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Coherent damping of excitations in nuclear collisions from the energy-weighted inelastic sum rule

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We calculate the energy-weighted inclusive inelastic cross section for the α-α scattering at medium energies. The damping of nuclear excitation energies at large momentum transfers, due to coherent multiple collisions, is found.

The structure of the nuclear ground state may be studied either in the elastic or in the inclusive inelastic scattering off the nucleus. The cross section of the latter is measured in poor energy resolution experiments[1] or can be obtained by integrating inelastic spectra of the scattered beam particle over its energy loss.[2] However, when the angular and momentum distributions of the scattered particle are available, one may also consider the inclusive cross sections weighted with various powers of the nuclear excitation energy. Such cross sections can provide interesting information on the shape of the energy loss spectra. In this paper we study the energy weighted sum rule for α-α scattering, challenged both by the existing experimental data[2] and by their recent theoretical analyses.[3,4] Despite its simplicity, the α particle requires a careful treatment due to the center-of-mass correlations implied by the constraint of translational invariance.[5] These correlations, neglected in Ref. 3 and shown to be important in Ref. 4, could be particularly effective in the energy weighted inclusive cross section, as may be expected from the classical example of the dipole sum rule.[6]

Let $T(q)$ be an operator describing the nuclear transitions induced by the incident particle at a given momentum transfer q. The elastic and the inclusive inelastic cross sections are then

$$\frac{d\sigma_{\text{el}}}{d^2q}=|\langle 0|T(q)|0\rangle|^2\ , \tag{1}$$

$$\frac{d\sigma_{\text{inel}}}{d^2q}=\sum_{n\neq 0}|\langle n|T(q)|0\rangle|^2=\langle 0|T^\dagger(q)T(q)|0\rangle-\frac{d\sigma_{\text{el}}}{d^2q}\ , \tag{2}$$

where $|0\rangle$ denotes the nuclear ground state.

The energy weighted inelastic cross sections can also be expressed in terms of ground state expectation values. We have

$$\frac{d(E\sigma)_{\text{inel}}}{d^2q}=\sum_n (E_n-E_0)|\langle n|T(q)|0\rangle|^2 =\tfrac{1}{2}\langle 0|T^\dagger[H,T]+[T^\dagger,H]T|0\rangle\ , \tag{3}$$

$$\frac{d(E^2\sigma)_{\text{inel}}}{d^2q}=\sum_n (E_n-E_0)^2|\langle n|T(q)|0\rangle|^2 =\langle 0|[T^\dagger(q),H][H,T(q)]|0\rangle\ , \tag{4}$$

where H is the Hamiltonian of the nuclear target:

$$H=-\frac{1}{2m}\sum_{j=1}^{A}\nabla_j^2+V \quad\text{with}\quad \nabla_j=\frac{\partial}{\partial \mathbf{r}_j}-\frac{1}{A}\sum_{k=1}^{A}\frac{\partial}{\partial \mathbf{r}_k}\ , \tag{5}$$

m being the nucleon mass and V representing the nuclear potential energy. The presence of the total nuclear momentum in the kinetic part of H is required by the translational invariance.[5]

If the incident particle is a nucleon or a nucleus, both V and T contain the nucleon-nucleon interaction, which at high energies is, to a large extent, local, spin, and isospin independent. It is then a plausible approximation to assume[3]

$$[V,T]=[V,T^\dagger]=0\ . \tag{6}$$

On the contrary, for electron or photon absorption $T(q)$ depends on isospin, and the commutator (6) implies a correction due to charge exchange forces present in the nuclear ground state.[7]

Under the assumption (6) the commutators in Eqs. (3) and (4) can be readily calculated. Luckily, only a part of them contributes to the expectation value, since the ground state wave function is real. We have

$$\frac{d(E\sigma)_{\text{inel}}}{d^2q}=\frac{1}{2m}\left\langle 0\left|\sum_{j=1}^{A}(\nabla_jT^\dagger)(\nabla_jT)\right|0\right\rangle\ , \tag{7}$$

$$\frac{d(E^2\sigma)_{\text{inel}}}{d^2q}=\frac{1}{(2m)^2}\left\langle 0\left|\sum_{j,k=1}^{A}(\nabla_k^2T^\dagger)(\nabla_j^2T)\right|0\right\rangle\ . \tag{8}$$

We will use the Glauber form[8] of the transition operator, which accounts for multiple scattering on constituent nucleons:

$$T(q)=-\frac{i}{2\pi}\int d^2b\,\exp(i\mathbf{q}\cdot\mathbf{b})\times\left\{1-\prod_{j=1}^{A}[1-\gamma_j(\mathbf{b}-\mathbf{s}_{jA}+\mathbf{S}_A)]\right\}\ , \tag{9}$$

where γ_j are the profiles of the target nucleons which depend on the coordinates $\mathbf{s}_{jA}$ in the plane of impact parameters. Their reference to the center-of-mass coordinates $\mathbf{S}_A$ of the target assures the translational invariance. This c.m. correction is easily factorized and affects only the elastic cross section, being irrelevant for the expectation values containing bilinear forms of $T(q)$.[4]

The profile of each target nucleon, as seen by the incident nucleus, can be expressed through the profiles γ_{NN} of the elementary nucleon-nucleon interaction as follows:

$$\gamma(b)=\left\langle 0_B\left|1-\prod_{k=1}^{B}[1-\gamma_{\text{NN}}(\mathbf{b}-\mathbf{s}_{kB}+\mathbf{S}_B)]\right|0_B\right\rangle\ , \tag{10}$$

where $|0_B\rangle$ denotes the ground state of the beam nucleus,

$\mathbf{s}_{kB}-\mathbf{S}_B$ being the intrinsic coordinates of its nucleons. Equation (10) means that the incident nucleus is treated as quasirigid during the collisions, and its virtual excitations[9] are neglected.

We assume that the nuclear ground state can be described by means of the independent particle model:

$$|\langle r_1,\ldots,r_A|0\rangle|^2=\prod_{j=1}^{A}\rho_A(r_j)\ , \tag{11}$$

where ρ_A is the single particle density. Then for the energy-weighted inelastic cross section one obtains

$$\frac{d(E\sigma)_{\rm inel}}{d^2q}=\frac{A-1}{2m}\frac{1}{(2\pi)^2}\int d^2b_1\,d^2b_2\exp[i\mathbf{q}(\mathbf{b}_1-\mathbf{b}_2)][1-S(b_1)-S^*(b_2)+U(\mathbf{b}_1,\mathbf{b}_2)]^{A-2}$$
$$\times\{[1-S(b_1)-S(b_2)+U(\mathbf{b}_1,\mathbf{b}_2)]W_1(\mathbf{b}_1,\mathbf{b}_2)-W_2(\mathbf{b}_1,\mathbf{b}_2)\}\ , \tag{12}$$

where

$$S(b)=\int d^3r\,\rho_A(r)\gamma(\mathbf{b}-\mathbf{s})\ , \tag{13}$$

$$\mathrm{U}(\mathbf{b}_1,\mathbf{b}_2)=\int d^3r\,\rho_A(r)\gamma(\mathbf{b}_1-\mathbf{s})\gamma^*(\mathbf{b}_2-\mathbf{s})\ , \tag{14}$$

$$W_1(\mathbf{b}_1,\mathbf{b}_2)=\frac{\partial^2}{\partial\mathbf{b}_1\cdot\partial\mathbf{b}_2}U(\mathbf{b}_1,\mathbf{b}_2)\ , \tag{15}$$

$$W_2(\mathbf{b}_1,\mathbf{b}_2)=\frac{\partial}{\partial\mathbf{b}_1}[S(\mathbf{b}_1)-U(\mathbf{b}_1,\mathbf{b}_2)]\cdot\frac{\partial}{\partial\mathbf{b}_2}[S^*(\mathbf{b}_2)-U(\mathbf{b}_1,\mathbf{b}_2)]\ . \tag{16}$$

The term W_2, though independent of A, is entirely due to the translationally invariant form of the gradient (5).

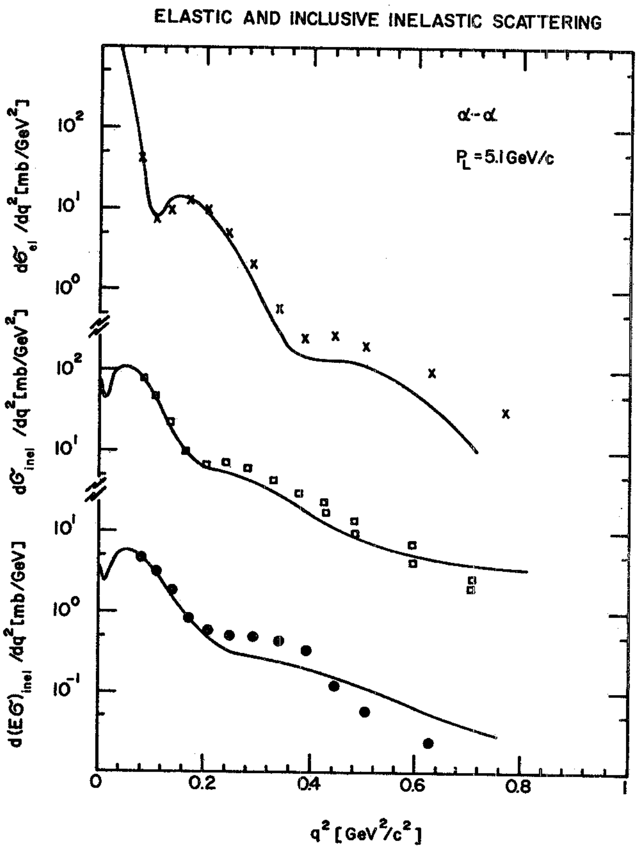

FIG. 1. The elastic, inclusive, and energy-weighted inclusive inelastic cross sections for the α-α scattering at $p_L=5.1$ GeV/c. The parameters used are (Ref. 12) $R=1.37$ fm, $\sigma=39.3$ mb, $\alpha=-0.4$, $a=3.4$ GeV^{-2}.

In our calculations we have used the Gaussian density for both the projectile and target nuclei:

$$\rho(r)=\pi^{-3/2}R^{-3}\exp\left[-\frac{r^2}{R^2}\right]\ . \tag{17}$$

The elementary nucleon profiles have been assumed spin and isospin independent as

$$\gamma_{\rm NN}(b)=[\sigma(1-i\alpha)/4\pi a]\exp(-b^2/2a)\ . \tag{18}$$

Thus our model contains four parameters, but none of them is free: R (nucleus radius), σ (total NN cross section), α (Re/Im ratio), and a (slope of the forward elastic amplitude). The Gaussian shape allows us to express analytically the functions S, U, W_1, and W_2.

Our calculations of the energy-weighted inelastic sum rule (12) for the α-α scattering together with the evaluated earlier[4] elastic (1) and inclusive inelastic (2) cross sections are compared with the experimental data[2,10,11] in Fig. 1. Without any adjustment of the parameters, we obtain a consistent description of all the data in a wide range of momentum transfer. The agreement with experiment successfully confirms the Glauber theory, which means that the collision of two nuclei at medium energies is dominated by multiple scattering of constituent nucleons.

We are aware of shortcomings of the calculations, especially regarding the approximation (11) of independent nu-

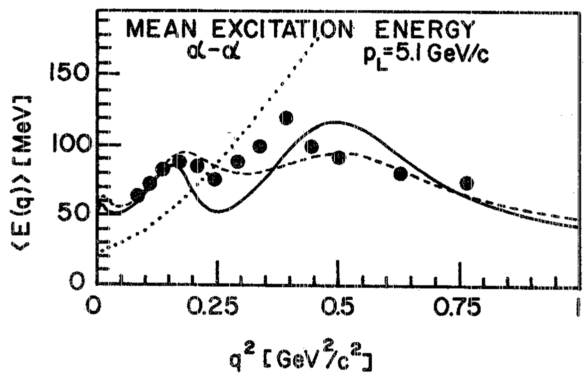

FIG. 2. The mean excitation energy at a given momentum transfer for the α-α scattering at $p_L=5.1$ GeV/c. For the dashed line only the first term in Eq. (10) is retained, while the solid line is the result of the complete calculation. In both cases, the full multiple scattering series of Eq. (9) is evaluated. The dotted line corresponds to the single scattering approximation, where only the first terms of the expansions (9) and (10) are considered. The same parameters as in Fig. 1 are used.

cleons. The inclusion of the nucleon-nucleon correlations might account for a correct magnitude of the inclusive inelastic cross section in the saddle region as well as of the elastic cross section at large momentum transfers.

A severe test of the theory is provided by the calculation of the mean excitation energy at a given momentum transfer q:

$$\langle E(q)\rangle = \frac{d(E\sigma)_{\rm inel}}{d^2q}\left[\frac{d\sigma_{\rm inel}}{d^2q}\right]^{-1} . \tag{19}$$

In Fig. 2 we have shown how $\langle E\rangle$ is affected by multiple collisions in the beam and in the target α particle. The latter are particularly important, since all four terms in Eq. (9) have to be retained. This is due to the strong c.m. correlations between nucleons.

Neglecting the multiple scattering both in the projectile and in the target, one obtains the mean excitation energy increasing quadratically with momentum transfer. It corresponds to an incoherent sum of elastic scatterings of the beam particle on quasifree nucleons of the target. This knock-out mechanism is usually assumed valid for deep-inelastic electron scattering from nuclei. However, from Fig. 2 we conclude that the quasielastic mechanism is far insufficient for explaining the intrinsic excitations in the collisions of strongly interacting particles, in particular, their damping and saturation at large momentum transfers. This points to a necessity of a coherent description of the target disintegration including multiple scattering of the projectile both in the initial and final states. The coherent damping of excitation (CODEX) has also been found in the p-^{4}He scattering; it occurs here at larger values $\langle E\rangle$ and q: $\langle E\rangle_{\rm max}=350$ MeV at $q=1.1$ GeV/c as compared to $\langle E\rangle_{\rm max}=120$ MeV at $q=0.7$ GeV/c for the α-^{4}He scattering. Therefore it will hardly be seen, being masked by the production processes.

The CODEX effect that we have found in the nucleus-nucleus collisions should also be observed in the subnuclear inelastic scattering. In the case of the proton-proton collisions this would virtually invalidate the quasielastic mechanism of Drell and Hiida,[13] where the incident proton scatters diffractively from only one constituent (the pion) of the proton target. In fact, their model turned out to be insufficient to account, at the same time, for the correct position and width of the inelastic bump.[14]

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