G. Preparata:

QCD AND THE DYNAMICS OF CONFINEMENT: A PROBLEM THAT IS BEING SOLVED

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1. INTRODUCTION

So far the only theoretical framework that has produced a _coherent_ picture of Quantum Chromo Dynamics (QCD) has been Perturbation Theory (PT).

The idea that PT must have something to do with any candidate theory of hadrons, such as QCD, is strongly motivated by the simple description that the parton model gives of physical processes at short light-cone distances, e.g. deep inelastic scattering. The discovery of Asymptotic Freedom (AF) in perturbative QCD more than ten years ago[1] has convinced the great majority of particle physicists that a deep theoretical reason had been found for the observed simple partonic behaviour, and that QCD represents the only believable candidate for a fundamental theory of hadrons and their interactions.

The overall success[**] of the research programme based on perturbative QCD has, in my opinion, played a considerable role in diverting the proper attention from what is and remains the crucial problem of QCD, colour confinement. The favourable preliminary indications from the numerical simulations of Lattice Gauge Theories (LGT)[2] have again convinced the community that the difficult problem of confinement most likely could only be attacked through the development of appropriate computational tools for the numerical simulations of LGT's, such as the new dedicated vector computers[3].

Thus the general assessment of the status of QCD today is that short distance physics is simple and can be systematically analysed through PT, while long distance physics - the physics of confined quarks and gluons - is rather messy and can be conquered (approximately) through the massive computations required by LGT's on lattices of increasing size.

One of the aims of this lecture is to put the above generally accepted view of QCD to a theoretical test of internal consistency by trying to determine whether PT does represent a stable dynamical realization of the QCD lagrangian. The question we wish to answer is then: is there in QCD a _finite_ length $d = \frac{1}{\mu} [\mu \lesssim 1 \text{ GeV}]$ such that for light-cone distances

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$|x^2| \leq d^2$, the physics is *asymptotically free*, i.e.

a) The perturbative ground state is a good approximation to the real ground state;

b) The physical Hilbert space consists simply of coloured quarks and gluons?

Needless to say the generally accepted *assumption* is that such μ does exist, and is related to the well known parameter Λ_{QCD}, that enters in the analysis of scaling violations in deep inelastic scattering. It should be quite clear, however, that within perturbative QCD, a theory with no mass scales (apart from the irrelevant quark masses), there is no way to give a sensible answer to this crucial question. One must go beyond PT.

We can picture the perturbative ground state (PGS) as a class of gauge-field configurations characterized by *small quantum fluctuations* $\eta_\mu{}^i(\vec{x})$ [i-1,2....N^2-1(SU(N)), μ-0,1,2,3] of the gauge-field $A_\mu{}^i(\vec{x})$ around the *classical ground state* $F_{\mu\nu}{}^i(\vec{x})$-0, and possessing minimum energy density

$$H(\vec{x}) - \frac{1}{2} \sum_{i=1}^{N^2-1} (\vec{E}^i(\vec{x})^2 + \vec{B}^i(\vec{x})^2) \qquad (1)$$

The object of Perturbation Theory is then the systematic investigation in powers of the coupling constant of the dynamics of the quantum fluctuations (Gluodynamics). A possible way to go beyond PT and the PGS was suggested in 1977 by G.K.Savvidy[4], who considered a larger class of field configurations characterized by quantum fluctuations around the special set of solutions of the classical Yang-Mills equations

$$B_k{}^i - \delta^{i\alpha} B u_k , \qquad (2)$$

i.e. a constant abelian chromomagnetic field in the space direction u, in the SU(N) direction α, and of magnitude B. Clearly B - 0 corresponds to the PGS. We shall call these particular states the "Savvidy states" $S_{u,\alpha}(B)$. From (1) and (2) their classical energy density E_{cl} is evidently:

$$E_{cl} - \frac{B^2}{2} . \qquad (3)$$

In the following section I will argue that the PGS [B = 0] is unstable, i.e. its energy density is higher than that of a Savvidy state with $B \neq 0$, and that the Savvidy state with minimum energy is associated with a magnetic field $B \simeq \Lambda^2$, Λ being the ultraviolet cut-off. This strange finding, that I have called the " essential instability" of the PGS, will be shown to be ruinous for AF and the consequent PT strategy In the final sections I will discuss how in QCD from the ashes of PT a full dynamical theory of confinement and hadronic behaviour emerges and is related to the Anisotropic Chromo-Dynamics (ACD) that I proposed in 1980[5].

2. THE "SAVVIDY STATES" LIE FAR BELOW THE PGS.

For Savvidy states three important questions arise:

Q_1 : Are the fluctuations small for small coupling constant g? (This is clearly a prerequisite for the applicability of a perturbative strategy).

Q_2 : Is their energy density lower than the perturbative ground state?

Q_3 : If this is the case, by how much?

I know of four different answers to these questions, that are summarized in Table I, which I will now comment upon.

Year	1977	1978	1985	1985
Q \ A	A_1	A_2	A_3	A_4
Q_1	YES	NO	NO	NO
Q_2	YES	YES	?	YES
Q_3	$\dfrac{\Lambda^4_{QCD}{}^{(*)}}{96\pi^2}$	?	$(+)$?	$-\Lambda^{4(++)}$

TABLE I

Synopsis of the 4 different answers to the 3 different questions formulated above. A_1 = Ref.[3], A_2 = Refs[6,7], A_3 = Ref.[8], A_4 = Refs[9,10]

The negative answer to Q_1 is completely uncontroversial, for if we write the gauge field $A_\mu{}^i = f_\mu{}^i + \eta_\mu{}^i$, where $\eta_\mu{}^i$ is the fluctuating field, for small η the Yang-Mills action can be expanded as

$$S[A] = S[f] + 1/2 \int \left. \frac{\delta^2 S[A]}{\delta\eta_\mu{}^i(\vec{x})\, \delta\eta_\nu{}^j(\vec{y})} \right|_{A=f} \eta_\mu{}^i(\vec{x})\, \eta_\nu{}^j(\vec{y}) + \dots \qquad (4)$$

(*) Λ_{QCD} is a typical hadronic scale of the order of a few hundred MeV.
(+) However it is claimed that PT is good at short distances.
(++) Λ is the ultraviolet cut-off.

The stability of the small fluctuations around the classical field $f_\mu{}^i$ is thus controlled by the positivity properties of the operator

$$0_{\mu\nu}{}^{ij}(\vec{x},\vec{y}) \to 1/2 \left. \frac{\delta^2 S[A]}{\delta\eta_\mu{}^i(\vec{x})\,\delta\eta_\nu{}^j(\vec{y})} \right|_{A=f} \tag{5}$$

The diagonalization of $0_{\mu\nu}{}^{ij}(\vec{x},\vec{y})$ was carried out more than 50 years ago by Landau, who showed that the energy levels of a charged massless particle in a constant magnetic field are described by three quantum numbers: p, the momentum in the direction of the constant field, n, nonnegative integer related to the confined transverse motion, and S_3 the spin projection along the field direction. The energy eigenvalues are given by the simple formula:

$$E_n(p)^2 = p^2 + gB(2n+1) - 2g\,B\,S_3 . \tag{6}$$

It is thus clear that for a particle with spin $|S| \geq 1$, the operator (5) cannot be positive definite. In particular for $|S| = 1$ gluons, as in Yang-Mills theories, there is a portion of the spectrum $[n = 0, S_3 = +1, p < (gB)^{1/2}]$ for which (6) is negative. These modes will be called unstable, or briefly U-modes. As first pointed out by Nielsen and Olesen[6] Savvidy's analysis, being a perturbative loop expansion, cannot be applied to the U-modes. On the other hand his analysis gives to leading order in g^2 the correct results for the other modes, the S-modes (for which $E_n(p)^2 > 0$).

According to (4) and (5) the existence of the U-modes prevents the fluctuating amplitudes associated to them from remaining small when the coupling constant $g \to 0$, for small U-modes amplitudes the action is at a maximum rather than a minimum, and we must expect the stabilization of the theory to occur for large amplitudes in such a way as to compensate the local maximum we have found with the neglected $0(g^2)$ terms in the expansion (4). One first unescapable consequence of this peculiar fact is that PT can only make sense in space-volumes whose size is $0\,(1/(gB)^{1/2})$, where an infrared cut-off gets generated large enough to exclude the U-modes from the spectrum. Thus the fate of AF and its perturbative strategy depends on whether B^* - the magnetic field for which the Savvidy states' energy density has a minimum - is finite or not. We shall see in a moment that the latter possibility holds in QCD.

As for the second question, neglecting the U-modes, in $SU(2)^{(*)}$ one computes[4,9] for the energy difference $\Delta E\,(B)$ between a Savvidy state $S_{u,cc}(B)$ and the PGS:

$$\Delta E\,(B) = E\,(B) - E\,(0) = B^2/2 - g^2 B^2/48\pi^2 \ln\,(\Lambda^2/gB) + 0\,[g^4 B^2 \ln\,(\Lambda^2/gB)] \tag{7}$$

(*) For simplicity we shall always refer to SU(2), but the generalization of our discussion to the interesting case of SU(3) poses no problem.

one should note the classical term $B^2/2$, the minus sign in front of the logarithm of the ultraviolet cut-off Λ, and the form of the correcting term which has been derived in Ref.11.

One immediately finds that (7) has a non-trivial $(B \neq 0)$ minimum for

$$gB^* \simeq \Lambda^2 \exp - 24\pi^2/11g(\Lambda)^2 . \tag{8}$$

By setting $gB^* = \Lambda^2_{QCD}$, a fixed hadronic scale, one can solve (8) for $g(\Lambda)^2$, and obtain the well-known AF result:

$$g(\Lambda)^2 \simeq 24\pi^2/11 \ln(\Lambda^2/\Lambda^2_{QCD}) . \tag{9}$$

If the physics were exhausted by S-modes, the generally accepted picture of QCD (freedom at short distances and confinement at long distances) would be strongly supported by the preceding analysis. Indeed the presence of a non-trivial magnetic structure in the ground state is a very good signal for confinement, while the finiteness of gB^* insures, as already stressed, the applicability of PT for distances $\lesssim 0 \, (1/ \sqrt{gB^*})$.

Thus apart from the authors of Ref.8, who take the rather sterile point of view that only the statements based on PT are presently meaningful, the consensus is that the answer to the second question is definitely positive.

We come now to the third question – the crucial one – which can be rephrased as Q_3 : what is the effect of the U-modes on $\Delta E(B)$?

Savvidy[4], unaware of the existence of the U-modes, has obviously nothing to say about them, while Nielsen and Olesen[7] do suspect that their effect is to lower further the energy difference (7), by a quantity they cannot determine, giving the Savvidy states a typical "spaghetti" structure, consisting of tubelike fluctuations of width $\sim 1/\Lambda_{QCD}$. The latter statement being a consequence of their unfailing belief in AF. Rather strangely the authors of Ref.8, clearly unhappy with the results that I will discuss in a moment, seem to ignore the potential danger for the PT strategy inherent in an instability which is admittedly non-perturbative, and reassure themselves by carrying out a perturbative calculation, and not surprisingly obtaining a result that is consistent with PT. Thus, they conclude, the effect of the U-modes on $\Delta E(B)$ must be nil. But how this miracolous rescue of AF should take place in practice they do not care to indicate.

The impasse posed by the non-perturbative nature of the dynamics of the U-modes has been overcome by the development of variational techniques within a class of (approximately) gauge-invariant wave-functionals[9,10]. The results of a rather laborious analysis[10] is that the U-modes contribute to the energy density of a Savvidy state the quantity

$$\Delta E_U(B) = - B^2/2 + O(g^2 B^2), \tag{10}$$

Thus completely cancelling in Eq. (7) the contribution of the classical energy density, leaving us with a total energy density's difference

$$\Delta E(B) \approx - 11g^2 B^2/48\pi^2 \ln(\Lambda^2/gB) + 0\,[g^4 B^2 \ln (\Lambda^2/gB)] \qquad (11)$$

whose minimum occurs at

$$gB \simeq \Lambda^2 , \qquad (12)$$

with the value

$$\Delta E(B^*) \simeq - \Lambda^4 . \qquad (13)$$

In the last part of this lecture I will discuss the consequences of Eqs. (11), (12) and (13) on the generally accepted picture of QCD.

3. PERTURBATIVE QCD IS DYNAMICALLY IRRELEVANT

We analyze now the consequence of the results just described on the dynamical relevance of perturbative QCD. Let us then summarize first the basic features of a Savvidy state of minimum energy density $S_{\vec{U},\alpha}(B^*)^{10,12}$:

(i)   it lies below the PGS by the quantity:

$$-\Delta E(B^*) \simeq 11/96\pi^2 \, (gB^*)^2 = 11/96\pi^2 \, \Lambda^4 \exp - 79/11 ; \qquad (14)$$

(ii) the stable gluonic modes acquire a mass $M^2 = 0\,(gB^*) \sim \Lambda^2$;

(iii) the expectation value of the chromomagnetic field is

$$<\vec{B}^i> = 6^{\,i\alpha}\vec{U}\,\overline{B} . \qquad (15)$$

with

$$\overline{B} = - 11/79 \,(8-\pi) \, g/16\pi^2 \,(gB^*) \sim \Lambda^2 ; \qquad (15')$$

(iv) the only finite mass fluctuations are the U-modes, which obey by the dispersion relation:

$$\omega\,(p_U) = [p_U^{\,2} + \mu^2]^{1/2} , \qquad (16)$$

with

$$\mu^2 = 4\,gB^* \exp - 8\pi^2/g^2 \sim \Lambda^2 \exp - 8\pi^2/g^2 ; \qquad (16')$$

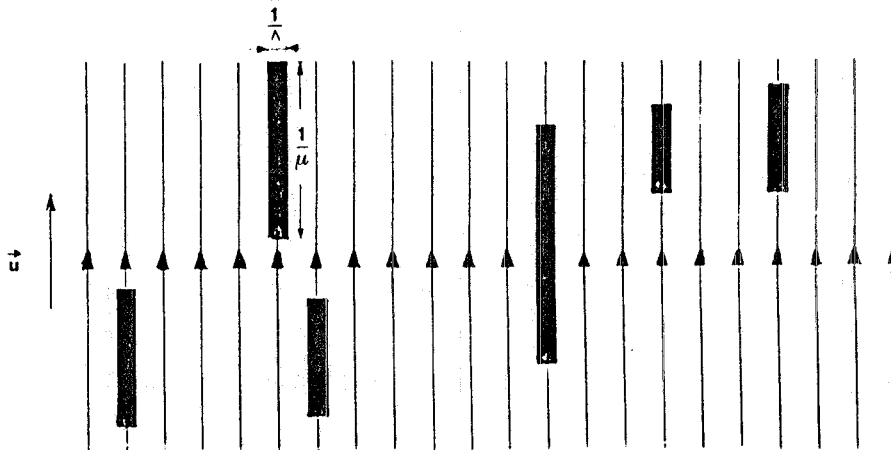thus leading to the picture of the Savvidy state depicted is in Fig.1

FIGURE 1
Schematic picture of the Savvidy state $S_{\alpha\vec{U}}(B^*)$. The lines of force are those of the background magnetic field while the thicker tubes are the dynamical fluctuations of the colour field (U-modes).

Upon the diverging ($\sim\Lambda^2$) homogeneous background field (15) there are fluctuations of the colour field (the U-modes) which have the shape of very thin ($\sim 1/\Lambda$) tubes, oriented in the direction of the chromomagnetic field, of length $\sim 1/\mu$ [Eqs.(16)], which look like "spaghettini" rather than "spaghetti".

It should now be clear that if energetically a configuration like this is immensely favoured over the PGS, perturbative QCD cannot describe, in any sensible way, the dynamics of the colour field, not even at very short distances. We may phrase concisely this fact by stating that: at no finite scale perturbative QCD can be a stable dynamical realization of QCD.

This does not mean, however, that some of the basic perturbative features, such as partonic behaviour, do not remain in the real ground state [*].

4. THE SAVVIDY STATES OF MINIMUM ENERGY CONFINE COLOUR

A given Savvidy state of minimum energy $S_{\alpha\vec{u}}(B^*)$ cannot represent the real QCD ground state, for it obviously violates both colour and rotational invariance. However, as I shall demonstrate in a while that it is an essential ingredient of the QCD ground state, it is worthwhile to investigate whether colour confinement holds in $S_{\alpha\vec{u}}(B^*)$. To see this, let's put a colour charge in the Savvidy state $S_{3\hat{z}}(B^*)$, the gauge field A_μ^i can then be written:

$$A_\mu^i(\vec{x}) = f_\mu^i(\vec{x}) + a_\mu^i(\vec{x}), \tag{17}$$

where

$$f_\mu^\alpha(\vec{x}) = \delta^{\alpha 3} g_{\mu 2} x_1 \bar{B}, \tag{17'}$$

$\bar{B}$, a divergent chromomagnetic field, is given by Eq.(15'), and $a_\mu^\alpha(\vec{x})$ is the part of the gauge-field that is generated by the external current $J_\mu^\alpha(x)^{ext}$, associated to our external charge. The finiteness of the difference $F_{\mu\nu}^\alpha(A) - F_{\mu\nu}^\alpha(f)$ [$F_{\mu\nu}^\alpha(g) = \partial_\mu g_\nu^\alpha - \partial_\nu g_\mu^\alpha + g \varepsilon^{\alpha\beta\gamma} g_\mu^\beta g_\nu^\gamma$] clearly requires that $a_\mu^\alpha(x)$ points in isospace in the direction of the background field (17'), i.e.

$$a_\mu^\alpha(\vec{x}) = \delta^{\alpha 3} a_\mu(\vec{x}), \tag{18}$$

thus rendering $a_\mu^\alpha(\vec{x})$ an effectively abelian gauge-field.

The colour dynamics in our "magnetic" medium $S_{3\hat{z}}(B^*)$ is thus described by the classical effective Lagrangian:

$$L_{eff}(x) = -1/4 \ f_{\mu\nu}(x) F^{\mu\nu}(x) + g a^\mu(x) J_\mu^3(x)^{ext}. \tag{19}$$

(*) Indeed in ACD the physics at short distances is free[13,15].

where $f_{\mu\nu} = (\vec{D}, \vec{H})$ and $F_{\mu\nu} = (\vec{E}, \vec{B})$ are the "magnitude" and the "intensity" tensors respectively, whose meaning is well known from the electrodynamics of continuous media; note that we have

$$\vec{B} = \bar{B}\hat{z} + \vec{B}_\perp \tag{20}$$

In a homogeneous medium, such as $S_{3\hat{z}}$ (B^*), we may set:

$$f_{\mu\nu}(x) = \in_{\mu\nu}{}^{\alpha\beta}(\hat{z})\, F_{\alpha\beta}(x) \tag{21}$$

where

$$F_{\alpha\beta}(x) = \partial_\alpha A_\beta(x) - \partial_\beta A_\alpha(x), \tag{21'}$$

$f_{\mu\nu}(x)$ obeys the second set of Maxwell equations:

$$\partial^\mu f_{\mu\nu}(x) = g\, J_\nu^{ext}(x), \tag{22}$$

and $\in_{\mu\nu}{}^{\alpha\beta}(\hat{z})$ is an "anisotropy" tensor, depending on the direction $\hat{z}$, and describing the electric and magnetic polarizability of the medium.

We can easily determine the structure of $\in_{\mu\nu}{}^{\alpha\beta}(\hat{z})$, by writing down the energy-momentum density associated with the effective lagrangian (19). We have :

$$H(\vec{x}) = 1/2\,(\vec{E}(\vec{x}).\vec{D}(\vec{x}) + \vec{B}(\vec{x}).\vec{H}(\vec{x})) \tag{23}$$

$$\vec{P}(x) = 1/2\,(\vec{D}(x) \times \vec{B}(x) + \vec{E}(x) \times \vec{H}(x)), \tag{24}$$

which in view of (20) can remain finite if and only if

$$\vec{D}(x) \parallel \hat{z} \quad \text{and} \quad \vec{H}(x) \perp \hat{z}, \tag{25}$$

From the conditions (25), setting the background field in a generic direction $\vec{u}$, we easily obtain:

$$\in_{\mu\nu}{}^{\alpha\beta}(\vec{u}) = -1/2\,(\delta_\mu{}^\alpha\, n_\nu n^\beta + \delta_\nu{}^\beta\, n_\mu n^\alpha - \delta_\mu{}^\beta\, n_\nu n^\alpha - \delta_\nu{}^\alpha\, n_\mu n^\beta) \tag{26}$$

with $n_\mu = (o, \vec{u})$. It is remarkable that the effective lagrangian (21) with the form (26) of the "anisotropy" tensor just coincides for $\vec{u}$ fixed with the lagrangian of Anisotropy Chromo Dynamics, that I proposed back in 1980[5]. According to (25) and the Gauss theorem a static charge g in $S_{3\hat{z}}$ (B^*) gives rise to the field structure depicted in Fig.2(a), while a charge-anticharge pair produces the configuration in Fig.2(b). This is just the typical manifestation of colour confinement. We may thus conclude that: <u>in a Savvidy state of minimum energy $S_{\alpha\vec{u}}$ (B^*) colour is confined.</u>
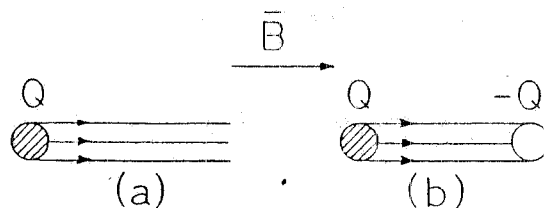

FIGURE 2
The lines of force of the electric displacement vector $\vec{D}$, for an isolated charge Q (a), and for a dipole (b).

5. FROM SAVVIDY STATES TO THE REAL QCD GROUND STATE.

As already emphasized any given Savvidy state of minimum energy $S_{\alpha\bar{u}}$ (B*) cannot be the real QCD ground state; however it seems extremely likely that the family of Savvidy states $(S_{\alpha\bar{u}}$ (B*)) has something to do with it. For of all known mechanisms of gluon condensation, such as instantons, merons and the like, the magnetic condensation occurring in Savvidy states is by far the most effective means to lower the energy of the PGS.

Let us then consider the family $(S_{\alpha\bar{u}}$ (B*)). One can easily prove that this system of states is essentially orthogonal, i.e. that the scalar product extended over any finite volume between any two different states vanishes. Indeed, the divergence of B* prevents any non-zero overlap between the different field configurations of any two Savvidy states.

The essential orthogonality of different Savvidy states, which prevents any local mixing among different states, allows us to view each state as the ground state of an independent quantum mechanical system and to express the real ground state |Ω>as

$$ |\Omega> = \prod_{\bar{u},\alpha} S_{\bar{u},\alpha} (B^*) , \qquad (27) $$

which is now both colour and rotation-invariant. On this ground state the dynamics is colour-confining (for, according to the above discussion, colour is confined in every Savvidy state), and its effective Lagrangian just coincides with the ACD-Lagrangian[12].

6. CONQUERING THE PHYSICS OF EHF.

The essential instability of the PGS, whose origin and physical consequences have been discussed in the preceding sections, leads then to a dynamical rearrangement of the QCD-Lagrangian into the ACD-Lagrangian, with one important additional dynamical element: the gluon-sector. Indeed, I have mentioned that in a given Savvidy state the S-modes get trapped in an infinite condensation process, while the non-trivial dynamics of the gauge field is carried by the U-modes only, which give rise to very thin ($\sim 1/\Lambda$), tubelike fluctuations upon an infinite ($\sim \Lambda^2$) magnetic background.

On the real ground state (27) such tubelike fluctuations become the components of dynamical fields which are [12]:

(i)   massive , as implied by the dispersion relation Eqs. (16). Note that μ should be simply related to the string-tension of the ACD confining potential;

(ii)   abelian , as determined by the absence of trilinear couplings among U-modes;

(iii)   confined , due to their coloured nature;

(iv)   vectors , due to their gauge-field nature.

Those who have some familiarity with the basic facts of hadronic behaviour, will have no difficulty in recognizing in the above properties a very good description of the phenomenological appearence of gluons.

By using the ideas of ACD already developed[13,14,15] we can:

(a) Calculate the spectrum of finite energy states (the hadrons) in the $q\bar{q}$, qqq, gg, $q\bar{q}q\bar{q}$, ... channels. The appropriate formalism of relativistic Schroedinger equations is already there[13].

(b) Compute transition amplitudes, form factors[17], etc. thus developing a full theory of hadronization.

(c) Study in a precise way the breaking of chirality and the dynamical role of the π-meson[14].

(d) Compute in detail the hadronic properties of e^+e^- annihilation into hadrons[15].

These are only a few of the areas where QCD-ACD can be put to test in the near future, with the theoretical tools already available. I hope that, once the deep connections between QCD and ACD is fully appreciated, there will be renewed interest in exploring and exploiting the theoretical potentialities of ACD.

7. CONCLUSIONS.

The essential quantum instability of the PGS of a non-abelian Yang-Mills theory, whose simple physical origin I have tried to clarify in this lecture, appears to induce a significant shift in our perception and assessment of Quantum Field Theory (QFT). So far the paradigm of QFT has evidently been QED, with its precise calculability and great transparency of physical interpretation. It is my feeling that it is the QED paradigm that has crucially contributed to the success and acceptation of perturbative QCD, which transposes to a non-abelian gauge theory fundamental points of QED, like the ground state and the small fluctuations around it, gluons and quarks. Even though, through the theoretical filter of AF, this picture has met with undeniable experimental successes, to sustain it in its present form does not appear reasonable. For the essential instability discussed above prevents QCD from having a perturbative realization which can be of any dynamical relevance. On the other hand it has been a great surprise and a pleasing bonus that out of the collapse of perturbative QCD a simple explanation has emerged of the confinement phenomenon, clearly extraneous to the present "paradigm" of QFT, and that the confined mode of QCD is closely related to the Anisotropic Chromo Dynamics. This latter fact promises to bring soon to a stage of theoretical maturity the field of hadronic physics - the confinement frontier - that can be thoroughly and deeply explored by facilities like EHF.

Finally it is conceivable that the "paradigm" shift which the findings discussed in this lecture might cause, could lead to a rather different assessment of the Standard Model and of the more recent developments of QFT, such as GUT's and Supersymmetry.

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