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# BHABHA SCATTERING NEAR THE $Z_0$

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ABSTRACT

A complete analysis of e.m. radiative corrections to Bhabha scattering near the $Z_0$ is presented. Compact analytic formulae are given which include exact one--loop results and soft-and-collinear photon effects resummed to all orders. Detection of back-to-back $e^+e^-$ pairs can be therefore used as a high precision monitor of luminosity at LEP/SLC energies.

$e^+e^-$ annihilation at LEP/SLC energies will provide precision tests for the standard electroweak model only if QED radiative corrections are under control at the level of $\lesssim 1\%$. Indeed first-order corrections(1-3), for example, reduce the $Z_0$ peak cross section by more than 50%, or shift the zero in the forward-backward asymmetry by about ($\pm$ 300) MeV, for an energy resolution of $(10^{-1}$-$10^{-2})$. It is therefore important that higher-order corrections are properly taken into account if the electroweak parameters have to be measured to the required accuracy(4).

Previous studies of these effects have been presented over the past few years, in particular for the process $e^+e^- \to \mu^+\mu^-$(1-4). The reaction $e^+e^- \to e^+e^-$, on the other hand, is particularly interesting for its large cross section and could provide a high precision monitor of the beam luminosity. Detailed studies of electro-weak radiative corrections to this process, which have been performed earlier[5-8], are all incomplete in some respects.

Indeed the calculation of electro-weak first order corrections, performed in ref.(5), does not extend to the energy range around the $Z_0$, because of the lack of finite width effects. Those were included in ref.(6), together with the complete treatment of soft photon effects, resummed to all orders. The analytical expressions for the box diagrams in the s and t channels however, were only given in the limit $s \sim M^2$, the left-over terms being of order $(\alpha/\pi)$. An attempt to improve these results has been made in ref.(7). Finally a treatment of collinear hard photon effects, quite relevant in calorimetric-type experiments, has been given in ref.(8).

The aim of the present paper is to give a final and complete description of QED radiative effects for Bhabha scattering, including exact analytical expressions for all one-loop diagrams, and soft and collinear hard photon effects resummed to all orders. Therefore our results include all double logarithmic terms of the form $(\alpha/\pi)\ln(s/m^2)\ln(\Delta, \Gamma/M)$, $(\alpha/\pi)\ln\delta^2\ln\Delta$, simple logs as $(\alpha/\pi)\ln(s/m^2)$, $(\alpha/\pi)\ln(\Delta, \Gamma/M, \delta^2)$, resummed to all orders, and all finite terms of orders $(\alpha/\pi)$. In the above M and $\Gamma$ are the mass and width of the $Z_0$ boson, and $\Delta$ and $\delta$ the energy and angular resolutions of the experiment, better defined below. Hard photon effects of order $(\alpha/\pi)(\Delta, \delta)$ have been neglected.

Our considerations apply to a typical experiment in which the following requirements are satisfied:

(i) The electron-positron pair should be detected back-to-back within a certain acollinearity angle J of a few degrees ($J \lesssim 5°$). The energy resolution $\Delta\omega$ depends upon J (see eq.(16)).

(ii) An electromagnetic calorimeter of finite and small angular resolution $\delta$ is centred along the electron and positron directions. In principle it does not discriminate between a charged particle and the accompanying collinear photons.

Then, using (i) and (ii), one would be sure that all but a fraction $\Delta \equiv \Delta\omega/E$ of the beam energy ($\sqrt{s} = 2E$) is taken by the electrons and the accompanying hard photons. For small $\Delta$ and $\delta$, fully analytic expressions can be used, neglecting hard-photon effects of order $[(\alpha/\pi)\Delta, (\alpha/\pi)\delta)]$.

We give a brief account of the derivation of our formulae. Our notations are as in ref. (6), unless stated explicitly. The relevant virtual graphs are shown in Fig. 1.

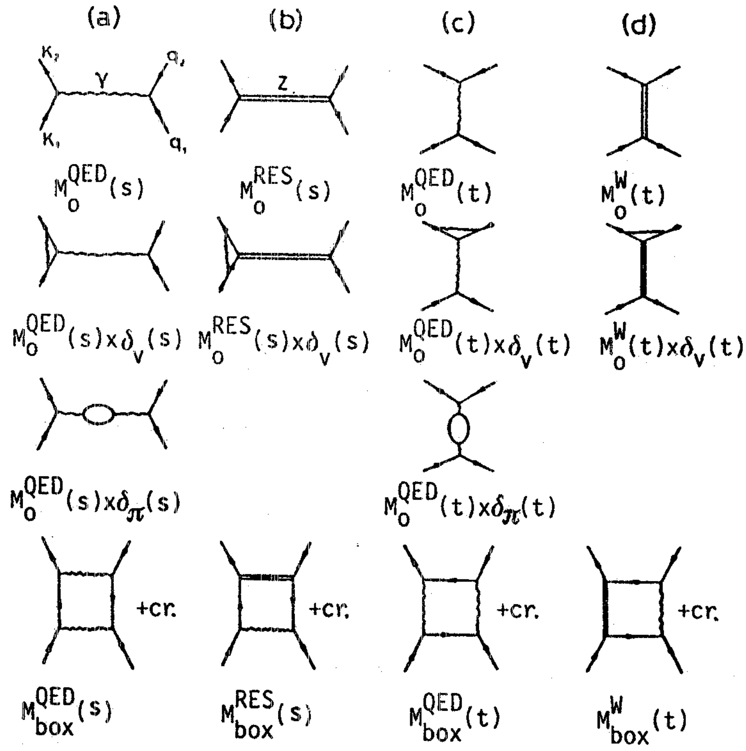


Fig. 1 - Virtual graphs in the s and t channel.

We have:

$$M(s,t) = M_0^{QED}(s)\left[1+\delta^{QED}(s)\right] - M_0^{QED}(t)\left[1+\delta^{QED}(t)\right] + M_0^{RES}(s)\left[1+2\delta_V(s)\right] - \tag{1}$$
$$- M_0^{W}(t)\left[1+2\delta_V(t)\right] + M_{box}^{QED}(s) - M_{box}^{QED}(t) + M_{box}^{RES}(s) - M_{box}^{W}(t),$$

where

$$M_0^{QED}(s) = (e^2/s)J_\mu(s)J_\mu'(s), \quad M_0^{QED}(t) = (e^2/t)J_\mu(t)J_\mu'(t),$$
$$M_0^{RES}(s) = \left[e^2/(s-M_R^2)\right]\left[f_V J_\mu(s) + f_A A_\mu(s)\right]\left[f_V J_\mu'(s) + f_A A_\mu'(s)\right],$$
$$M_0^{W}(t) = \left[e^2/(t-M^2)\right]\left[f_V J_\mu(t) + f_A A_\mu(t)\right]\left[f_V J_\mu'(t) + f_A A_\mu'(t)\right],$$
$$M_{box}^{QED}(s) = (2\alpha^2/s)\left\{J_\mu(s)J_\mu'(s)\left[V_1^\gamma(s)+2\pi i V_2^\gamma(s)\right] + A_\mu(s)A_\mu'(s)\left[A_1^\gamma(s)+2\pi i A_2^\gamma(s)\right]\right\}, \tag{2}$$
$$M_{box}^{QED}(t) = (2\alpha^2/t)\left\{J_\mu(t)J_\mu'(t)\left[V_1^\gamma(t)+2\pi i V_2^\gamma(t)\right] + A_\mu(t)A_\mu'(t)\left[A_1^\gamma(t)+2\pi i A_2^\gamma(t)\right]\right\},$$
$$M_{box}^{RES}(s) = \frac{\alpha}{2\pi}\left\{M_0^{RES}(s)\left[V_1^Z(s)+2\pi i V_2^Z(s)\right] + M_5^{RES}(s)\left[A_1^Z(s)+2\pi i A_2^Z(s)\right]\right\},$$
$$M_{box}^{W}(t) = \frac{\alpha}{2\pi}\left\{M_0^{W}(t)\left[V_1^Z(t)+2\pi i V_2^Z(t)\right] + M_5^{W}(t)\left[A_1^Z(t)+2\pi i A_2^Z(t)\right]\right\},$$

with

$$J_\mu(s) = \bar{v}(k_2)\gamma_\mu u(k_1), \quad J'_\mu(s) = \bar{u}(q_1)\gamma_\mu v(q_2), \quad A_\mu(s) = \bar{v}(k_2)\gamma_\mu\gamma_5 u(k_1),$$

$$A'_\mu(s) = \bar{u}(q_1)\gamma_\mu\gamma_5 v(q_2), \quad J_\mu(t) = \bar{u}(q_1)\gamma_\mu u(k_1), \quad J'_\mu(t) = \bar{v}(k_2)\gamma_\mu v(q_2), \tag{3}$$

$$A_\mu(t) = \bar{u}(q_1)\gamma_\mu\gamma_5 u(k_1), \quad A'_\mu(t) = \bar{v}(k_2)\gamma_\mu\gamma_5 v(q_2),$$

and

$$f_V = (4\sin^2\theta_W - 1)/4\sin\theta_W\cos\theta_W, \quad f_A = -1/4\,\sin\theta_W\cos\theta_W,$$

$\theta_W$ being the weak mixing angle. Moreover the following notations are used:

$$s = (k_1+k_2)^2 = 4E^2, \quad t = (k_1-q_1)^2 = -s\tfrac{1}{2}(1-\cos\theta), \quad u = (k_1-q_2)^2 =$$

$$= -s\tfrac{1}{2}(1+\cos\theta), \quad z = \cos\theta, \quad a = \sin\tfrac{1}{2}\theta, \quad b = \cos\tfrac{1}{2}\theta,$$

$$\beta_e = (2\alpha/\pi)\left[\ln(s/m^2)-1\right], \quad \beta_{int} = (4\alpha/\pi)\ln(a/b).$$

The matrix elements $M_5^{RES}(s)$ and $M_5^W(t)$ are defined as $M_5^{RES,W} = M_0^{RES,W}(\gamma_\mu \to \gamma_\mu\gamma_5, \gamma_\mu\gamma_5 \to \gamma_\mu)$.

The weak boson is taken as a resonance of mass M and width $\Gamma$, with $M_R^2 = M^2 - iM\Gamma$ and phase shift $\delta_R(s)$ where $\mathrm{tg}\,\delta_R(s) = M\Gamma/(M^2-s)$.

The radiative factors $\delta$ in eq.(1) are defined as follows (see Fig.1):

$$\delta^{QED}(x) = 2\delta_v(x) + \delta_\pi(x), \quad (x = s,t), \tag{4}$$

with the vertex and vacuum polarization parts given by ($\lambda$ is the photon mass)

$$\delta_v(s) \equiv \delta_v^R(s)+i\delta_v^I(s) = \Big\{-\frac{1}{2}\beta_e\ln(2E/\lambda)+(\alpha/2\pi)\left[\frac{1}{2}\ln^2(s/m^2)-\ln(s/m^2)\right] +$$

$$+ \frac{3}{8}\beta_e+(\alpha/\pi)(\pi^2/3-\frac{1}{4})\Big\} + i(\alpha/\pi)\left[\pi\ln(2E/\lambda)-\frac{3}{4}\pi\right], \tag{5}$$

$$\delta_v(t) = -\frac{1}{2}\beta_e\ln(2E/\lambda)-(2\alpha/\pi)\ln a\ln(2E/\lambda)+(\alpha/2\pi)\left[\frac{1}{2}\ln^2(s/m^2)-\ln(s/m^2)\right] +$$

$$+ \frac{3}{8}\beta_e+(\alpha/\pi)(\frac{3}{2}\ln a-\ln^2 a)+(\alpha/\pi)(\pi^2/12-\frac{1}{4}),$$

and

$$\delta_\pi(s) \equiv \delta_\pi^R(s)+i\delta_\pi^I(s) = \frac{\alpha}{3\pi}\sum_{i=1,q} Q_i^2\left[\ln(s/m_i^2)-\frac{5}{3}\right]+i(-\frac{\alpha}{3}\sum_{i=1,q} Q_i^2), \tag{6}$$

$$\delta_\pi(t) = \frac{\alpha}{3\pi}\sum_{i=1,q} Q_i^2\left[\ln(-t/m_i^2)-\frac{5}{3}\right],$$

with

$$Q_1^2 = 1, \quad Q_i^2 = \frac{4}{3}(\text{up}), \quad Q_i^2 = \frac{1}{3}(\text{down}).$$

The $\gamma Z$ box diagrams contributions $M^{RES}_{box}(s)$ and $M^{W}_{box}(t)$ can be casted in a very compact formula, which holds generally for every s and t, and in the limit $M^2 \to \lambda^2$ reduces to the known results for the $\gamma\gamma$ box diagrams $M^{QED}_{box}(s)$ and $M^{QED}_{box}(t)$. One has[9]

$$M^{RES}_{box}(s) = -2\alpha^2 \left\{ \left[f(s,t,u)-f(s,u,t)\right]\left[f_V J_\mu(s)+f_A A_\mu(s)\right]\left[f_V J'_\mu(s)+f_A A'_\mu(s)\right] + \right.$$
$$\left. + \left[f(s,t,u)+f(s,u,t)\right]\left[f_V A_\mu(s)+f_A J_\mu(s)\right]\left[f_V A'_\mu(s)+f_A J'_\mu(s)\right]\right\} \tag{7}$$

where

$$f(s,t,u) = \frac{1}{M_R^2-s}\left\{\left[\ln\frac{(ut)^{\frac{1}{2}}}{\lambda^2} + \ln(1-\frac{s}{M_R^2})^2\right]\ln(\frac{u}{t}) + \left[sp(1+\frac{u}{M_R^2})-sp(1+\frac{t}{M_R^2})\right]\right\}+$$
$$+\frac{u-t-M_R^2}{u^2}\left\{\ln(1-\frac{s}{M_R^2})\ln(-\frac{t}{s})+sp(1+\frac{t}{M_R^2})-sp(1-\frac{s}{M_R^2})\right\} + \frac{1}{u}\left\{(\frac{M_R^2}{s}-1)\ln(1-\frac{s}{M_R^2})+\ln(-\frac{t}{M_R^2})\right\}, \tag{8}$$

with

$$sp(x) = -\int_0^x \frac{dt}{t}\ln(1-t),$$

and $s \equiv s + i\varepsilon$. Then for $M_R^2 \to \lambda^2$ one recovers the $\gamma\gamma$ box diagrams results[10]

$$f(s,t,u) - f(s,u,t) \to -\frac{1}{s}\left[V_1^\gamma(s) + 2\pi i\, V_2^\gamma(s)\right]$$
$$f(s,t,u) + f(s,u,t) \to -\frac{1}{s}\left[A_1^\gamma(s) + 2\pi i\, A_2^\gamma(s)\right] \tag{9}$$

with(*)

$$V_1^\gamma(s) = -8\ln(a/b)\ln(2E/\lambda)-z(b^{-4}\ln^2 a+a^{-4}\ln^2 b)+b^{-2}\ln a-a^{-2}\ln b \equiv$$
$$\equiv -8\ln(a/b)\ln(2E/\lambda)+V_{1f}^\gamma(s),$$
$$V_2^\gamma(s) = 2\ln(a/b)-\frac{1}{2}z(b^{-4}\ln a+a^{-4}\ln b)-z/(1-z^2), \tag{10}$$
$$A_1^\gamma(s) = -z(b^{-4}\ln^2 a-a^{-4}\ln^2 b)+b^{-2}\ln a+a^{-2}\ln b,$$
$$A_2^\gamma(s) = -\frac{1}{2}z(b^{-4}\ln a-a^{-4}\ln b)+1/(1-z^2).$$

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(*) The functions $V_2^\gamma(s)$ and $A_2^\gamma(s)$ differ from the definition adopted in ref.(6) by a factor $2\pi$.

On the other hand, with the definition of $M_{box}^{RES}(s)$ given in eq.(2), one obtains

$$V_1^Z(s)+2\pi i V_2^Z(s) = (M_R^2-s)\left[f(s,t,u)-f(s,u,t)\right] \equiv$$
$$\equiv 4\ln(\frac{b}{a})\ln\frac{(M_R^2-s)^2}{\lambda^2 s} + V_{1f}^Z(s)+2\pi i V_2^Z(s), \qquad (11)$$
$$A_1^Z(s)+2\pi i A_2^Z(s) = (M_R^2-s)\left[f(s,t,u)+f(s,u,t)\right].$$

The expressions for the t-channel box diagram $M_{box}^{QED}(t)$ and $M_{box}^{W}(t)$ can be easily obtained by applying the crossing relation $s\leftrightarrow t$ in eq.(7), or equivalently in eqs. (9) and (11), with $M_R^2 \to M^2$ and $t \to t+i\varepsilon$, when necessary. One recovers then the $\gamma\gamma$ results(6)(*)

$$V_1^\gamma(t) = 8\ln b\ln(2E/\lambda)+8\ln a\ln b+\frac{1}{4}\pi^2(1-b^4)+\left[(1-b^4)/b^4\right]\ln^2 a +$$
$$+(1-b^4)\ln^2(a/b)+(a^2/b^2)\ln a+a^2\ln(a/b) \equiv 8\ln b\ln(2E/\lambda)+V_{1f}^\gamma(t),$$
$$A_1^\gamma(t) = -\frac{1}{4}\pi^2(1-b^4)+\left[(1-b^4)/b^4\right]\ln^2 a-(1-b^4)\ln^2(a/b)+(a^2/b^2)\ln a-a^2\ln(a/b), \qquad (12)$$
$$V_2^\gamma(t) = 2\ln(2E/\lambda)+\frac{1}{2}\left\{4\ln a-\left[(1-b^4)/b^4\right]\ln a+a^2/2b^2\right\} \equiv 2\ln(2E/\lambda)+V_{2f}^\gamma(t),$$
$$A_2^\gamma(t) = -\frac{1}{2}\left\{\left[(1-b^4)/b^4\right]\ln a+a^2/2b^2\right\}.$$

and, similarly to eq.(11),

$$V_1^Z(t)+2\pi i V_2^Z(t) = (M^2-t)\left[f(t,s,u)-f(t,u,s)\right] \equiv$$
$$\equiv 4\ln\frac{\sqrt{s}}{\lambda}(2\ln b+i\pi)+V_{1f}^Z(t)+2\pi i V_{2f}^Z(t) \qquad (13)$$
$$A_1^Z(t)+2\pi i A_2^Z(t) = (M^2-t)\left[f(t,s,u)+f(t,u,s)\right].$$

Let us briefly comment the above formulae. The s-channel $\gamma Z$ box diagrams have been first calculated(1) in the approximation $s\sim M^2$, where

$$M_{box}^{RES}(s) = (2\alpha/\pi)M_0^{RES}(s)\left\{\ln(b/a)\ln\left[(M_R^2-s)^2/M_R^2\lambda^2\right]+\right.$$
$$\left.+\frac{1}{2}sp(a^2)-\frac{1}{2}sp(b^2)-\ln^2 a+\ln^2 b\right\}, \qquad (14)$$

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(*) As for $V_2^\gamma(s)$ and $A_2^\gamma(s)$, notice the different definition of $V_2^\gamma(t)$ and $A_2^\gamma(t)$, with respect of ref.(6), by a factor $2\pi$. Notice also a sign misprint in ref. (6), in the expressions of $V_{2f}^\gamma(t)$ and $A_2^\gamma(t)$.

and $V_2^Z(s)$, $A_1^Z(s)$, $A_2^Z(s)$ are of order $(s-M^2)$.

Similarly the $\gamma Z$ box diagrams in the t channel have been previously computed[6] in the same soft limit, namely not including those contributions which vanish in in the limit of $k\to 0$, and are of the same order of magnitude of the left-over weak corrections. This amounts to approximate $M^W_{box}(t)$ as

$$M^W_{box}(t) = M^W_0(t)(\alpha/\pi)\{\ln\left[(-t+M^2)^2/M^2\lambda^2\right](i\pi+2\ln b)+2\ln^2 b-sp(b^2)- \\ - 4\ln a\ln b+\frac{2}{3}\pi^2-sp((s+M^2)/M^2)\} . \qquad (15)$$

It is clear that the exact expression for the $\gamma Z$ box diagrams, given in the simple forms of eqs.(8,11,13), allows for the complete evaluation of all $(\alpha/\pi)$ e.m. contributions to the process under consideration. A similar attempt has been made in ref.(7). Although the box contributions have been put in closed form in terms of two functions, analogous to the non-infrared parts of $V_i$ and $A_i$, their analytical expressions are rather cumbersome and include many simple and double logarithms of $(s-M^2)$. This concludes the discussion of the virtual contributions to one-loop corrections.

The analysis of the photon emission contributions follows closely that of ref. (6), as far as soft effects are concerned. Let us first define operatively the energy resolution $\Delta\omega$, in an experiment where the electron-positron pair is detected almost back-to-back.

For a given acollinearity angle J, the maximum energy $k_{max}$ taken by undetected soft photons, which defines the energy resolution $\Delta\omega$, is given by

$$k_{max} \equiv \Delta\omega = \frac{\sqrt{s}}{1+\cos J}\left\{-(1-\cos J)+2\left[(1-\cos J)\ \frac{1}{2} - \frac{m^2}{s}\ (1+\cos J)\right]^{1/2}\right\} . \qquad (16)$$

Then for J=1°, 3° and 5°, one obtains $\Delta\equiv\Delta\omega/E$ = (1.7)%, (5.1)% and (8.3)%, respectively.

The first order bremasstrahlung contributions, in the soft-photon approximation, can be grouped, following ref.(1) in three different classes: "QED-like" terms, pure resonant terms and interference with the resonating amplitude. To this aim it is useful to define the various lowest-order cross section as follows:

$$d\sigma_0\left[\gamma(s),\gamma(s)\right] = (\alpha^2/4s)(1+z^2) \equiv d\sigma_0(1), \qquad (17a)$$

$$d\sigma_0\left[\gamma(s),\gamma(t)\right] = -\ (\alpha^2/4s)2(1+z)^2/(1-z) \equiv d\sigma_0(2), \qquad (17b)$$

$$d\sigma_0\left[\gamma(t),\gamma(t)\right] = (\alpha^2/4s)\left[2/(1-z)^2\right]\left[(1+z)^2+4\right] \equiv d\sigma_0(3), \qquad (17c)$$

$$d\sigma_0\left[\gamma(s),Z(t)\right] = -(\alpha^2/4s)2R'(t)(1+z)^2(f_V^2+f_A^2) \equiv d\sigma_0(4), \tag{17d}$$

$$d\sigma_0\left[\gamma(t),Z(t)\right] = (\alpha^2/4s)\left[2/(1-z)\right]2R'(t)\left[(f_V^2+f_A^2)(1+z)^2+4(f_V^2-f_A^2)\right] \equiv d\sigma_0(5), \tag{17e}$$

$$d\sigma_0\left[Z(t),Z(t)\right] = (\alpha^2/4s)2R'^2(t)\left\{(1+z)^2\left[(f_V^2+f_A^2)^2+4f_V^2f_A^2\right]+ + 4\left[(f_V^2+f_A^2)^2-4f_V^2f_A^2\right]\right\} \equiv d\sigma_0(6), \tag{17f}$$

$$d\sigma_0\left[Z(s),\gamma(s)\right] = (\alpha^2/4s)2R'(s)\left[f_V^2(1+z^2)+f_A^2 2z\right] \equiv d\sigma_0(7), \tag{17g}$$

$$d\sigma_0\left[Z(s),\gamma(t)\right] = -(\alpha^2/4s)2R'(s)\left[(1+z)^2/(1-z)\right](f_V^2+f_A^2) \equiv d\sigma_0(8), \tag{17h}$$

$$d\sigma_0\left[Z(s),Z(t)\right] = -(\alpha^2/4s)R'(s)2R'(t)(1+z)^2\left[(f_V^2+f_A^2)^2+4f_V^2f_A^2\right] \equiv d\sigma_0(9), \tag{17i}$$

$$d\sigma_0\left[Z(s),Z(s)\right] = (\alpha^2/4s)\left[R'^2(s)+I'^2(s)\right](f_V^2+f_A^2)^2\left\{1+z^2+ +\left[4f_V^2f_A^2/(f_V^2+f_A^2)^2\right]2z\right\} \equiv d\sigma_0(10), \tag{17j}$$

$$R'(t) = \frac{1}{2}s/(M^2-t), \qquad R'(s)+iI'(s) \equiv s/(s-M_R^2). \tag{17k}$$

Then in terms of these elementary cross sections the bremsstrahlung terms read as

$$d\sigma(1\gamma) = \delta^{QED}(1\gamma)\sum_{i=1}^{6} d\sigma_0(i)+\delta^{int}(1\gamma)\sum_{i=7}^{9} d\sigma_0(i)+\delta^{RES}(1\gamma)d\sigma_0(10), \tag{18}$$

with

$$\delta^{QED}(1\gamma) = (2\beta_e+2\beta_{int})\ln(2E/\lambda)+(2\beta_e+2\beta_{int})\ln\Delta - - (2\alpha/\pi)B(m^2)+(2\alpha/\pi)F(a,b), \tag{19a}$$

$$\delta^{int}(1\gamma) = (2\beta_e+2\beta_{int})\ln(2E/\lambda)+(\beta_{int}+\beta_e)\ln\Delta + \mathrm{Re}(\{\exp\left[i\delta_R(s)\right]/\cos\delta_R(s)\}\cdot \cdot(\beta_e+\beta_{int})\ln\left\{\Delta\left[1+(\Delta s/M\Gamma)\exp\left[i\delta_R(s)\right]\sin\delta_R(s)\right]^{-1}\right\}) - - (2\alpha/\pi)B(m^2)+(2\alpha/\pi)F(a,b), \tag{19b}$$

$$\delta^{RES}(1\gamma) = (2\beta_e+2\beta_{int})\ln(2E/\lambda)+\beta_e\ln\Delta - \beta_e\delta(s,\Delta\omega)\cot\delta_R(s) + + (\beta_e+2\beta_{int})\ln\left|\Delta\left\{1+(\Delta s/M\Gamma)\exp\left[i\delta_R(s)\right]\sin\delta_R(s)\right\}^{-1}\right| - - (2\alpha/\pi)B(m^2)+(2\alpha/\pi)F(a,b), \tag{19c}$$

and

$$B(m^2) = \frac{1}{3}\pi^2 - \ln(s/m^2) + \frac{1}{2}\ln^2(s/m^2), \quad F(a,b) = 2\ln^2 a + sp(b^2) - 2\ln^2 b - sp(a^2),$$

$$\delta(s,\Delta\omega) = \mathrm{arctg}\,\emptyset_a + \mathrm{arctg}\,\emptyset_b, \quad \emptyset_a = (2\Delta\omega\sqrt{s} + M^2 - s)/M\Gamma, \quad \emptyset_b = (s - M^2)/M\Gamma. \tag{20}$$

In experiments where the electrons are observed as a single particle track, then one obtains the final corrected cross section by simply adding the virtual and real corrections from eqs.(1) and (18) respectively, exponentiating the soft part as usual(1,11). When however collinear hard radiation ($k \approx \Delta\omega$) from the final particles is also detected, as in calorimetric-type experiments, one has to include further corrections, as explicitly indicated in ref.(8). We will first consider the former case. Then, as in ref.(6) we obtain(*)

$$d\sigma_{tot}(e^+e^- \to e^+e^-) = \sum_{i=1}^{10} d\sigma_0(i)(C^{(i)}_{infra} + C^{(i)}_F), \tag{21}$$

where(1)

$$C^{(i)}_{infra} = (\Delta)^{(2\beta_e + 2\beta_{int})} \qquad (i=1,\ldots,6), \tag{22a}$$

$$C^{(i)}_{infra} = (\Delta)^{(\beta_e + \beta_{int})}\left[1/\cos\delta_R(s)\right] \mathrm{Re}(\exp\left[i\delta_R(s)\right]\left\{\Delta\{1+(\Delta s/M\Gamma)\right. \cdot$$

$$\cdot \exp\left[i\delta_R(s)\right]\sin\delta_R(s)\}^{-1}\}^{\beta_e}\left\{\Delta\{\Delta+(M\Gamma/s)\exp\left[-i\delta_R(s)\right]/\sin\delta_R(s)\}^{-1}\right\}^{\beta_{int}})$$

$$(i = 7,8,9), \tag{22b}$$

$$C^{(10)}_{infra} = \Delta^{\beta_e}\left|\Delta\{1+(\Delta s/M\Gamma)\exp\left[i\delta_R(s)\right]\sin\delta_R(s)\}^{-1}\right|^{\beta_e}\left|\Delta\{\Delta+(M\Gamma/s)\right. \cdot$$

$$\left.\cdot \exp\left[-i\delta_R(s)\right]/\sin\delta_R(s)\}^{-1}\right|^{2\beta_{int}}\left[1 - \beta_e\,\delta(s,\Delta\omega)\cot\delta_R(s)\right], \tag{22c}$$

and the finite factors $C^{(i)}_F$(**) also include now the contributions of order $(\alpha/\pi)$ previously neglected

$$C^{(1)}_F = \frac{3}{2}\beta_e + (2\alpha/\pi)(\frac{1}{3}\pi^2 - \frac{1}{2}) + (2\alpha/\pi)F(a,b) + 2\delta^R_\pi(s) + (\alpha/\pi)\{V^\gamma_{1f}(s)) +$$

$$+\left[2z/(1+z^2)\right]A^\gamma_1(s)\}, \tag{23a}$$

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(*) Notice the rearrangement of the factors with respect to the analogous equation of ref.(6).

(**) The vacuum polarization corrections due to $\delta^R_\pi(s)$ can be resummed by introducing(4) the running coupling constant $e^2(s) = e^2/\left[1-\delta^R_\pi(s)\right]$ in $M^{QED}_0(s)$.

$$C_F^{(2)} = \frac{3}{2}\beta_e+(2\alpha/\pi)(\frac{1}{12}\pi^2-\frac{1}{2})+(2\alpha/\pi)(\frac{3}{2}\ln a-\ln^2 a)+(2\alpha/\pi)F(a,b)+ \\ +\delta_\pi^R(s)+\delta_\pi(t)+(\alpha/2\pi)\left[V_{1f}^\gamma(s)+A_1^\gamma(s)+V_{1f}^\gamma(t)+A_1^\gamma(t)\right], \tag{23b}$$

$$C_F^{(3)} = \frac{3}{2}\beta_e-(2\alpha/\pi)(\frac{1}{6}\pi^2+\frac{1}{2})+(4\alpha/\pi)(\frac{3}{2}\ln a-\ln^2 a)+(2\alpha/\pi)F(a,b)+2\delta_\pi(t)+ \\ +(\alpha/\pi)\left\{V_{1f}^\gamma(t)+\left[(b^4-1)/(b^4+1)\right]A_1^\gamma(t)\right\}, \tag{23c}$$

$$C_F^{(4)} = \frac{3}{2}\beta_e+(2\alpha/\pi)(\frac{1}{12}\pi^2-\frac{1}{2})+(2\alpha/\pi)(\frac{3}{2}\ln a-\ln^2 a)+(2\alpha/\pi)F(a,b)+\delta_\pi^R(s)+ \\ +(\alpha/2\pi)\left[V_{1f}^\gamma(s)+A_1^\gamma(s)+V_{1f}^Z(t)+A_1^Z(t)\right], \tag{23d}$$

$$C_F^{(5)} = \frac{3}{2}\beta_e-(2\alpha/\pi)(\frac{1}{6}\pi^2+\frac{1}{2})+(4\alpha/\pi)(\frac{3}{2}\ln a-\ln^2 a)+(2\alpha/\pi)F(a,b)+\delta_\pi(t)+(\alpha/2\pi)\left[V_{1f}^\gamma(t)+ \right. \\ \left. +V_{1f}^Z(t)\right]+(\alpha/2\pi)\left\{\left[(f_V^2+f_A^2)b^4-(f_V^2-f_A^2)\right]/\left[(f_V^2+f_A^2)b^4+(f_V^2-f_A^2)\right]\right\}\left[A_1^\gamma(t)+A_1^Z(t)\right], \tag{23e}$$

$$C_F^{(6)} = \frac{3}{2}\beta_e-(2\alpha/\pi)(\frac{1}{6}\pi^2+\frac{1}{2})+(4\alpha/\pi)(\frac{3}{2}\ln a-\ln^2 a)+(2\alpha/\pi)F(a,b)+(\alpha/\pi)V_{1f}^Z(t)+ \\ +(\alpha/\pi)A_1^Z(t)\left\{b^4\left[(f_V^2+f_A^2)^2+4f_V^2f_A^2\right]-(f_V^2-f_A^2)^2\right\}/\left\{b^4\left[(f_V^2+f_A^2)^2+4f_V^2f_A^2\right]+(f_V^2-f_A^2)^2\right\}, \tag{23f}$$

$$C_F^{(7)} = \frac{3}{2}\beta_e+(2\alpha/\pi)(\frac{1}{3}\pi^2-\frac{1}{2})+(2\alpha/\pi)F(a,b)+\delta_\pi^R(s)+\left[I'(s)/R'(s)\right]\delta_\pi^I(s)+(\alpha/2\pi)\left\{V_{1f}^\gamma(s)+ \right. \\ \left. +V_{1f}^Z(s)+2\pi\left[(I'(s)/R'(s)\right]\left[V_2^\gamma(s)-V_2^Z(s)\right]\right\}+(\alpha/2\pi)\left\{\left[f_A^2(1+z^2)+ \right.\right. \\ \left.\left. +f_V^2 2z\right]/\left[f_V^2(1+z^2)+f_A^2 2z\right]\right\}\left\{A_1^\gamma(s)+A_1^Z(s)+2\pi\left[I'(s)/R'(s)\right]\left[A_2^\gamma(s)-A_2^Z(s)\right]\right\}, \tag{23g}$$

$$C_F^{(8)} = \frac{3}{2}\beta_e+(2\alpha/\pi)(\frac{1}{12}\pi^2-\frac{1}{2})+(2\alpha/\pi)(\frac{3}{2}\ln a-\ln^2 a)+(2\alpha/\pi)F(a,b)+\delta_\pi(t)+ \\ +(\alpha/2\pi)\left[V_{1f}^\gamma(t)+V_{1f}^Z(s)+A_1^\gamma(t)+A_1^Z(s)\right]+\alpha\left[I'(s)/R'(s)\right]\left[V_{2f}^\gamma(t)-V_2^Z(s)+A_2^\gamma(t)-A_2^Z(s)+\frac{3}{2}\right], \tag{23h}$$

$$C_F^{(9)} = \frac{3}{2}\beta_e+(2\alpha/\pi)(\frac{1}{12}\pi^2-\frac{1}{2})+(2\alpha/\pi)(\frac{3}{2}\ln a-\ln^2 a)+(2\alpha/\pi)F(a,b)+ \\ +(\alpha/2\pi)\left[V_{1f}^Z(s)+V_{1f}^Z(t)+A_1^Z(s)+A_1^Z(t)\right]+\alpha\left[I'(s)/R'(s)\right]\left[V_{2f}^Z(t)- \right. \\ \left. -V_2^Z(s)+A_2^Z(t)-A_2^Z(s)+\frac{3}{2}\right], \tag{23i}$$

$$C_F^{(10)} = \frac{3}{2}\beta_e+(2\alpha/\pi)(\frac{1}{3}\pi^2-\frac{1}{2})+(2\alpha/\pi)F(a,b)+\frac{\alpha}{\pi}\left\{V_{1f}^Z(s)+\left[4f_V^2f_A^2(1+z^2)+ \right.\right. \\ \left.\left. +(f_V^2+f_A^2)^2 2z\right]/\left[(f_V^2+f_A^2)^2(1+z^2)+8f_V^2f_A^2 z\right]A_1^Z(s)\right\}. \tag{23j}$$

We consider now the case of calorimetric-type measurements, where collinear hard radiation ($k \gtrsim \Delta\omega$) from the final particles is detected within a small cone of half opening angle $\delta$ ($\delta \ll 1$). Then one has to add the following correction factor[8,12,13,14] to each term in the r.h.s. of eq.(21), taken to first order in $\alpha$:

$$\delta^{coll}(i) = d\sigma_0(i)\,\frac{4\alpha}{\pi}\left[(\ln\frac{E}{\Delta\omega} - \frac{3}{4})\ln(\frac{E\delta}{m}) - \frac{1}{2}\ln(\frac{E}{\Delta\omega}) + \frac{1}{2}(\frac{9}{4} - \frac{\pi^2}{3})\right]. \tag{24}$$

Then in agreement with the Kinoshita-Lee-Nauenberg theorem on the mass singularities[15], the m-dependence coming from the final electron-positron pair disappears after adding eq.(24) to eq.(21) and the overall correction factor to the Born cross-sections can be simply obtained from eq.(21), to first order in $\alpha$, by the substitution

$$\beta_e(\ln\Delta + \frac{3}{4}) \rightarrow \frac{2\alpha}{\pi}\left[\ln\frac{4}{\delta^2}\,(\ln\Delta + \frac{3}{4}) + (\frac{3}{2} - \frac{\pi^2}{3})\right]. \tag{25}$$

From the known results on the exponentiation of soft and collinear divergences[16], one then obtains the final result,

$$d\tilde{\sigma}_{tot}(e^+e^- \rightarrow e^+e^-) = \sum_{i=1}^{10} d\sigma_0(i)\left[\tilde{C}^{(i)}_{infra} + \tilde{C}^{(i)}_F\right] \tag{26}$$

where

$$\tilde{C}^{(i)}_{infra} = C^{(i)}_{infra}\cdot\Delta^{\beta_\delta - \beta_e}\ , \qquad \tilde{C}^{(i)}_F = C^{(i)}_F + \frac{3}{4}(\beta_\delta - \beta_e) + \frac{2\alpha}{\pi}(\frac{3}{2} - \frac{\pi^2}{3})$$

with

$$\beta_\delta = (4\alpha/\pi)\ln(2/\delta).$$

So far large-angle hard bremsstrahlung effects have not been considered. As long as the electron-positron pair is detected back-to-back with good collinearity, the accuracy of the formulae given above is of order $(\alpha/\pi)(\Delta,\delta)$. Hard photon effects have to be taken into account otherwise[17]. Finally, weak interactions have been only considered to renormalize the mass and the width of the vector boson.

To conclude we have presented a complete analysis of e.m. radiative corrections to Bhabha scattering near the $Z_0$. Our results, in fully analytic form, include the exact contributions of one-loop diagrams and the whole series of double and simple logarithms from soft and collinear divergences in exponentiated form. The process of $e^+e^-$ scattering, with the electron-positron pair detected almost back-to-back, can be therefore used as a high precision monitor of luminosity at LEP/SLC energies.

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