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A NEW DEFINITION OF THE DIFFRACTIVE LIMIT WITH APPLICATIONS TO pp AND $p\bar{p}$ SCATTERING

E. Etim, A. Malecki* and L. Satta
INFN - Laboratori Nazionali di Frascati, P.O.Box 13 - 00044 Frascati (Italy)

ABSTRACT

Multiple-dip structures in pp and $p\bar{p}$ elastic differential cross sections, predicted by many models, but unconfirmed experimentally may be due simply to an error. Glauber-type models, with composite Gaussian wave functions are in general not directly applicable in the limit in which the amplitudes factorise or partially factorise in their "hard" and "soft" components. A wrong combination of wave functions with eigenvalues may be involved. The description of diffraction scattering with Gaussian and related wave functions involves a completely different kind of limit in which the corresponding cross sections have only single-dip structures. The clarification of this point requires the introduction of the concept of equivalence of states with respect to an operator, which generalises that of degeneracy. It is required in order to be able to formulate precisely the idea of quasi-elastic scattering. Factorisation is still possible but must be used with more complicated wave functions defined over the unit circle.

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* On leave of absence from Institute of Nuclear Physics, Cracow, Poland.

Quasi-elastic scattering is a well known example of a situation where "hard" and "soft" processes are assumed to be factorised. Underlying the assumption is the idea of "closeness" of states, that is, the approximate degeneracy of certain states having dominant internal quantum numbers corresponding to those of the elastic channel. The property of factorisation of the scattering amplitude into its "hard" and "soft" components is well defined(1). The concept of "closeness" of states is not. There is in it, however, an implication that the wealth of the set of "close" states reflects on the compositeness of the particles involved in the collision. On the other hand "closeness" is related to the existence of soft processes which are responsible for the approximate degeneracy of the states. It is this connection with internal structure on the one hand and with factorisation on the other, which makes of the concept of "closeness" an important dynamical input. It would therefore be useful to have for it a serviceable formal definition and show how it characterises quasi-elastic and, in particular, diffraction scattering.

This is the purpose of this paper. The problem is an old one. In hadron-hadron scattering it is associated with the names of Good and Walker(2). There are many different but essentially equivalent formulations(3-8).

The mediation of these approximately degenerate states in elastic scattering does not exhaust inelastic shadowing. We shall, however, show that if there are sufficiently many such states and their unitarity contributions are not truncated but properly summed over, they do build up the elastic diffraction cross section well beyond the dip and in agreement with data for all CM energies. Comparison is made with experimental data on pp and $p\bar{p}$ elastic differential cross sections $d\sigma(s,t)/dt$(9-12), as functions of the invariant momentum transfer t and for CM energies $\sqrt{s}$ from 5-600 GeV.

Let us start by recalling the well known approximation

$$T = 2iD_0; \quad D^{\dagger}_0 = D_0 \tag{1}$$

for the scattering operator T, which describes high energy scattering in terms of purely imaginary amplitudes. The operator D_0 is said to be "diffractive" or purely absorptive. Unitarity implies that it is a projection operator

$$D_0^2 = D_0 \tag{2}$$

A given state $| i >$ may then be decomposed into its "diffractive" and "non-diffractive" components, $| d_i >$ and $| n_i >$, respectively, defined by(5,6)

$$| d_i > = D_o | i >$$
$$| n_i > = (1 - D_o) | i > \qquad (3)$$

They are eigenstates of T, that is

$$T | d_i > = 2i | d_i > = T | i > \qquad (4a)$$
$$T | n_i > = 0 \qquad (4b)$$

On the basis of these equations it seems natural to approximate[5,6] the state $| i >$ by its component $| d_i >$, in this so-called limit of diffraction scattering. The approximation is motivated by Eq. (4a) which says that as far as the scattering operator is concerned, the two states are essentially degenerate. This approximation and the idea behind it may be generalised and formulated as an equivalence relation in Hilbert space.

To see this consider, in place of Eq. (1), the unitary transformation

$$T = e^{iA} T_o e^{-iA} \qquad (5)$$

of the complete scattering operator T to the basis of physical states $| i >$ in which the operator T_o is diagonal, i. e.

$$T_o | i > = \eta_i | i > \quad ; \quad i = 1, 2, 3, ... \qquad (6)$$

Introduce the operator

$$D = \frac{1}{2} (1 - e^{iA}) \qquad (7)$$

It satisfies the unitarity equation

$$DD^+ = \frac{1}{2} (D + D^+) \qquad (8)$$

of which Eq. (2) is obviously a special case. The main idea in using (5) and (6) to describe quasi elastic scattering is that T_o gives the hard scattering part of the amplitude while the operator e^{iA} or equivalently D is responsible for the soft scattering corrections. This is arranged by means of an extension of the concept of degeneracy of the eigenstates of T_o which allows some of these eigenstates to be identified as equivalent or "close" even if they correspond to different eigenvalues. Here is the formal definition: two eigenstates $|\, i >$ and $|\, j >$ of T_o, degenerate or not, will be said to be asymptotically equivalent with respect to the operator D ($|\, i > \equiv |\, j >$ modulo D), if there exists a state $|\, \lambda >$ and non-zero complex functions $\varphi_{i\lambda}$ and $\varphi_{j\lambda}$ such that one has

$$\frac{D\,|\, i >}{\varphi_{i\lambda}} = \frac{D\,|\, j >}{\varphi_{j\lambda}} := |\, \lambda > \qquad (9)$$

This is an equivalence relation in Hilbert space. By iterating it, it is easy to establish that the state $|\, \lambda >$ is necessarily an eigenstate of D. Let λ be the corresponding eigenvalue, that is

$$D\,|\, \lambda > = \lambda\, |\, \lambda > \qquad (10)$$

If the eigenvalue λ is degenerate, one has

$$|\, \lambda > = \sum_{\alpha} \mu_{\alpha}\, |\, \lambda,\alpha > \qquad (11)$$

and hence, from eqs. (9)-(11)

$$|\, i > = \sum_{\alpha}\, \varphi_{i\lambda}(\alpha)\, |\, \lambda,\alpha > + \sum_{\beta} \varphi_{io}(\beta)\, |\, 0,\beta > \qquad (12a)$$

$$\varphi_{i\lambda}\;(\alpha) := < \lambda,\, \alpha\, |\, i > = \mu_{\alpha}\;\; \varphi_{i\lambda} \qquad (12b)$$

where the states $|\, 0,\, \beta >$ correspond to zero modes or states of "soft quanta". They satisfy

$$D\,|\, 0,\, \beta > = < 0,\, \beta\, |\, D^{+} = 0 \qquad (13)$$

Compared to the physical states $|\, i >$, the states $|\, \lambda,\, \alpha >$ with no soft quanta are "bare states". Physical states may thus be characterised by the quantum numbers of D and T_o even though D and T_o do not commute. The state $|\, i > \equiv |\, \lambda,\, \eta >$ so characterised satisfies the equations

$$T_o\, |\, \lambda,\, \eta > = \eta\;\; |\, \lambda,\, \eta > \qquad (14a)$$

$$|\, \lambda,\, \eta > = \sum_{\alpha}\, \varphi_{\eta\lambda}\,(\alpha)\, |\, \lambda,\alpha > + \sum_{\beta} \varphi_{\eta o}\,(\beta)\, |\, 0,\beta > \qquad (14b)$$

Eqs. (9), (12) and (13) generalise Eqs. (3) and (4) while the constraint

$$(D^2 - \lambda D) \mid \lambda, \eta > = 0 \tag{15}$$

generalises Eq. (2). It follows from Eqs. (14) that the zero mode state $\mid 0,\eta >$ is a physical state if

$$T_o \mid 0,\beta > = \sum_{\beta'} \varphi_{\beta o} (\beta') \mid 0,\beta' > \tag{16a}$$

$$\eta \, \varphi_{\eta o} \, (\beta) = \sum_{\beta'} \varphi_{\eta o} (\beta') \, \varphi_{\beta' o} (\beta) \tag{16b}$$

Eqs. (13) and (16a) then imply

$$[\, T_o, D \,] \mid 0, \beta > = 0 \tag{17}$$

At this point one observes that the concept of equivalence may be extended to operators[13]. Two operators A and B are said to be equivalent modulo D if their commutators with D are proportional to D. In formulae, $A \equiv B \bmod D$ if

$$\begin{aligned} [\, A, D \,] &= F_A \, D \\ [\, B, D \,] &= F_B \, D \end{aligned} \tag{18}$$

where F_A and F_B are functions (in general operators) depending on A and B respectively. The equivalence classes are algebras. In fact from the Jacobi identity, one has

$$\begin{aligned} &C := [A, B] \\ &[C, D] = F_c \, D \\ &F_c = [A, F_B] - [B, F_A] - [\, F_A, F_B] \end{aligned} \tag{19}$$

Operators which have the property in Eq. (18) form the set of symmetries of D[13]. An immediate application of this concept is that one obtains Eq. (16a) directly from (13), without the mediation of (14), if it is assumed that the scattering operator T_o is a symmetry of D. Eq. (5) then implies that $T \equiv T_o \bmod D$ since one has

$$[\, T, D \,] = [\, T_o, D \,] := FD \tag{20}$$

Taking matrix elements of (20) between the states $\mid \lambda, \alpha >$ one has

$$< \lambda', \alpha' \mid T_o \mid \lambda, \alpha > = \frac{\lambda}{\lambda - \lambda'} < \lambda', \alpha' \mid F \mid \lambda; \alpha > \tag{21}$$

Hence if F is a c-number or more generally if $< \lambda, \alpha' | F | \lambda, \alpha > \neq 0$ then the scattering operator has singular matrix elements between equivalent (i.e. degenerate) bare states. The divergence is compensated by the zero modes when one passes to the dressed physical states $| \lambda, \eta >$. In fact rewriting (5) as

$$T = T_0 + 2D\,T_0 + 2T_0\,D^+ + 4D\,T_0\,D^+ \tag{22}$$

and taking matrix elements of T between the states $| i > := | \lambda, \eta_i >$ and $| f > := | \lambda', \eta_f >$, and making use of (14), one finds

$$T_{fi} := < f | T | i > = \eta_i\,\delta_{fi} + \delta_{\lambda\lambda'}\,2\varphi^*_{f\lambda}\,\varphi_{i\lambda}\,[\,i(\Delta\eta_{i\lambda} - \Delta\eta_{f\lambda})\,\mathrm{Im}(\lambda) + (\Delta\eta_{i\lambda} + \Delta\eta_{f\lambda})\,\mathrm{Re}(\lambda)] \tag{23}$$

where

$$\Delta\eta_{i\lambda} := \eta_\lambda - \eta_{i\lambda} \tag{24a}$$

$$\eta_\lambda := \sum_k \eta_k\,|\varphi_{k\lambda}|^2 = < \lambda | T_0 | \lambda > = < \lambda | T | \lambda > \tag{24b}$$

We have also made use of Eq. (8) in the form

$$| \lambda |^2 = \mathrm{Re}\,(\lambda) \tag{25}$$

in terms of the eigenvalues of D.

In the elastic limit Eq. (23) becomes

$$T_{ii} = \eta_{i\lambda} + | 2\lambda\,\varphi_{i\lambda} |^2\,\Delta\eta_{i\lambda} \tag{26}$$

The eigenvalue λ functions therefore as a coupling constant. In the "weak coupling" limit

$$| \lambda |^2 \to 0, \qquad \mathrm{Re}\,(\lambda) << | \mathrm{Im}\,(\lambda) | \tag{27}$$

Eq. (23) is in a form in which "hard" and "soft" processes are completely factorised, that is

$$T_{fi} \xrightarrow[| \lambda |^2 \to 0]{} \eta_i\,\delta_{fi} + i\,\varphi^*_{f\lambda}\,\varphi_{i\lambda}\,\mathrm{Im}\,(\lambda)\,(\eta_f - \eta_i) \tag{28}$$

This approximation corresponds to keeping only the single commutator between T_0 and A in the expansion of T in Eq. (5) in terms of multiple commutators.

We shall argue that it is the uncritical use of this approximation, in one form or the other, which gives rise to the multiple-dip structures predicted by many models in the elastic differential cross sections $d\sigma_{pp}(s,t)/dt$ and $d\sigma_{p\bar{p}}(s,t)/dt$. Only one dip is observed experimentally.

It is reproduced at the correct place by a proper use of Eq. (26). To illustrate this we compare Eq. (26) with pp and $p\bar{p}$ data(9-12) in Fig. 1 for three CM energies, $\sqrt{s} = 4.5$,

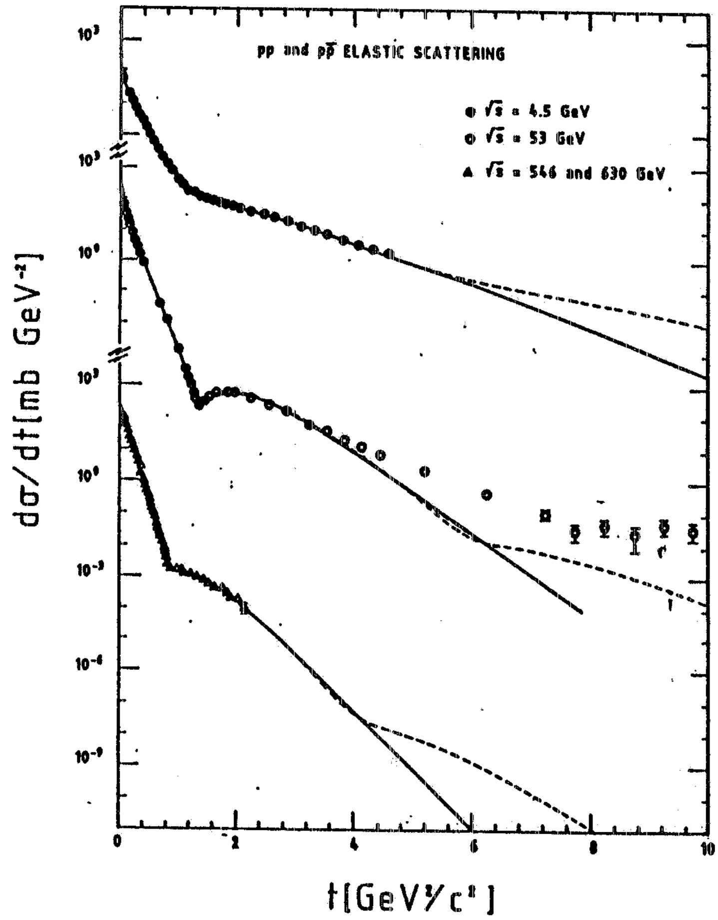

FIG. 1 - Plots of the pp and $p\bar{p}$ elastic differential cross sections $d\sigma(s,t)/dt$ against the momentum transfer t, for three CM energies $\sqrt{s} = 4.5$, 53 and 456-630 GeV. Full circles (•) are pp, 4.5 GeV data(9,10); open circles (o) are pp 53 GeV data(10) and triangles are $p\bar{p}$ data at 546 and 630(12) GeV. Dashed curves are theoretical predictions for small values of the coupling parameter in Eq.(26) of the text. Full curves are the predictions for large values of λ. Ratio of real to imaginary parts of the amplitudes are taken into account by means of the replacements $\sigma_{pp}(s) \to \sigma_{pp}(s)\,(1+i\,\alpha_p)$, $\sigma_{pq}(s) \to \sigma_{pq}(s)\,(1+i\,\alpha_q)$. The parameters α_p, α_q were not least squares fitted because values different from zero were found to give satisfactory fits to the data. The values used are shown in Table I.

53 and 546 - 630 GeV. We take for the state $|\,i > \equiv |\,\lambda, \eta >$, equivalent to the proton, the state $|\,p, n\,q >$ with one proton and n quanta (e.g. pions, quarks, etc.) (n = 1,2,3 ...). For the wave functions of these states we take, in the impact parameter representation(8),

$$|\,\varphi_{i\lambda}\,|^2 := |\,\varphi_n\,(b_1,\ldots b_n)\,|^2 = P_n \prod_{i=1}^{n} |\,\psi\,(b_i)|^2 \qquad (29a)$$

$$\psi(b) = \frac{e^{-b^2/4 R_o}}{\sqrt{(2\pi R_o)}} \tag{29b}$$

$$P_n = e^{-\bar{n}} \frac{(\bar{n})^n}{n!} \tag{29c}$$

For simplicity, we assume that the n quanta are Poisson-distributed. R_o is a free parameter. For the elastic amplitudes $\eta_i := \eta_{pn}(s,b,b_1,\ldots b_n)$, we take, following Glauber[14]

$$\eta_{pn}(s,b,b_1,\ldots b_n) = 1 - (1-\eta_{pp}(s,b)) \prod_{i=1}^{n} (1-\eta_{pq}(s,b+b_i)) \tag{30a}$$

$$\eta_{pp}(s,b) = \frac{\sigma_{pp}(s)}{4\pi\, a_p(s)} e^{-b^2/2a_p(s)} \tag{30b}$$

$$\eta_{pq}(s,b) = \frac{\sigma_{pq}(s)}{4\pi\, a_q(s)} e^{-b^2/2a_q(s)} \tag{30c}$$

where $\sigma_{pp}(s)$ and $\sigma_{pq}(s)$ are the pp and pq total cross sections and $a_p(s)$, $a_q(s)$ the corresponding slopes. Substituting from Eqs. (29) and (30) into (26) and taking Fourier transforms, one finds, for the differential cross section

$$\frac{d\sigma(s,t)}{d|t|} = \pi\, |T(s,t)|^2 \tag{31a}$$

$$T(s,t) = i \int_0^\infty db\, b\, J_o(b\sqrt{t})\, T(s,b) \tag{31b}$$

$$T(s,b) = \eta_{pp}(s,b) + |2\lambda|^2 \frac{(1-\eta_{pp}(s,b))}{(1-e^{-\bar{n}})} (1 - e^{-h(s,b)}) \tag{31c}$$

where

$$h(s,b) = \frac{\bar{n}\sigma_{pq}(s)}{4\pi R(s)} e^{-b^2/2R(s)} \tag{32a}$$

$$R(s) = R_o + a_q(s) \tag{32b}$$

We shall neglect $e^{-\bar{n}}$ with respect to unity so that the free parameters in Eqs. (31) are $a_p(s)$, R(s), $|\lambda|^2$ and $\sigma_q(s) := \bar{n}\, \sigma_{pq}(\sigma).\sigma_{pp}(s)$ is essentially fixed by $a_p(s)$. The values of these parameters for the fits in Fig. (1) are shown in Table I. The dashed curves in Fig. (1)

TABLE I - Values of the parameters used in the fits in Fig. 1. Small λ values refer to the dashed curves and large λ values to the full curves. The parameters (α_p, α_q) are the ratios of the real to imaginary parts of the scattering amplitudes.

$\sqrt{s}$ (GeV)	$\lvert\lambda\rvert^2$	a_p (fm^2)	σ_p (mb)	R (fm^2)	σ_q (mb)	$\lvert\lambda\rvert^2\sigma_q$ (mb)	α_p	α_q
4.5	0.542	0.367	32.87	0.194	19.72	10.7	−0.5	−0.5
	615.26	0.305	32.20	0.178	1.1×10^{-2}	6.97	−0.38	0.56
53	0.824	0.484	40.71	0.304	4.31	3.55	0.018	0.72
	605.21	0.481	40.63	0.317	6.44×10^{-3}	3.89	0.085	−0.83
546	1.0	0.567	56.75	0.435	6.77	6.77	0.191	0.60
	225.49	0.575	55.99	0.461	3.66×10^{-2}	8.25	0.187	0.502

correspond to small values of λ and the full curves to large values of λ. The latter values of λ, as can be seen from the Table, are in conflict with the unitarity constraint in Eq. (25). The conflict is only apparent and will be presently explained. But first we point out that the dashed curves are characterized by multiple dips and that these are absent in the full curve, except for the single one observed experimentally. Moreover, note from Table I that the product $|\lambda|^2 \cdot \sigma_q(s)$ is approximately constant over the energy range $\sqrt{s}$=4.5-600 GeV. This is consistent with taking the Bjorken-type limit: $B_j(|\lambda|^2|\sigma_q(s)) := |\lambda|^2 \to \infty, \sigma_q(s) \to 0$ and $|\lambda|^2 \cdot \sigma_q(s)$ fixed, in Eq. (31c). Large values of λ seem thus very natural when the cross sections $\sigma_{pq}(s)$ tend to zero. Such vanishing cross sections may be expected asymptotically if q is point-like. But there is a further justification: intuitively diffraction should correspond to the situation in which $\Delta\eta_p := \langle\eta_{pn}\rangle - \eta_{pp} \to 0$ asymptotically, but with nevertheless a finite contribution to the cross section. This is possible in Eq. (23), if $|\lambda|^2 \to \infty$. We therefore characterise the diffractive limit of the amplitude T_{fi} in Eq. (23) as the Bjorken-type limit

$$B_j(|\lambda|^2|\,\Delta\eta)\cdot T_{fi}\,(|\lambda|^2, \Delta\eta_i, \Delta\eta_f) := T_{fi}^{(o)} \qquad (33)$$

This limit is not inconsistent with the unitarity constraint in Eq. (25). The eigenvalue in the latter equation is of the operator D with eigenfunctions defined over the unit circle. The large values of λ in the fits and in eq. (33) refer not to D but to the related unbounded Hermitian operator A with eigenvalues and eigenfunctions defined over the real line. The relationship between the two sets of eigenfunctions is given by the well known transformation[15]

$$z \rightarrow \bar{z} = -i\ (1+z)/(1-z) \qquad (34)$$

which maps the unit circle on to the real line. An explicit model for the operator A is given by the familiar soft radiation or coherent state models[6,7]. The Gaussian wave function used in the calculations[8] is not an eigenfunction of D but the ground state eigenfunction of A in such a model. The eigenfunctions of A are Hermite polynomials multipled by a Gaussian. The ground state of A does not necessarily correspond to a small value of λ.

Eigenfunctions of D are much more complicated and will be given elsewhere. Using them, the corresponding eigenvalues are indeed small and satisfy Eq. (25). Since the use of Gaussian wave functions in theoretical descriptions of diffraction scattering is, for reasons of simplicity, so common-place we conclude that wave function representations are erroneously being mixed when λ is forced to be small, in particular to satisfy Eq. (25). The predicted multiple-dip structures in the differential cross sections seem to be a consequence of this error. They disappear leaving only the experimentally observed dips when the correct combination of wave functions and eigenvalues is used.

By the same token the fits given by the full curves may be improved and extended to large values of t by assuming A to be given by the coupling of a classical current to a quantised field. We shall present the results of this model using both eigenfunctions of A and D in a forthcoming publication. Details of our approach will also be given then.

The approach of this paper is not entirely new. One arrives at it easily through the well known extension of the idea of symmetries of an operator[13] and the associated generalised concept of degeneracy or equivalence of states modulo the operator in question. A similar extension of the concept of degeneracy of states has been attempted by Carruthers[16]. Motivated by the problem of violations of KNO scaling[17], he introduced mixed states by means of Ansätze on the density of states. The resulting transformations in Hilbert space are similar to ours: $|\,i > \rightarrow |\,\lambda >$, given in Eq. (12). He operates directly on probabilities, not on probability amplitudes as we do, his interest being in gaining better understanding of the negative binomial distribution and such-like that can accommodate violations of KNO scaling. Our main idea is that the concept of equivalence of states is a mathematical realisation of quasi-elastic scattering.

The relationship of our approach to the geometrical model of Chou and Yang[18] is not very direct. There is a basic and similar statistical assumption common to both. The geometrical model is essentially a classical description using an averaged hadronic matter density.

Our definition of the diffractive limit is however new. It does not require the amplitude to be purely imaginary. It is consistent with a composite structure of hadrons in terms of some elementary constituents.

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