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UNDERSTANDING THE YANG MILLS GROUND STATE: THE ORIGIN OF COLOUR CONFINEMENT

Giuliano Preparata*
INFN - Laboratori Nazionali di Frascati, P.O.Box 13, 00044 Frascati (Italy)

ABSTRACT

The essential magnetic instability of the perturbative ground state of a non-abelian Yang Mills theory, recently discovered, is shown to lead to a family of degenerate states, the Savvidy states, where the Yang Mills fields undergo an infinite (when the ultraviolet cut-off $\Lambda \rightarrow \infty$) condensation process. These states build up the real Yang Mills ground state, in which colour is confined and governed by the effective lagrangian of Anisotropic Chromo-Dynamics (ACD), proposed by the present author a few years ago. This appears to solve the problem of confinement in QCD.

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* On leave of absence from Dipartimento di Fisica, Università di Bari, Bari (Italy).

Two are the notions that are universally believed to play a key role for deciphering the code of hadronic behaviour: colour confinement at long distances and freedom at short distances. A theory that could be proved to engender both phenomena would most likely be able to describe in depth and successfully the multiform and fascinating world of hadrons. Most physicists believe that Quantum Chromo Dynamics (QCD) is such a theory; but has this been proved? There is at present a large consense that, as far as short distances are concerned, Asymptotic Freedom (AF) and the calculational strategy based on Perturbation Theory (PT) are a faithful representation of QCD. The situation with confinement at large distances is much more problematic, many hopes being presently attached to the numerical studies of Lattice Gauge Theories.

It is the purpose of this paper to show that both problems can be attacked and solved directly, with results that deviate considerably from the theoretical expectations upon which the prevailing QCD strategies have so far been based.

The key element of this work is an observation made in 1977 by G.K. Savvidy[1] that there exist states of the gauge-fields, characterized by quantum fluctuations around a constant chromo-magnetic field, in a given space direction $\vec{u}$ and along a fixed direction α in group space(*), that for suitable values of the intensity B of the chromo-magnetic field are lower in energy density than the perturbative ground state (B=0). I propose to call states of this type Savvidy states. As shown later by Nielsen and Olesen[2] these states are affected by peculiar instabilities, due to the fact that the operator (S is the Yang Mills action)

$$O^{\alpha\beta}_{\mu\nu}(x,y) = -\frac{1}{2} \left. \frac{\delta S[A]}{\delta^{\alpha}_{\mu}(x)\, \delta\eta^{\beta}_{\nu}(y)} \right|_B \qquad (1)$$

(*) It is well known that such a field is a solution of the classical Yang Mills equations.

controlling the small fluctuations $\eta_\mu^a(x)$ of the gauge fields around the classical "background" field B, has a spectrum which is not positive definite. Indeed the operator (1) has been first diagonalized more than 50 years ago by Landau as the solution of the relativistic quantum mechanical problem of the motion of a charged particle in a constant magnetic field. As well known, the spectrum of (1) is described by two quantum numbers (p, the momentum along the direction of the magnetic field B; and n, an integer related to the excitation of a harmonic oscillator) and is given by:

$$E_n(p)^2 = p^2 + gB(2n+1) - 2gBS_3 \qquad (2)$$

where S_3 is the spin projection along the magnetic field and g is the charge. It is clear that for n=0, S_3=1 and $|p| \leq (gB)^{\frac{1}{2}}$, the eigenvalues (2) are negative, so that we are bound to find severe instabilities(*). We shall call these Landau states unstable modes or U-modes, while all other states shall be called S-modes.

In spite of a number of vigorous attempts[3] to understand the effect of such instabilities, this problem has remained unsolved until recently, when new variational techniques within the Schröndinger approach to the dynamics of gauge-fields were untilized[4,5]. The results of this analysis have been quite surprising.

When calculating the difference $\Delta E(B)$ between the energy density of a Savvidy state with chromomagnetic field B and the PT ground state it was found that the classical term $B^2/2$ is completely compensated[5] by the negative contribution of the U-modes, yielding the following result [from now on we shall always work in SU(2)]

(*) In fact for the modes for which (2) is negative the action S is a maximum rather than a minimum, and a situation of small fluctuations, as envisaged in PT, cannot hold.

$$\Delta E(B) = \frac{g^2 B^2}{4\pi^2} \left[\frac{17}{6} - \frac{11}{12} \ln\left(\frac{\Lambda^2}{gB}\right) \right] + O(g^4 B^2 \ln \Lambda^2/gB), \tag{3}$$

whose minimum occurs at (Λ is the ultraviolet cut-off)

$$gB^* = \Lambda^2 \exp - \frac{79}{22} , \tag{4}$$

with the value:

$$\Delta E(B^*) = - \frac{11}{96\pi^2} (gB^*)^2 = - \frac{11}{96\pi^2} \Lambda^4 \exp - \frac{79}{11} . \tag{5}$$

Eqs. (3), (4) and (5) do not need many comments. According to them there exist Savvidy states, i.e. well defined and physically realizable configurations of the gauge fields, characterized by a divergent ($\sim\Lambda^2$) "background" field (see Eq. (4)), and by a divergent ($\sim\Lambda^4$) difference of energy density with the PT ground state (see Eq. (5)). As a result the perturbative ground state - and PT -, contrary to the common belief, cannot resume their dynamical relevance even at very short distances, for according to (5) this may only happen at distances $O(1/\Lambda)$. I have referred to this state of affairs as the "essential instability" of the perturbative ground state(5).

In the Savvidy states one may also calculate the expectation value of the gauge covariant field strengths $F^{\alpha}_{\mu\nu}$. The only non vanishing components are (we take the background field along the z-direction)

$$\langle F^{\alpha}_{12} \rangle = - \eta^{\alpha} \bar{B} \tag{6}$$

$$\bar{B} \cong - \frac{11}{79} (8-\pi) \frac{g}{16\pi^2} (gB^*) , \tag{6'}$$

which according to (4) diverges like Λ^2. Eqs. (6) are also remarkable for the absence of the classical term B*, arising from the expectation value of the abelian part $\partial_\mu A^\alpha_\nu - \partial_\nu A^\alpha_\mu$ of $F^\alpha_{\mu\nu}$, which gets cancelled by the contribution of the U-modes to the non-abelian part $g\epsilon^{\alpha\beta\gamma} A^\beta_\mu A^\gamma_\nu$. This is just the parallel of Eq. (3).

Another interesting result is that the energy $\omega(p)$ of the U-modes satisfies the dispersion relation:

$$\omega(p) = \left[p^2 + 4gB^* \exp - \frac{8\pi^2}{g^2} \right]^{\frac{1}{2}} \tag{7}$$

typical of a non-interacting relativistic massive boson with mass

$$\mu^2 = 4gB^* \exp - \frac{8\pi^2}{g^2} , \tag{7'}$$

which may be rendered finite by a suitable relationship between the bare coupling constant $g^2(\Lambda)$ and the cut-off Λ(*).

As for the S-modes they all acquire divergent masses O(gB*), thus becoming dynamically inert. Indeed in a Savvidy state, as a result of the essential magnetic instability of the perturbative ground state, all S-modes undergo an infinite (when $\Lambda \to \infty$) condensation phenomenon, that "freezes" them in a highly correlated magnetic state (see Eqs. (6)). In this way the S-modes drop out for ever of the dynamics of the Yang Mills fields, leaving behind a medium with very extreme magnetic properties.

(*) As is well known relationships of this kind are at the basis of the renormalization of renormalizable field theories.

A straightforward analysis(*) of the magnetic properties of the S-modes' condensate (Eqs. (6)) shows that it behaves like an infinite magnetic superconductor, with its vortex lines in the direction of the magnetic field. In such a medium external colour charges give rise to the field configurations reported in Fig. 1, typical of a confinement situation. Also the effective

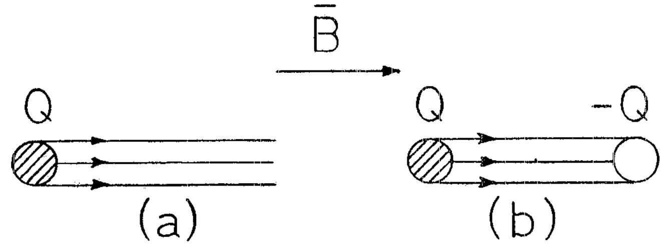

FIG. 1 - The lines of force of the electric displacement vector $\bar{D}$, for an isolated charge Q (a), and for a dipole (b).

Lagrangian describing the dynamics of external colour currents in the Savvidy state has precisely the structure postulated by Anisotropic Chromo Dynamics (ACD) that I proposed in 1980[7] to incorporate colour confinement in a Lagrangian field theory(**).

However the individual Savvidy state, in spite of its confining properties, is a rather unlikely candidate for the ground state of the Yang Mills theory, due to its violating both rotational and colour invariance. On the other hand it is quite clear that the whole family of Savvidy states must have something to do with the ground state. Indeed it appears extremely improbable that to lower

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(*) This analysis will be carried out in full in a future publication[6].

(**) In Ref. (7) it was just suggested that ACD was the effective chromodynamical theory in a state that has exactly the properties that have been found for the Savvidy states. Consultation of Refs. (7) and (8) might be helpful to the reader unfamiliar with the dynamical situation envisaged in this paper.

the energy of the gauge field system from its perturbative value there exist mechanisms more effective than the "gluon-condensation" found in the Savvidy states.

Let us therefore consider the infinity of degenerate Savvidy states of minimum energy (5), characterized by the orientation $\vec{u}$ and the isospace-direction η^{α} of the magnetic field $\bar{B}$ (see Eqs. (6')), and denote the generic element of the family $|\vec{u},\alpha\rangle$. These states have the peculiar property of being <u>essentially orthogonal,</u> i.e. their scalar product, even restricted to small but finite spatial domains, vanishes as a consequence of the divergence of $\bar{B}$(*).

The <u>essential orthogonality</u> of the different Savvidy states, which prevents any local mixing among different states, allows us to view each state as the ground state of an <u>independent quantum mechanical system,</u> and to write the real ground state $|\Omega\rangle$ as

$$|\Omega\rangle = \prod_{\vec{u},\,\alpha} |\,\vec{u},\alpha\,\rangle, \tag{8}$$

which is now both Lorentz and colour invariant. On this ground state the dynamics is colour-confining (for colour is confined, as we have just remarked, on each state $|\vec{u},\alpha\rangle$), and its effective Lagrangian coincides with the ACD - Lagrangian.

It should be stressed that the dynamics of a Yang Mills theory is not exhausted by that of ADC. The U-modes on each Savvidy state, that are absent in ACD, form in fact the basis of dynamical, confined and massive (see Eqs. (7)) gluon-fields which are basically abelian, lacking any self-coupling.

(*) This situation is quite different from the orthogonality in other well known analogous systems, such as the Heisenberg ferromagnet, where it is the simple cosequence of the infinite volume limit.

Leaving to a future publication(6) a full account and derivation of many of the results reported above, I wish to conclude this letter by summarizing what has been achieved.

The essential instability of the PT ground state found in Ref. (5), that exposes the irrelevance of PT for describing the short distance dynamics of Yang Mills fields, strongly indicates at the same time the importance of Savvidy states where an infinite "gluon condensation" phenomenon occurs. These states, in which colour is confined, have been shown to be the ground states of dynamically independent quantum mechanical systems, whose product builds up the real ground state. In the real ground state colour confinement continues to hold, and its dynamics is described by the ACD Lagrangian. Finally "gluons" acquire a mass, get confined, and become effectively abelian.

We need now to follow through the ACD developments of the last five years(8), and recall a number of the most important experimental facts about hadrons, to perhaps realize that a non-negligible step has been made towards understanding the basic aspects of hadronic behaviour.

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