 \ |\delta| \ A^{4/3} \mu_N^2. \qquad (7.18)$$
$$B(E2)\uparrow \simeq 0.003 \ A^2 \ |\delta| \ e^2 \ fm^4 . \qquad (7.19)$$

For ^{156}Gd, $\omega = 3.2$ MeV, $B(M1)\uparrow = 5.8 \ \mu_N^2$, and $B(E2)\uparrow = 18 \ e^2 \ fm^4$, to be compared with the experimental values $\omega = 3.1$ MeV, $B(M1)\uparrow = 2.3 \pm 0.5 \ \mu_N^2$ and $B(E2)\uparrow = 40 \pm 6 \ e^2 \ fm^4$.

8. - TRIAXIALITY IN THE SCHEMATIC RPA

In the triaxial case we assume that in the intrinsic frame the nucleons move in an anisotropic potential with frequencies

$$\omega_i = \omega_0 \exp \ (-\alpha_i) \qquad (8.1)$$
where
$$\alpha_i = \alpha \cos \ (\gamma - i \ 2\pi/3) \qquad (8.2)$$

We assume as before a schematic interaction of the form

$$V_i = \chi \ F_i \ (1) \ F_i \ (2) \qquad (8.3)$$

where the fields F_i are given by (second of Refs. 10)

$$F_1 = \tau_3 \, r^2/(i\sqrt{2}) \, (Y_{21} + Y_{2,-1})$$
$$F_2 = \tau_3 \, r^2/\sqrt{2} \, (Y_{21} - Y_{2,-1})$$
$$F_3 = \tau_3 \, r^2/(i\sqrt{2}) \, (Y_{22} - Y_{2,-2}) \tag{8.4}$$

The x_1- and x_2- eigenmodes are

$$\omega_i = \cos\gamma[\, 1-(-1)^i \, 1/\sqrt{3} \, \tan\gamma\,]\,\omega\,, \quad i = 1, 2 \tag{8.5}$$

where ω is the RPA eigenvalue in the axial limit (γ=0). The splitting between the two levels is

$$\Delta\omega = 2/\sqrt{3} \sin\gamma\,\omega \tag{8.6}$$

The M1 strenghts are

$$B_i(M1)\!\uparrow = 1/2 \cos\gamma \, [\, 1-(-1)^i \, 1/\sqrt{3} \tan\gamma\,]\, B(M1)\!\uparrow\,, \quad i = 1, 2 \tag{8.7}$$

where $B(M1)\!\uparrow$ is the value in the axially symmetric limit $\gamma=0$.

The above equations agree with Eqs. (5.3) and (5.4) of the TRM.

A third mode absent in axial nuclei emerges with an energy

$$\omega_3 = 2/\sqrt{3} \sin\gamma\,\omega \tag{8.8}$$

and a M1 strenght

$$B_3(M1)\!\uparrow = 2/\sqrt{3} \sin\gamma\, B(M1)\!\uparrow \tag{8.9}$$

The two quantities vanish in the axial limit consistently with the fact that in this limit the state must disappear. For very small values of γ, the x_3- mode is very low in energy and weakly excited, so that it is very unliked to be observed, if it exists at all.

9. - REALISTIC CALCULATIONS

A number of realistic calculations have been performed for ^{156}Gd. They all give a value of the total strength more than twice the experimental one. All these calculations with one exception give an M1 strength concentrated into 2 regions.

In ref. (21) wave functions of a deformed Wood-Saxon potential with a l-s term are used. The two-body potential contains paring, quadrupole and spin-spin interactions (Third plot of Fig. 3). In spite of the orbital nature of the main state, the authors question its interpretation as a scissors mode.
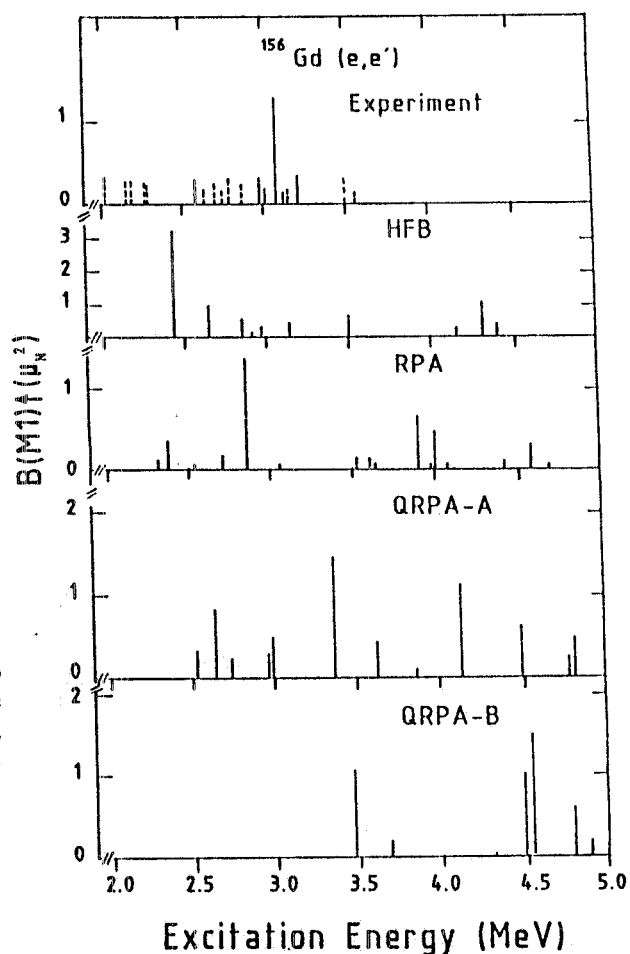
FIG. 3 - Experimental M1 strength distribution in ^{156}Gd (upper part, where full lines correspond to collective rotational excitations) compared to the theoretical predictions of ref. (11) (second plot), ref. (21) (third plot), ref. (22) (fourth plot), ref. (23) (fifth plot).

In ref. (22) wave functions of a deformed oscillator with a Skyrme interaction are used (Fourth plot of Fig. 3), and in ref. (23) wave functions of an axially symmetric Wood-Saxon potential with a quadrupole interaction (Fifth plot of Fig. 3).

The exception is ref. (11), which gives a strength of the main level much higher than the strength of the other fragments (second plot of Fig. 3). The distinctive feature of the calculation is the addition of a quadrupole pairing to a quadrupole interaction. Wave functions of a Nillson potential with a l-s term are used. A Hartree-Bogoliubov plus RPA calculation shows that the quadrupole paring plays an important role in bringing collectivity into the M1-state, while the l-s produces a significant fragmentation with a result similar to the experimental one.

Finally a calculation[24] on nuclei of mass around A=130 shows a pattern of fragmentation analogous to the papers of the first group (Fig. 4). The wave functions are linear combinations of particle-number and spin-projected 0 q-p and 2 q-p determinants obtained from an optimal HFB mean field. The interaction is a sligthly renormalized Bruckner G-matrix.
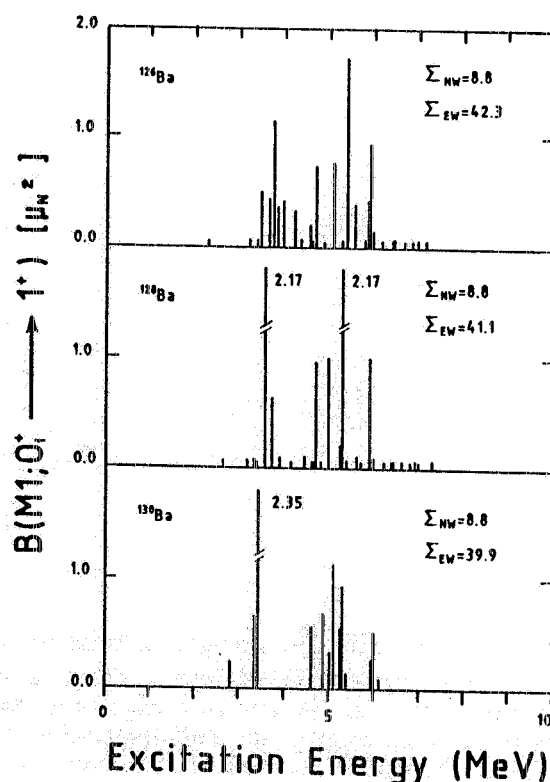
FIG. 4 - Predicted B(M1)↑ strength distributions for selected Ba isotopes. Free values for the proton (g_l^P=5.587, g_l^P=1.0) and neutron (g_s^n= -3.383, g_e^n=0.0) gyromagnetic factors are used. In addition non-weighted (Σ_{NW}) and energy-weighted (Σ_{EW}) sum rules are presented.

10. - THE IBA

I will only sketch how the M1 state in the IBA is related to the scissors mode of the TRM and then I will report some IBA predictions.

The Hamiltonian used is

$$H = \varepsilon_d\, n_d + K\, (Q_\pi + Q_v) \cdot (Q_\pi + Q_v) + \lambda M \tag{10.1}$$

The first term accounts for the pairing, the second one for the quadrupole and the third one for Majorana interaction. The operators Q and M are given in second quantized form using the boson creation and annihilation operators for bosons of angular momentum zero and two (s and b bosons) as

$$Q_\rho = (s^+_\rho\, \tilde{d}_\rho + d^+_\rho\, s_\rho)^{(2)} + \chi_\rho\, (d_\rho^+\tilde{d}_\rho)^{(2)}, \tag{10.2}$$

$$M = (s^+_\pi d^+_v + d^+_\pi s^+_v)^{(2)} \cdot (s_v\, \tilde{d}_\pi + \tilde{d}_v s_\pi)^{(2)} - 2 \sum_{k=1,3} (\tilde{d}_\pi\, \tilde{d}_v)^{(k)} \cdot (d^+_\pi d^+_v)^{(k)} \tag{10.3}$$

with $\rho = (\pi, v)$ denoting proton and neutron bosons, respectively.

If the structure constants of the quadrupole operator are equal, i.e. $\chi \equiv \chi_\pi = \chi_v$, the Hamiltonian is symmetric under the interchange of proton and neutron variables. This symmetry is related to the boson

quantum number F. Bosons are assumed to have F-spin $F=1/2$ with projection $F_z = 1/2$ for proton and $F_z=-1/2$ for neutron bosons. With the help of this new quantum number the boson states can be labeled according to their symmetry in the proton neutron degree of freedom. The low lying symmetric states have $F_{max}= (N_\pi+N_\nu)/2$ while the mixed symmetric $J^\pi=1^+$ states have $F=F_{max}-1$. The Majorana operator (10.3) used in the Hamiltonian (10.1) reduces to a simple form in the presence of this symmetry, namely

$$M = F_{max} (F_{max} + 1) - F(F+1) \qquad (10.4)$$

The parameters of the pairing and quadrupole part have been fixed by the well known low energy spectra and the strength of the Majorana force has been determined from the excitation energy of the $J^\pi=1^+$ state.

By use of coherent states of the form

$$| \psi_\alpha > = exp (\sum_\rho (\alpha_{s\rho} s_\rho^+ + \alpha_{d\rho} d^+_\rho) | 0 > \qquad (10.5)$$

a classical Hamiltonian can be constructed

$$H_c (\alpha_{s\pi}, \alpha_{s\nu}, \alpha_{d\pi}, \alpha_{d\nu}) = < \psi_\alpha | H | \psi_\alpha >. \qquad (10.6)$$

A procedure analogous to the one adopted for the TRM can be followed at this stage. By transforming to the intrinsic deformation parameters β_π γ_π and β_ν γ_ν and to Euler angles for the whole nucleus, the intrinsic part of the energy becomes a function of the intrinsic deformation parameters and of three angles θ_1 θ_2 θ_3 describing the relative orientation of protons versus neutrons. If both neutrons and protons posses axial symmetry and equal deformation $\gamma_\pi=\gamma_\nu=0$, $\beta_\pi=\beta_\nu=\beta$ and a single angle θ is needed to specify the relative orientation. For small values of θ the TRM Hamiltonian of Eq. (4.11) is obtained.

Let me now list the predictions of the model for ^{164}Dy

$$B(M1)\uparrow= 4.1 \ \mu^2_N$$
$$B(E2)\uparrow= 102 \ e^2 \ fm^4 \qquad (10.7)$$
$$B(M3)\uparrow= 0.07 \ \mu^2_N \ b^2$$

The B(M3) estimate is very uncertain, however, due to the difficulty of making the appropriate fermion-boson mapping. The value reported above has been obtained in ref. (9) by fitting the gyromagnetic factors.

11. - EXPERIMENTAL RESULTS

A state with the properties of the $J^\pi=1^+$, $K=1$ state predicted by the TRM has been first discovered[5] in a high resolution (e,e') experiment on ^{156}Gd. It is now confirmed in three regions of the periodic table, i.e. the deformed rare earth nuclei[5,6] (Figs. 5, 6), the $f_{7/2}$-shell nuclei[7] ^{46,48}Ti (Fig. 7) and the actinides[8] (Fig. 8).
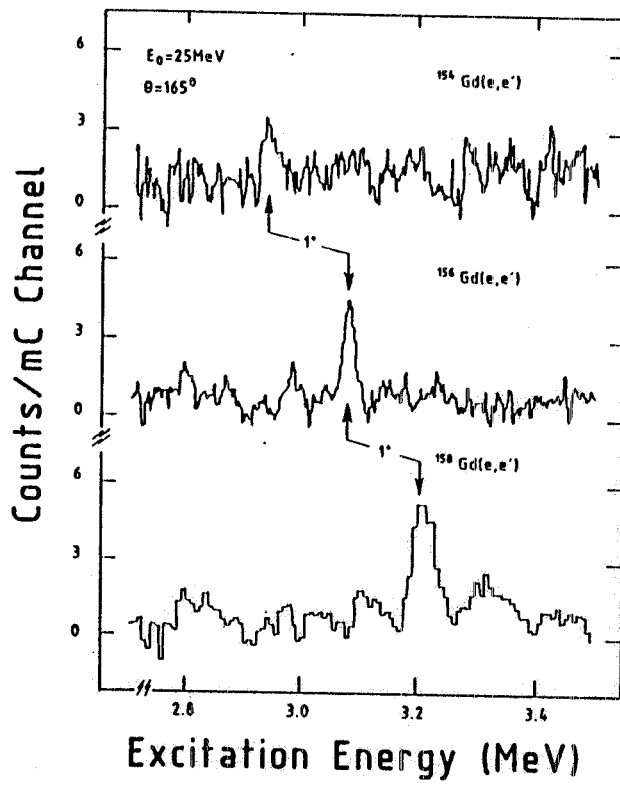
FIG. 5 - Inelastic electron scattering spectra. Peaks denoted by arrows are interpreted as $J^{\pi}=1^+$ state
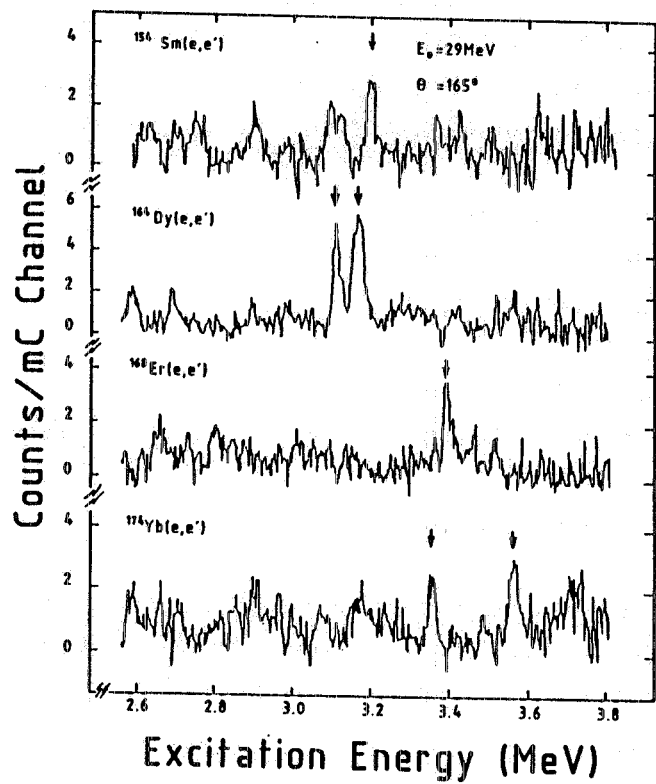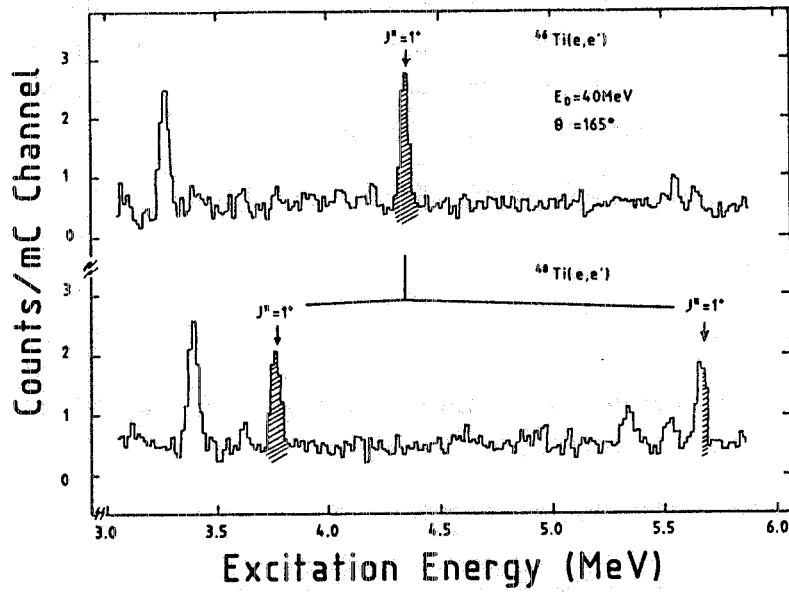
FIG. 6 - See caption of Fig. 5.

FIG. 7 -Inelastic electron scattering spectra on ^{46,48}Ti. In the triaxial nucleus ^{48}Ti two 1+ states are excited. They are separated by an energy of roughly 1.8 MeV.
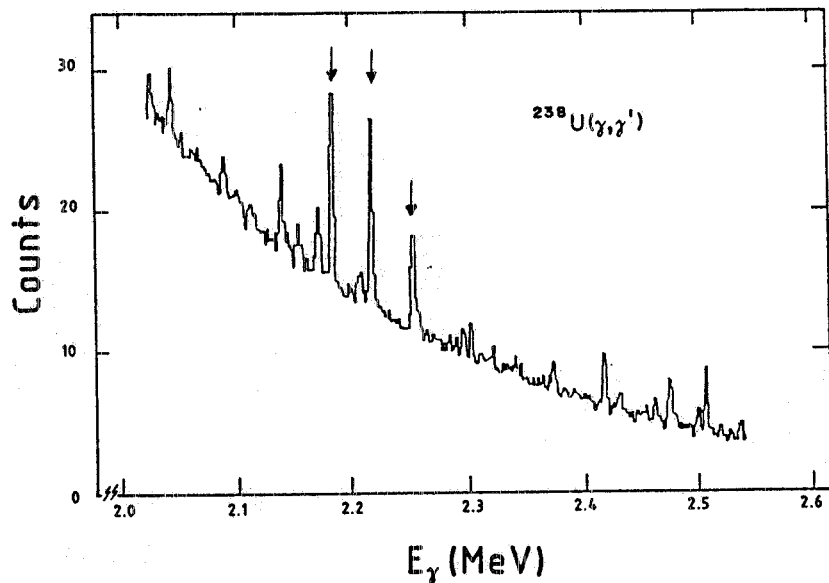

FIG. 8 - See caption of Fig. 5.

Two features of these states give special support to their interpretation as the scissors mode.

The first one is that the electron scattering form factors are in good agreement with the predictions which assume an orbital excitation mode and no deviations have been observed so far (see for instance Fig. 9).

The second is that the (p,p') reactions do not appreciably excite these states[7]. Since the intermediate energy proton scattering at small angles excites magnetic dipole states only through the spin part of the nucleon-nucleon interaction[26], this finding is consistent with the orbital nature of the M1-states.

The comparison between (e,e') and (p,p') experiments allows a quantitative evaluation of the orbital and spin contribution to the strength. In electron scattering one measures

$$B(M1) \uparrow = 3/(16\pi) \mid 1/2\, g_s < f \| \sum_\kappa \sigma_\kappa \tau_\kappa \| i > + g_l < f \| \sum_\kappa l_\kappa \tau_\kappa \| i > \mid^2$$

for a $\Delta T=1$ transition. Hence

$$\sqrt{B(M1)} \uparrow = \mid \pm \sqrt{B(\sigma)} + \sqrt{B(l)} \mid,$$

where the $\pm$ accounts for the uncertainty in the relative phase between $\sqrt{B(l)}$ and $\sqrt{B(\sigma)}$.

The (p,p') cross-section at q~0 can be written

$$d\sigma/d\Omega \mid_{q \sim 0} = (\mu/(2\pi))^2\, k_f/k_i\, N_D \mid V_{\sigma\tau} \mid^2 \mid < f \| \sum_\kappa \sigma_\kappa \tau_\kappa \| i > \mid^2$$

where μ is the total energy of the projectile, N_D a distortion factor and $V_{\sigma\tau}$ the σ–τ nucleon-nucleon interaction. The (p,p') cross-section is therefore proportional to $B(\sigma)$.
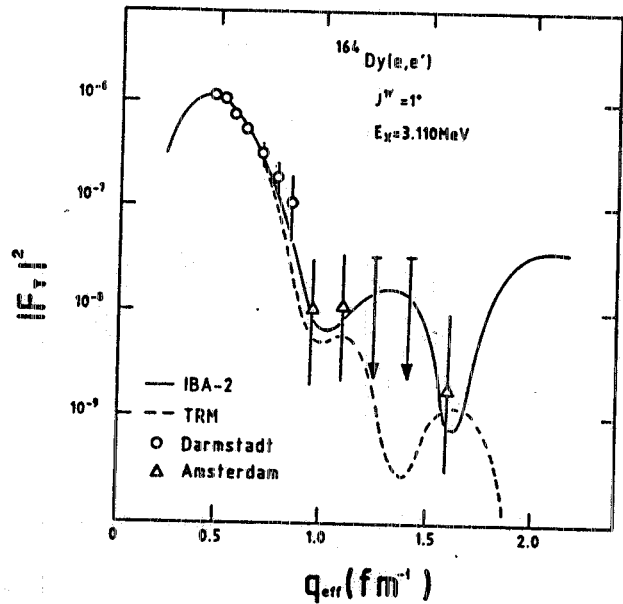

FIG. 9 - Transverse form factors of the transition to the $J^\pi=1^+$ state in ^{164}Dy at Ex=3.11 MeV. Compared to the data are the IBM and TRM prediction.

The result of the comparison is reported in Table I, which shows that the orbital contribution is always dominant with the exception ^{46}Ti. In this connection we must note that Zamick[27] has been the first to point out that low lying $J^\pi=1^+$ states should exist also in medium light nuclei, but with important spin effects. He has calculated that the $B(l)/B(\sigma)$ ratio in the $f_{7/2}$-shell should be of order 1, while the experimental value is 3.

Having established the nature of the state let us have a quantitative loock at energy and strength.

The energy scales rather well according to the RPA result

$$\omega = 66 \ |\delta| \ A^{-1/3} \text{ MeV},$$

with the exception of ^{46}Ti, while the TRM predicts a larger value. In this connection we must recall that the coupling to the quadrupole oscillations is essential to lower the RPA energy.

The experimental strength in ^{156}Gd is $B(M1)\uparrow = 2.3 \pm 0.5 \ \mu_N^2$. This is less than half the schematic RPA with pairing and the realistic calculations value, and much smaller than the TRM prediction.

TABLE I - $B(M1)\uparrow$ strength and ratio $\sqrt{B(1)}/\sqrt{B(\sigma)}$ from (e,e') and (p,p') scattering: [b] assuming the positive sign; [c] assuming the negative sign.

Nucleus	Ex (MeV)	$B(M1)\uparrow$ (μ^2_N)	$\sqrt{B(1)}/\sqrt{B(\sigma)}$ (range)
^{154}Sm	3.2	0.8±0.2	>(0.8-1.4)[b] >(0.8-3.4)[c]
^{156}Gd	2.18 3.075	1.3±0.2	>(1.3-1.7)[b] >(3.3-3.7)[c]
^{164}Dy	3.11 3.16	1.5±0.3 +1.4±0.4 =2.9±0.5	>(1.8-2.3)[b] >(3.4-4.3)[c]
^{46}Ti	4.32	1.0±0.2	1.2-2.5[b] 3.2-2.4[c]

We must remind, however, that the expression for the strength is exactly the same in the TRM and in the RPA, so that this difference is only due to the different values of ω and J used in the two cases. The agreement with the IBA is instead good.

Let us now come to the pattern of fragmentation. The investigation of the energy spectra above 4 MeV has not yet been completed, but preliminary results show no M1-strength in this region[9]. All the strength seems therefore concentrated very closely around the main level (Fig. 3) in fragments of very small individual strength, with the exception of ^{164}Dy, ^{157}Yb, ^{48}Ti and ^{238}U. Apart from this latter nucleus, in these cases we actually have a splitting rather than a fragmentation. It was in fact this observation in ^{164}Dy and ^{174}Yb which suggested a relation to triaxiality[10]. Now recent results with higher resolution have shown that there are three levels in ^{164}Dy rather than two, at energies 3.111, 3.159 and 3.173 MeV with $B(M1)\uparrow$ 1.3, 1.25 and 1.1 μ_N^2 respectively[28]. These results are no longer compatible with a splitting due to triaxiality.

In conclusion the pattern of fragmentation is reproduced only by the calculation of ref. (11), which seems and indication of the importance of the quadrupole pairing.

It remains only to mention that there exists a candidate for the $J^\pi=2^+$ member of the band[29], with a strength

$B_2(E2) \uparrow = 40 \pm 6 \, e^2fm^4$.

in reasonable agreement with the RPA and IBA predictions.

12. - CONCLUSION

We have seen that there is a new collective mode in deformed nuclei, which is described in essentially the same way in different models. It is remarkable that the expression for the B(M1) in the TRM and in the RPA coincide, as it is the case for the eigenstate equation in the TRM and in the IBA.

While the general features of this mode are well understood, there are a few points which deserve further investigation.

From the experimental side it is necessary

i) to complete the measurement of the M1 strenght at higher energy.

ii) to study the members $J = 2^+$, 3^+ of the band, in particular with respect to the orbital and spin contribution to the strength. Let me remind in this connection that in the latest IBA analysis the M3 mode is not expected to be a dominantly orbital mode(9).

iii) to study triaxial nuclei.

From the theoretical side I think it would be very interesting to investigate the effect of a different proton-neutron deformation on the total strenght and fragmentation of the M1-mode. A quantitive prediction of these features of the mode remains in fact the main open problem.

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