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FOURIER ANALYSIS OF COSMIC RAYS ARRIVAL TIMES

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1. INTRODUCTION

The Nusex and Soudan underground detectors, have reported (ref. 1,2) the distribution of arrival times of muons pointing to the Cygnus X3 direction, displaying a characteristic 4.8 hours modulation which coincides with the Binary period of the X-ray emission.

This effect would indicate, either the existence of a new neutral and stable particle, or anomalous interactions of photons or neutrinos, producing showers with a muon content equal to that of hadronic cascades.

Because the number of events coming from CygX3 direction is not significantly different from that found in any off source region, the evidence for the signal arises from the phase plot structure.

For this reason it is very important to understand if collected 'off source' data are really randomly distributed in time.

To attribute a confidence level to the detected signal we have to make some well established hypothesis on the background temporal structure, from which depends critically the statistical significance attributed to the signal.

We describe a method to identify time correlations in samples of cosmic ray muons, which allows, in case of negative result, to prove the random temporal distribution of the analyzed data.

2. ANALYSIS METHOD

The analysis is performed on the squared modulus of the Fourier transform of the pulse temporal sequence  $x(t) = \sum_{n=1}^N \delta(t-t_n)$ , e.g.

$$F(\omega) = [ (\sum_k N_{e1} \cos \omega t_k)^2 + (\sum_k N_{e1} \sin \omega t_k)^2 ] / N \quad (\text{ref. 3,4,5}),$$

that is an estimator of the signal distribution among the various frequencies

The frequency resolution depends on the total observation time  $\Delta\omega = 2\pi / (T_N - T_1)$ , and the number of structures that can be identified between  $\omega_1$  and  $\omega_2$  is  $(\omega_2 - \omega_1) / \Delta\omega$  (ref.3,4)

If arrival times of the events are uniformly distributed, the  $F(\omega)$  values are exponentially distributed, and  $P(F(\omega) \geq F(\omega_0)) = e^{-F(\omega)}$  (ref.3,5).

Using the knowledge of the distribution function, a confidence level can be assigned to the hypothesis that the muon sample exhibits a time modulation

The probability for a uniform succession of events of  $F(\omega)$  being greater or equal to the calculated value, e.g.  $e^{-F(\omega)}$ , is the error on the assumption that the event sample has a periodicity  $P = 2\pi/\omega$

Spectra of  $F(\omega)$  have been examined in several frequency ranges, for uniform samples and for samples where periodic signals of different shapes have been added to various background levels. By these simulations the lowest value of the signal to background ratio to which the method is sensitive, has been determined

In fig.1 is shown the Fourier spectrum of a muon sample generated assuming a generalized Poisson distribution, with mean  $\lambda(t) = \lambda_0 + \lambda_1 \cos \omega t$  and essentially no background ( $\lambda_0 - \lambda_1 \approx 0$ ). This spectrum and that corresponding to higher harmonics have been compared to those produced by a periodic rectangular signal, lasting 10% of the whole period (always without background), which is shown in fig. 2

In the latter case the signal appears also in the higher harmonics, suggesting that the values of  $F(\omega)$  for frequencies multiple of the fundamental should be also considered

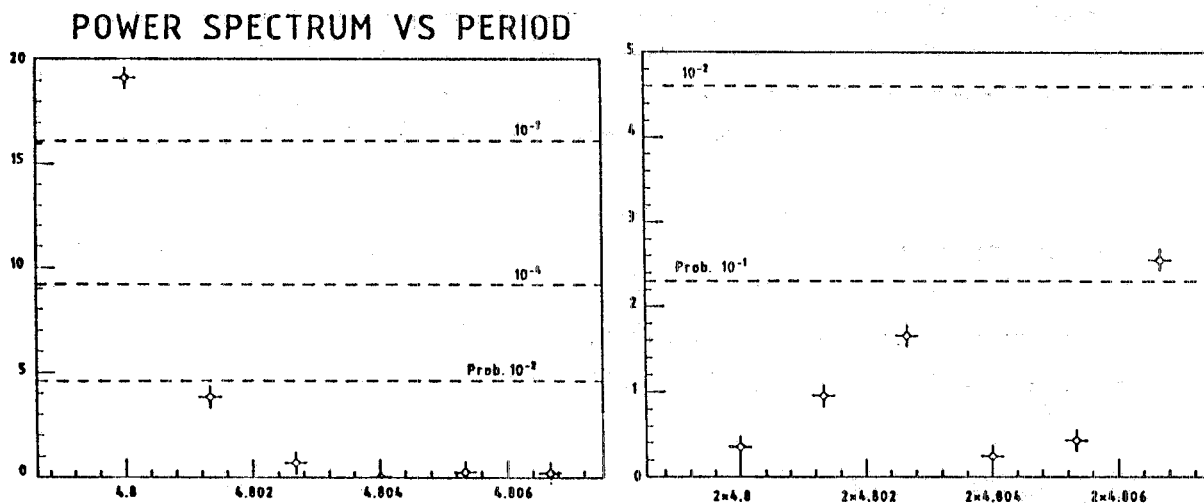


FIG. 1 - Power spectra of a generalized Poisson signal with sinusoidal shape, referring to fundamental and first harmonic. Contiguous points have been chosen so that  $\Delta\omega = 2\pi/T$ , where T is the total sampling time.

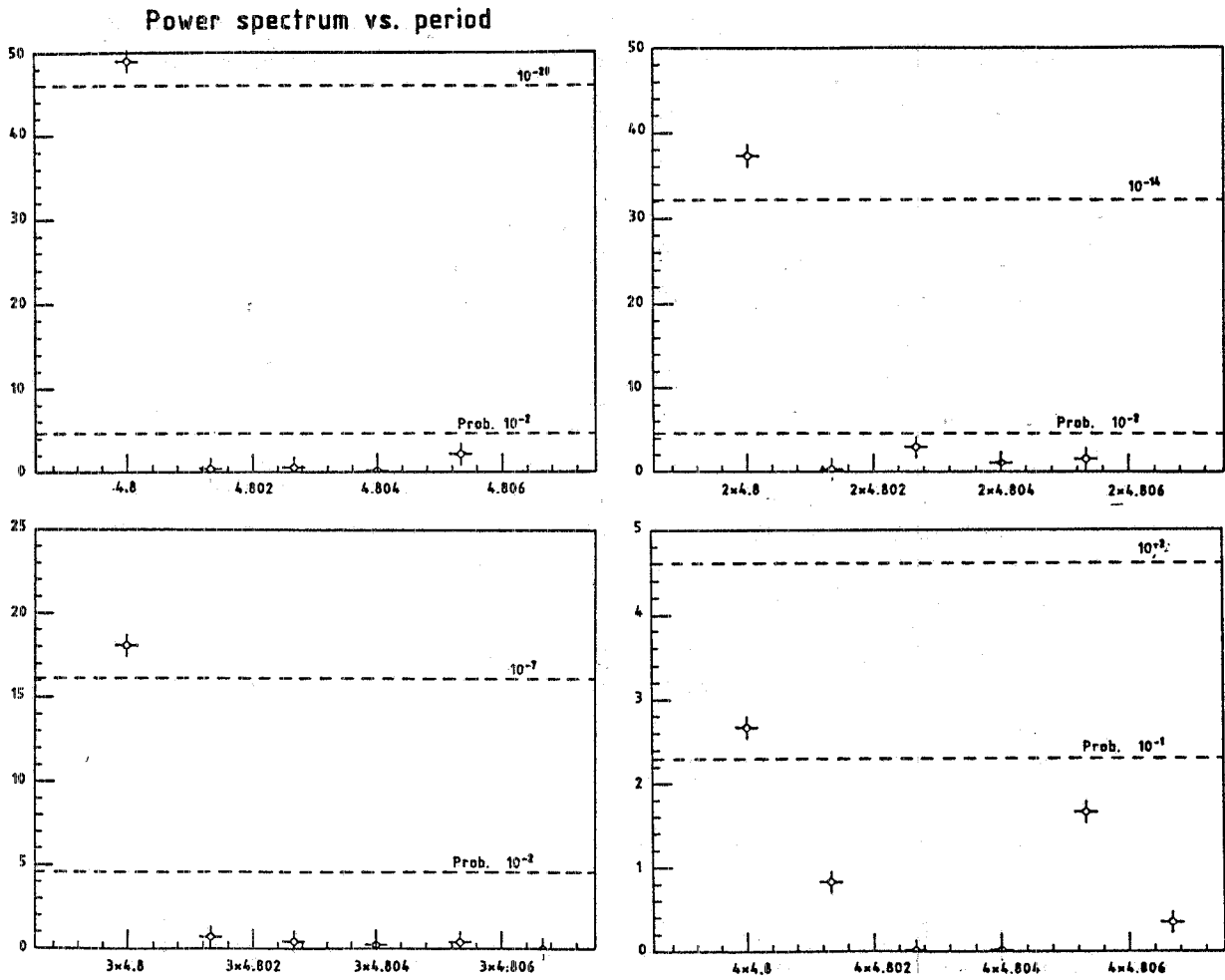


FIG. 2 - Power spectra of a generalized Poisson signal with rectangular periodic shape, referring to fundamental and its successive harmonic.

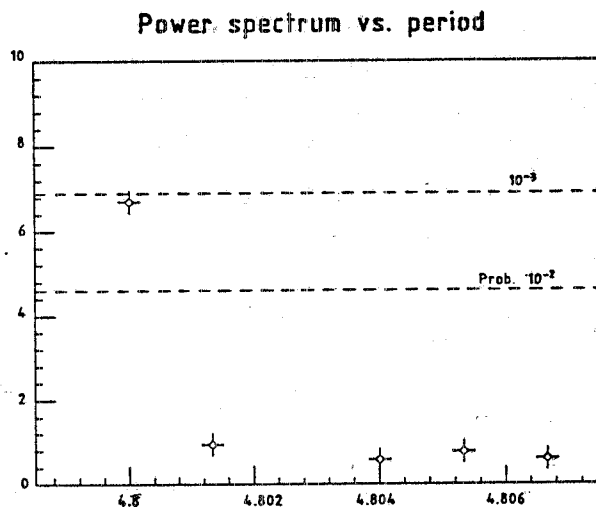


FIG. 3 - Power spectrum of a sinusoidal signal as in Fig.1, superimposed to a background uniformly distributed in time. The relative contribution to the sample, integrated over a period, is 1:1.

A method to take into account these further enhancements could be to sum up the  $F(\omega)$  values corresponding to several harmonics. This sum, performed over  $n$  frequencies, has a probability of exceeding the actual  $S_n$  value, given by

$$P(\geq S_n) = (e^{-S_n} S_n^{n-1}) / (n-1)! \quad \text{with } S_n = \sum_{k=1}^N F(k\omega).$$

The number of harmonics among which the signal is distributed, is higher the shorter signal is with respect to the period. If the actual width of the signal is unknown, we can consider all the harmonics whose amplitude is greater than, for instance, 80% of fundamental

Fig. 3 shows the spectrum of a sinusoidal signal superimposed to an uncorrelated background, in the ratio 1:1. The periodic component is identified with a confidence level of 99.9%

Figs. 4 and 5 show the spectra of a rectangular signal lasting 10% of the period embedded in a constant background 5 and 10 times the periodic component. In the first case 25 events, over 155 collected, come from the time modulated signal; in the second case they are 15, over 155.

By using the fundamental frequency and its first harmonic, the characteristic period is picked out with an error probability less than  $10^{-6}$  in the first case, and  $\leq 10^{-2}$  in the second

To understand the relevance of these numbers and the method efficiency, it is useful to consider the result of phase plot analysis

Fig. 6 shows the phase plot corresponding to the case of fig. 5, e.g. 15 signal events over 155

By phase plot analysis, once optimized the binning at 10 bins, we find an excess of 15 events in the first bin and obtain an error probability of  $3 \cdot 10^{-4}$  to the hypothesis of the signal existence. However in the case of phase plot analysis, we have to take into account the number of trials performed to obtain the binning optimization

It is worth mentioning that this analysis allows the elimination of known and uninteresting periodic components, which could blot out weak signals. A typical example is the appearance of an enhancement in  $F(\omega)$  at the sidereal day frequency, when events coming from a given region of the celestial sphere are selected. This enhancement can be cancelled by constructing a function  $O(t)$ , which is zero whenever the apparatus does not look at the selected region, and 1 during "data taking". Its Fourier transform  $O(\omega)$ , is introduced into  $F(\omega)$ , and the analyzed function becomes

$$F^1(\omega) = | (\sum_{k=1}^N \cos \omega t_k) + i (\sum_{k=1}^N \sin \omega t_k) - \lambda_0 O(\omega) |^2 / N,$$

with  $\lambda_0 = N/T_d$ , and  $T_d$  is the total data taking time (ref. 5)

Fig. 7 shows the differences between spectra, without and with  $O(\omega)$  subtraction

A typical distribution of  $F(\omega)$  values referring to the analysis of a muon sample of the Nussex data, with 1000 scanned frequencies, has been reported in fig. 8. The straight line in semilogarithmic plot, represents the expected distribution for a uniform sequence of arrival times

There is a good agreement between the actual data distribution and the expectation values, providing a prove of the temporal uniformity of the data sample

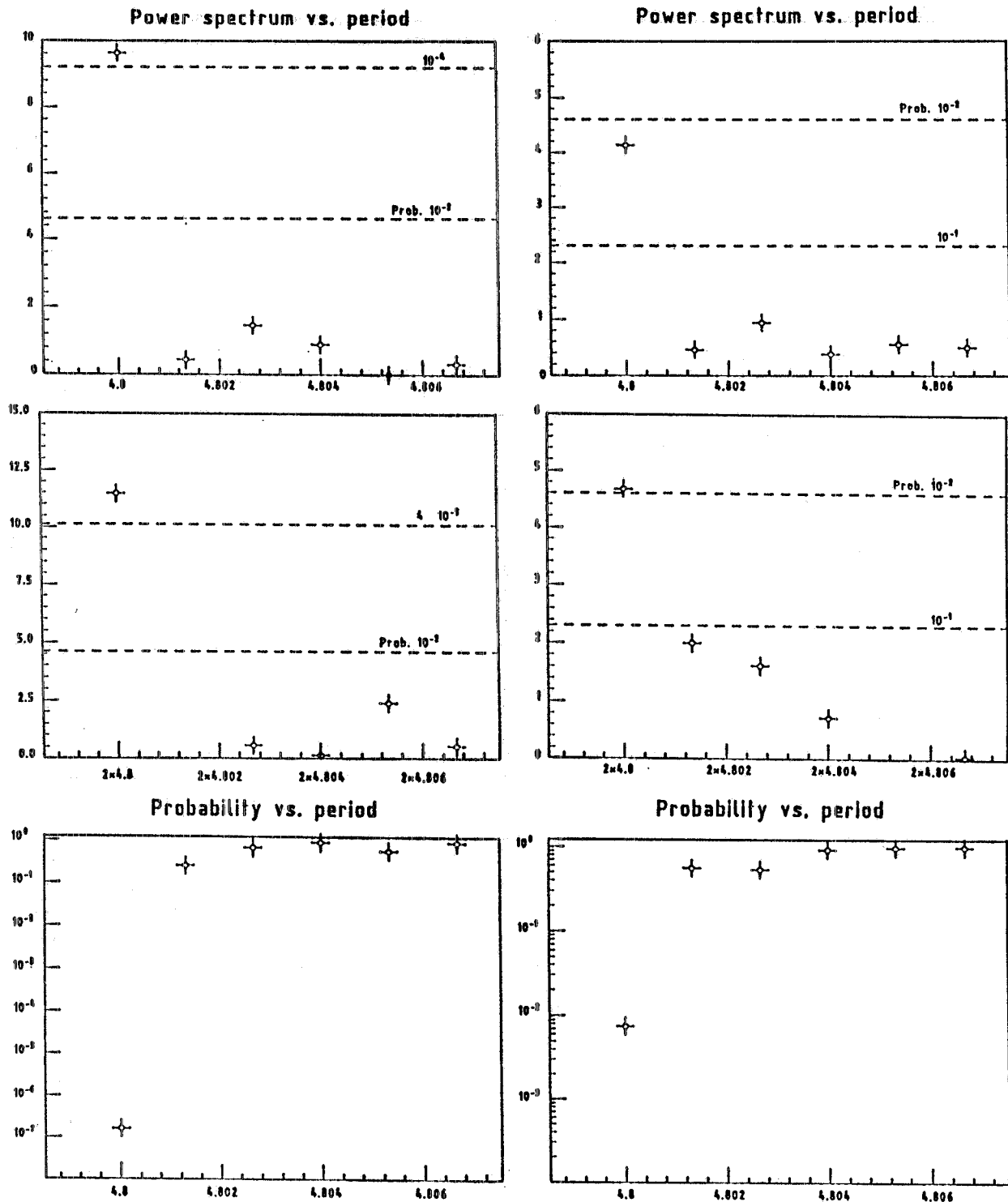


FIG. 4

FIG. 5

FIG. 4- Power spectrum of a rectangular periodic signal as in Fig.2, superimposed to a background uniformly distributed in time. Twentyfive events, over 155, come from the time modulated signal. It is also shown the spectrum referring to the first harmonic, together with the probability that the  $F(\omega)$  value belongs to the distribution of a time-uniform sample. This value has been computed considering the power of fundamental and first harmonic.

FIG. 5 - As in Fig.4, with a signal of 15 events, over 155.

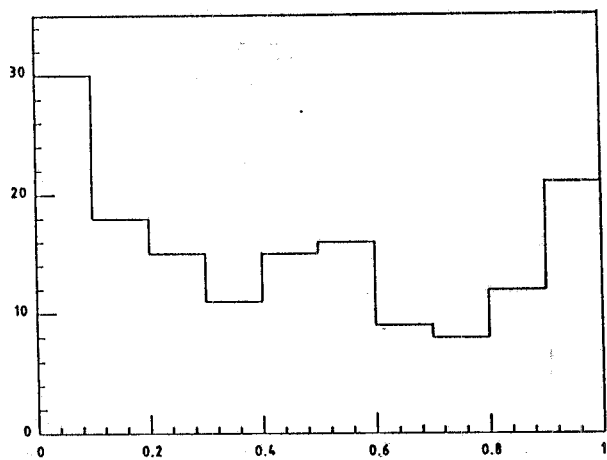


FIG. 6 - Phase plot of the sample-wise power spectrum has been shown in Fig.5.

Power spectrum vs. period

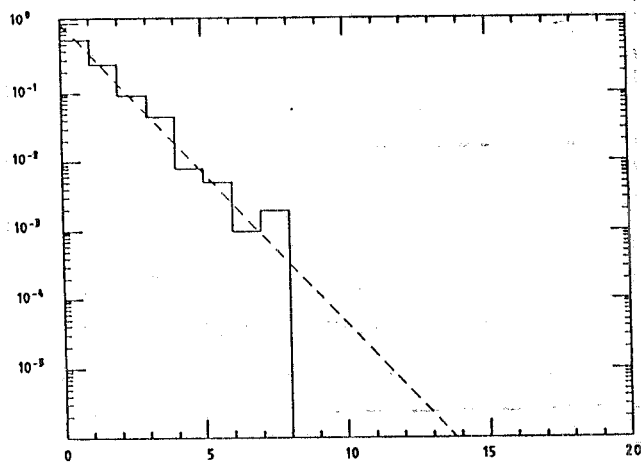
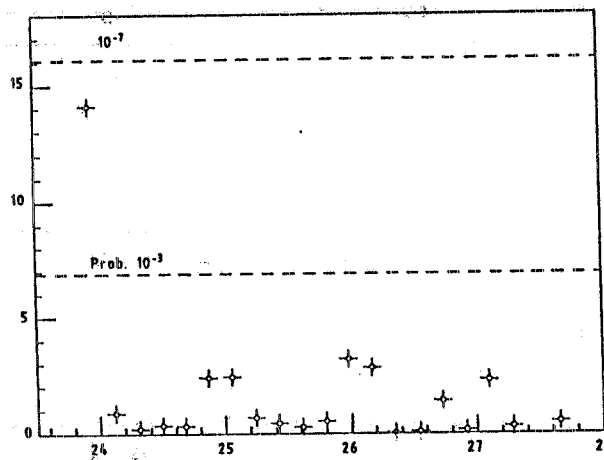


FIG. 8 - Typical distribution of  $F(\omega)$  vs  $\omega$  values for a cosmic ray data sample (Nusex data) referring to 1000 scanned frequencies. The straight line in the semilogarithmic plot, represents the expected distribution for a uniform sequence of arrival times.

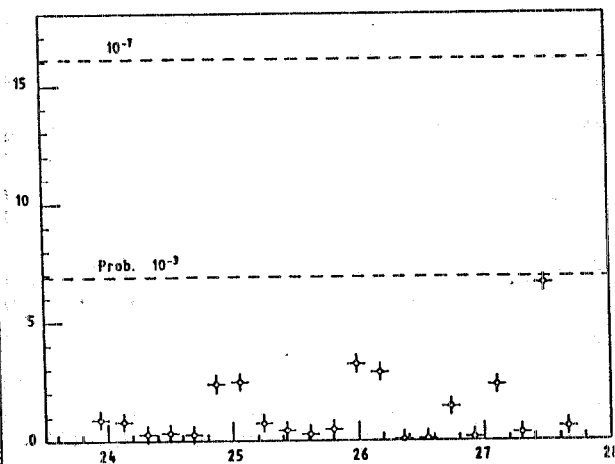


FIG. 7 - Power spectrum of events sampled in a temporal window recurring every 24 hours, as appears before subtracting the Fourier transform of the data-taking times, and after the correction.

In the case of known source emission characteristics, the method provides a test of the signal statistical significance by the calculation of the power  $F(\omega)$ , referring to fundamental and successive harmonics, including, for example, the time evolution of the source period.

In this case the frequency can be a function of the muon arrival time, and the analysis is commonly known as momenta analysis of a circular distribution, with the momentum order coinciding with the harmonic which  $F(\omega)$  refers

We performed this analysis on Nusex muon sample coming from a window of  $10^0$   $10^0$  centered on CygX3 position, after applying the heliocentric correction to the arrival times of the muons.

By the X-ray measurements is known that the Binary period is a weak function of the time, and  $P=P_0+(\delta P/\delta t)_0(t-t_0)$ , where  $P_0=0.1996830$ ,  $t_0=J. D. 2440949.8986$ , and  $(\delta P/\delta t)_0=1.18 \cdot 10^{-9}$ .

We performed a sum of  $F(\omega)$  over fundamental and first harmonic, obtaining a  $F^1(\omega)$  value equal to 5.4, corresponding to a probability of  $2 \cdot 10^{-2}$ , to find the same  $F^1(\omega)$  value in the hypothesis of random distribution of the arrival times

### 3. CONCLUSIONS

The described method can be used in cosmic ray analysis, in three ways:

- 1) to test the temporal uniformity of the data sample;
- 2) to individuate time correlations: in this case by a preliminary scanning of  $n$  frequencies we are able to identify temporal structures whose statistical significance is better than  $10^{-(n+1)}$  and which differ more than  $\Delta\omega=2\pi/T$  ( $T$ -total observation time). The provided information can be used to direct successive, more specific analysis.
- 3) to perform a statistical test on periodic signals with defined characteristics, known as momenta analysis of a circular distribution.

We studied the method performing various simulations of the periodic signal shape and background levels, and we analyzed arrival times of Nusex muon samples.

These latter applications allowed to test the uniformity of the background time distribution (fig.7), while considering the time variations of CygX3 period as known by X-ray measurements, we obtained for Nusex modulated signal from this source, a confidence level equal to 98%.

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