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THE EFFECT OF INTRA-BEAM SCATTERING ON THE DESIGN OF LOW EMITTANCE STORAGE RINGS FOR FEL OPERATION

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The increase in energy spread and beam size and the decay rate due to intra-beam scattering in a storage ring are evaluated in the case of a ring designed for the generation of high intensity coherent radiation in the VUV region.

1. Introduction

The emittance is one of the most important parameters in the design of storage rings for FEL operation. This quantity is intimately related to the gain of the FEL in the small signal regime through the value of the current density in the undulator: the lower the emittance, the higher is the bunch density, and therefore the gain.

Such a feature is required also in the recently proposed [1] high gain sources of coherent radiation, where a high density of the electron bunch and a low energy spread, at a proper energy, may give rise to collective instabilities with the production of intense radiation in the VUV and soft X-ray wavelength range.

Intra-beam scattering is a well-known subject in the physics of electron storage rings [2]: stored electrons perform betatron oscillations in a plane perpendicular to the direction of motion, and can be deflected by collisions with other electrons in the same bunch. A fraction of the transverse oscillation energy is transferred by the collision into longitudinal energy, and this fraction must be multiplied by the relativistic factor $\gamma = E/m_0c^2$ when the transformation is made from a reference frame moving with the bunch center of mass to the laboratory frame. Since γ is of the order of 10^3 for storage rings around 1 GeV (the interesting range for FEL operation), the energy gained or lost in the collision may be not negligible with respect to the total energy of the electrons: if the difference is larger than the energy acceptance of the ring, the electrons are lost in the collision, and a decay rate of the stored current can be calculated. If the energy difference is smaller than the energy acceptance, an additional energy spread in the bunch, with a related increase in beam emittance and bunch length, has to be taken into account. This process, called the "multiple Touschek effect", has a strong dependence on the energy of the electrons, since the scattering cross section scales like $1/E^2$ and the volume of the bunch increases with energy.

2. Multiple Touschek effect and bunch lengthening

Ref. [2] describes the theory of collisions within the bunch without loss of stored electrons. The relative energy spread σ_p characterizing the Gaussian distribution of electron energies can be written as

$$\sigma_p^2 = \sigma_{pq}^2 + \sigma_{pc}^2, \tag{1}$$

where σ_{pq} is the contribution of quantum emission of synchrotron radiation, and σ_{pc} comes from the collisions: both processes are in competition with the damping of synchrotron and betatron oscillations, and an increase in damping (obtained, as an example, with the insertion of damping wigglers in the storage ring) is the only way to counteract the effects of the collisions. The quantity

$$\delta = \sigma_p/\sigma_{pq} \tag{2}$$

is a measure of the contribution of collisions to the total energy spread.

The collision rate depends on the value of the total energy spread, and the value of δ can be found by solving the equation [3]

$$d(z^3 - z^2 - s) = (t + \ln z)^2, \tag{3}$$

with

$$z = \delta^2, \tag{4}$$

$$d = \frac{128}{27} \frac{\pi J_\varepsilon \gamma^6 \sigma_{pq}^6}{r_0 N_b R_b R_{av}} \sqrt{A_x A_z A_s A_{x'}}, \tag{5}$$

$$s = \frac{2}{27} \frac{\pi^2}{d}, \tag{6}$$

$$t = -\frac{2}{3}(0.5772 + \ln u), \tag{7}$$

$$\mu = \frac{r_0 N_b^{1/3}}{\sqrt{\pi}\gamma^2 \sigma_{pq}^3 A_{x'} (A_x A_z A_s)^{1/6}}, \tag{8}$$

where the symbols have the following meaning:

J_ϵ = synchrotron damping partition number (~ 2),
σ_{pq} = rms energy spread due to radiation only,
r_0 = classical electron radius = 2.82×10^{-15} m,
N_b = number of stored electrons per bunch,
R_b = bending radius in meters,
R_{av} = average radius of the ring in meters,
A_x = $(\sigma_{xq}/\sigma_{pq})^2$,
A_z = $(\sigma_{zq}/\sigma_{pq})^2$,
A_s = $(\sigma_{sq}/\sigma_{pq})^2$,
$A_{x'}$ = $(\sigma_{x'q}/\sigma_{pq})^2$,
σ_{xq} = rms horizontal beam size (radiation only),
σ_{zq} = rms vertical beam size (radiation only),
σ_{sq} = rms bunch length (radiation only),
$\sigma_{x'q}$ = rms horizontal angular spread (radiation only).

The contribution of the vertical betatron oscillations has been neglected, since the coupling factor between horizontal and vertical oscillations can be made very small in storage rings without vertical bendings.

A_x, A_z and $A_{x'}$ are functions of the azimuthal position along the storage ring, so that an average has to be made to get the value of δ: the beam sizes, divergences and bunch length scale linearly with δ.

In storage rings for FEL operation a crucial parameter for the gain of the system is the peak current in the bunch. In most of the operating storage rings, bunch lengthening has been observed, as an increasing function of peak current, together with an increase in energy spread (this phenomenon has also been called the microwave instability). However, the amplitude of the betatron oscillations is not affected by this instability, so that there is no increase in beam size where the dispersion function of the ring vanishes.

The bunch length in this regime can be written as [1]

$$\sigma_{sl} = R_{av} \left\{ \frac{\sqrt{2\pi} I_0}{hV} \left| \frac{Z_n}{n} \right| \right\}^{1/3}, \tag{9}$$

where I_0 is the stored current per bunch, V is the total voltage in the rf cavity and $|Z_n/n|$ the effective longitudinal coupling impedance of the ring. Of course, the bunch length cannot be smaller than the radiation length σ_{sq}, multiplied by δ when the multiple Touschek effect is taken into account. It is a threshold behaviour, with the bunch length given by eq. (9) when $\sigma_{sl} > \delta\sigma_{sq}$. The energy spread in this regime is enhanced by the same factor, namely

$$\sigma_{pl}/\sigma_{pq} = \sigma_{sl}/\sigma_{sq}. \tag{10}$$

3. Decay rate due to intra-beam collisions

It has been mentioned in the introduction that, if the energy transfer into the longitudinal direction is larger than the energy acceptance of the ring, the colliding particles cannot be kept inside the stable region in synchrotron phase space, and get lost on the walls of the vacuum chamber. The current in the bunch decays like

$$dN_b/dt = -\alpha N_b^2, \tag{11}$$

and the time τ when $N_b = N_{b0}/2$ (N_{b0} being the initial number of electrons) is given by

$$\tau = \frac{1}{\alpha N_{b0}}, \tag{12}$$

with

$$\alpha = \frac{4\pi r_0^2 c}{\gamma^2 \eta^2} \left\langle \frac{J(\eta, p_\perp)}{8\pi^{3/2}\sigma_x\sigma_z\sigma_s} \right\rangle, \tag{13}$$

where $\eta = \Delta E_{max}/E$ is the energy acceptance of the ring, σ_x, σ_z and σ_s are the beam dimensions (taking the multiple Touschek effect into account) and $J(\eta,\ p_\perp)$ is given by the integral

$$J(\eta,\ p_\perp) = \int_\eta^\infty dx \frac{\sqrt{1+x^2}}{x} \times \left[\left(1 + \frac{x^2}{1+x^2}\right)\left(1 + \eta^2 \frac{1+x^2}{x^2}\right) + \eta^2 \frac{x^2}{1+x^2}\left(1 - \eta\frac{\sqrt{1+x^2}}{x}\right) + \eta^2 \frac{4x^2+1}{x^2(1+x^2)} \times \ln\left(\eta\frac{\sqrt{1+x^2}}{x}\right)\right] \frac{e^{-(x/p_\perp)^2}}{2\sqrt{\pi}\, p_\perp}, \tag{14}$$

with

$$p_\perp = \gamma\sigma_{x'} = \gamma\delta\sigma_{pq}\sqrt{A_{x'}}. \tag{15}$$

Also in this case an average has to be taken over the length of the ring, since $p_\perp$ and the beam sizes depend on the azimuthal position. The decay rate may be very fast for low emittance, low energy storage rings of the kind of those being proposed for FEL operation: however, the beam enlargement due to multiple Touschek effect counteracts the loss of particles, and if a sufficient energy acceptance is provided, by means of a large voltage in the rf cavities, long lifetimes can be obtained. If the beam growth is limited, for example by the use of damping wigglers, the Touschek lifetime may become an important limitation to the performance of the machine.

A computer code has been written in the Frascati National Laboratories, which calculates the increase in beam size and energy spread due to intra-beam scattering, following the method described in section 2, and taking into account the effect of bunch lengthening when the threshold condition is satisfied. When the factor δ has been found, the program evaluates the decay rate corresponding to a given acceptance. The program needs, as input data, the complete magnetic structure of the ring, the bunch lengthening factor σ_{s1}/σ_{sq} (which is neglected if $\sigma_{s1} < \delta\sigma_{sq}$) and the energy acceptance of the ring.

4. Intra-beam scattering in a low emittance storage ring

It has recently been proposed [1] to generate coherent radiation in the VUV and soft X-ray wavelength range by means of a collective instability of an electron beam passing through an undulator with a large number of periods. The beam quality required for the start-up of the lasing action can only be provided by a storage ring with a bypass operated at low repetition rate, in order to restore the equilibrium properties of the beam after each passage through the undulator.

The efficiency of the energy transfer [1] from the electron beam to the electromagnetic radiation is given by the parameter

$$\rho = \left[\frac{K\gamma_0^2 \Omega_p}{4\gamma_R^2 \omega_0}\right]^{2/3}, \qquad (16)$$

where K is the undulator parameter

$$K = 66.0\ B_0 \lambda_0, \qquad (17)$$

where B_0 (T) is the maximum field in the undulator and λ_0 its period; γ_0 is the energy of the electron beam and γ_R is the resonant energy

$$\gamma_R = \left[\frac{\lambda_0(1+K^2)}{2\lambda}\right]^{1/2}. \qquad (18)$$

ω_0 is the undulator frequency

$$\omega_0 = \frac{2\pi c}{\lambda_0}, \qquad (19)$$

and Ω_p the relativistic plasma frequency

$$\Omega_p = \left[\frac{4\pi r_e n_0 c^2}{\gamma_0^3}\right]^{1/2}, \qquad (20)$$

with n_0 the particle density in the bunch

$$n_0 = \frac{N_b}{(2\pi)^{3/2}\sigma_x\sigma_z\sigma_s}. \qquad (21)$$

The electron beam transfers a fraction ρ of its energy to the electromagnetic wave after a number of undulator periods of the order of $1/\rho$, if the following condition is satisfied

$$\sigma_\varepsilon^* \ll \rho, \qquad (22)$$

σ_ε^* being an effective energy dispersion coming from the quadratic combination of the energy dispersion σ_p of the electrons and the inhomogeneous broadening contribution σ_ε

$$\sigma_\varepsilon = \frac{1}{2}\left(\frac{2\pi}{\lambda_0}\right)^2 \frac{K^2}{1+K^2}\varepsilon_z\beta_z + \frac{\gamma^2}{1+K^2}\left(\frac{\varepsilon_x}{\beta_x} + \frac{\varepsilon_z}{\beta_z}\right), \qquad (23)$$

ε_x and ε_z being the horizontal and vertical emittances of the electron beam, and β_x and β_z the values of the betatron functions at the undulator.

It is clear that the most important requirement to the design of a storage ring for this kind of FEL operation (the same considerations hold, however, for other rings designed for operation at visible wavelengths) is a very low emittance, which will enhance ρ and limit the contribution of σ_ε. The methods used in the design of low emittance storage rings are described in another paper presented at this conference [5].

Intra-beam scattering and bunch lengthening may become therefore a very serious limitation to the performance of storage ring FELs. Both these effects become important at low energy (< 1 GeV) storage rings: on the other hand, the gain decreases and the energy dispersion increases with the electron energy.

In order to estimate the contribution of these effects,

Table 1
Parameters of the model storage ring

Energy	500 MeV
Bending radius	5 m
Circumference	127.4 m
Periodicity	2
Number of achromatic bends	8
rf voltage	1 MV
Harmonic number	100
Average current	100 mA
Number of bunches	1
Momentum compaction	6.3×10^{-3}
Horizontal emittance	4.0×10^{-9} m·rad
Vertical emittance	4.0×10^{-11} m·rad
Horizontal betatron tune	7.6
Vertical betatron tune	5.2
Horizontal chromaticity	−19.2
Vertical chromaticity	−8.3
Zero current energy dispersion	1.9×10^{-4}
Zero current bunch length	1.7 mm
Synchrotron tune	1.4×10^{-2}
Rf acceptance	4.5×10^{-2}
Free length of the undulator straight section	4 m
Horizontal beta at undulator	1.31 m
Vertical beta at undulator	1.44 m
Coupling factor ($\varepsilon_z/\varepsilon_x$)	10^{-2}
Rms horizontal beam size at undulator	7.2×10^{-4} m
Rms vertical beam size at undulator	7.6×10^{-6} m
Rms horizontal divergence at undulator	5.5×10^{-5} rad
Rms vertical divergence at undulator	5.2×10^{-6} rad

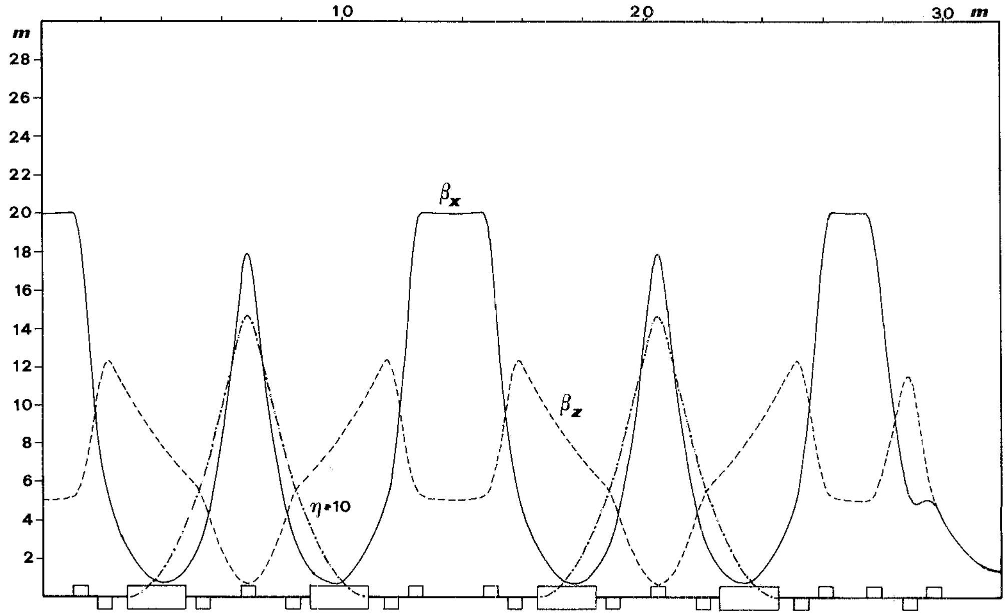

Fig. 1. Optical functions of the model storage ring.

a low emittance optics has been designed, following the guidelines of ref. [1], and the computer code described in the preceding section has been used to find the parameters of the ring with Touschek effect and bunch lengthening taken into account.

The most interesting parameters of the storage ring are listed in table 1. The bypass has not been included in the optics and it has been assumed that its optical functions can be made equal to those of the storage ring. The energy of the ring has been left as a free parameter, since the effect of intra-beam scattering and bunch lengthening has been investigated up to 1 GeV.

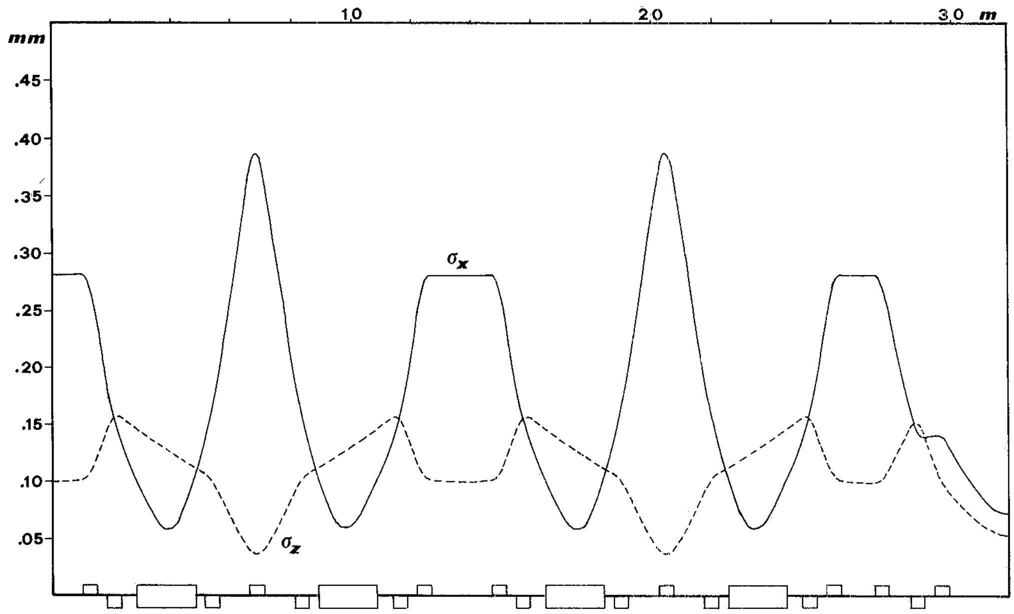

Fig. 2. Rms beam sizes σ_x (no coupling) and σ_z (full coupling) of the model storage ring.

The values listed in table 1 are referred, however, to an energy of 0.5 GeV and do *not* take into account intra-beam scattering and bunch lengthening.

Fig. 1 shows the optical functions β_x, β_z and the dispersion η in one quarter of the ring, while beam dimensions (the horizontal beam size is without coupling while the vertical one is in full coupling to be represented on the same scale) are displayed in fig. 2. The storage ring has a twofold symmetry and each superperiod is symmetric with respect to its center. The straight section for the undulator is 4 m long and the beta functions at its center can be rather easily shaped by means of a quadrupole triplet at each side.

The undulator straight is an insertion in the machine optics and the superperiodicity two comes from the fact that, for symmetry reasons, the same insertion has to be put on the opposite side of the undulator straight. If the insertions are not taken into account, the periodicity of the ring is 8, each period being a "Chasman–Green" achromatic bend [5]. The emittance of this lattice is a factor ~ 2 larger than the minimum achievable with this kind of magnetic arrangement, avoiding in this way large chromaticity and extreme sensitivity to alignment errors.

Due to the large peak current (100 mA in a single bunch, corresponding to 2.65×10^{11} stored electrons) bunch lengthening cannot be neglected. The same approach as in ref. [1] has been adopted, namely three values for $|Z_n/n|$ (see eq. (9)) have been chosen for comparison, $|Z_n/n| = 0.1\ \Omega$, $1\ \Omega$, and $10\ \Omega$ corresponding to bunch lengths of 1.3 cm, 2.7 cm and 5.9 cm respectively.

Fig. 3 shows the behaviour of the increase δ in energy spread (see eq. (2)) due to intra-beam scattering, as calculated by the program described in section 3, as a function of the storage ring energy, for different longitudinal coupling impedances. It can be seen that the effect is very large (> 3) below 0.5 GeV, irrespective of bunch lengthening, which tends to counteract the Touschek effect to some extent.

Fig. 4 shows the horizontal emittance, which scales with δ^2 and with E^2. There is a flat minimum of ε_x for $|Z_n/n| = 1\ \Omega$ and $10\ \Omega$ around 1 GeV, while for $|Z_n/n| = 0.1\ \Omega$ the minimum is shifted towards higher energy.

The decay rate τ, assuming an rf system operating at a peak voltage of 1 MV with an harmonic number of 100 is displayed in fig. 5. Obviously, the harmful effect of increased beam size due to intra-beam scattering is an advantage from the point of view of decay rate, since the bunch volume is increased (see eq. (13)). For a coupling impedance of the order of 1 Ω or less the lifetime decreases with energy, and a minimum is reached around 1 GeV. For $|Z_n/n| = 10\ \Omega$ the minimum occurs around 0.8 GeV, and then the lifetime increases again, following the natural behaviour of the Touschek lifetime without intra-beam scattering.

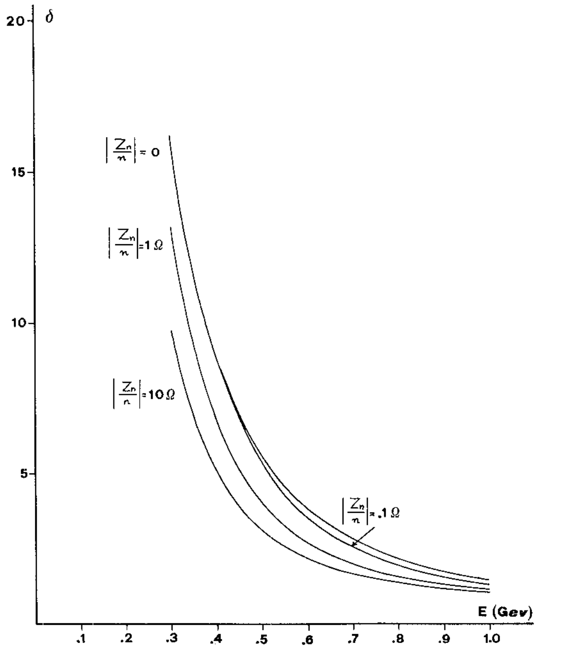

Fig. 3. Energy spread enhancement δ due to intra-beam scattering for different coupling impedances.

The possibility of generating high frequency coherent radiation depends on condition (22). In order to estimate the impact of bunch lengthening and intra-beam scattering on this condition, a model undulator has been

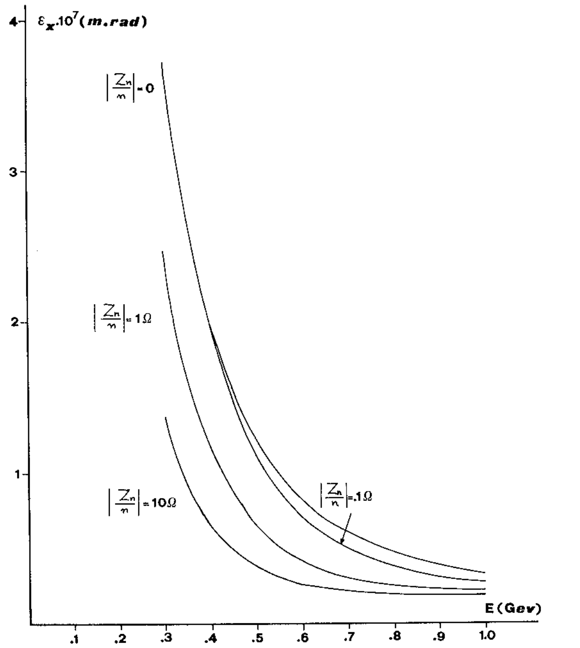

Fig. 4. Horizontal emittance as a function of energy.

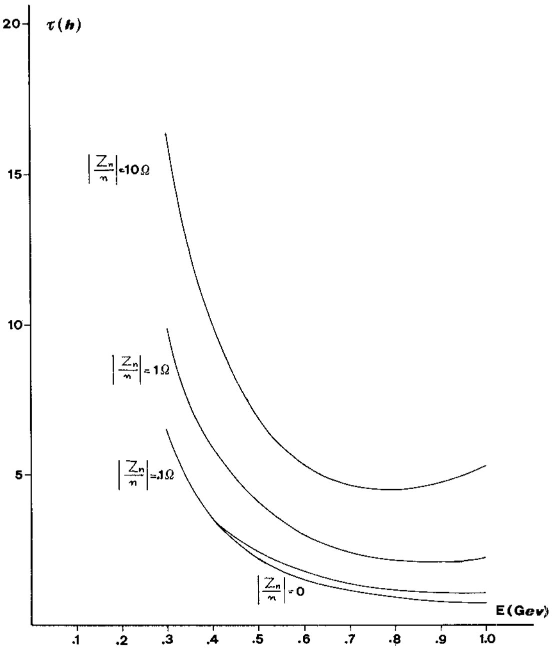

Fig. 5. Decay rate due to the Touschek effect ($V_{rf} = 1$ MV, harmonic number = 100).

assumed [1] with

$K = 2.8, \quad \lambda_0 = 2.5$ cm.

The output wavelength as a function of energy is shown

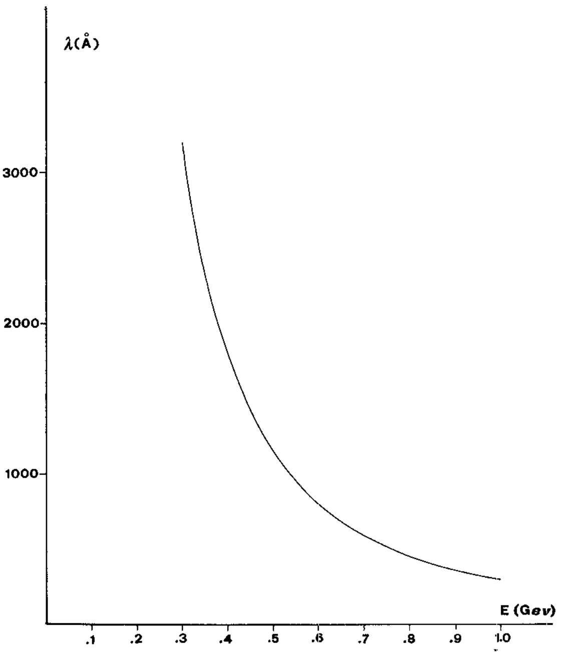

Fig. 6. Output wavelength from the model undulator ($K = 2.8$, $\lambda_0 = 2.5$ cm).

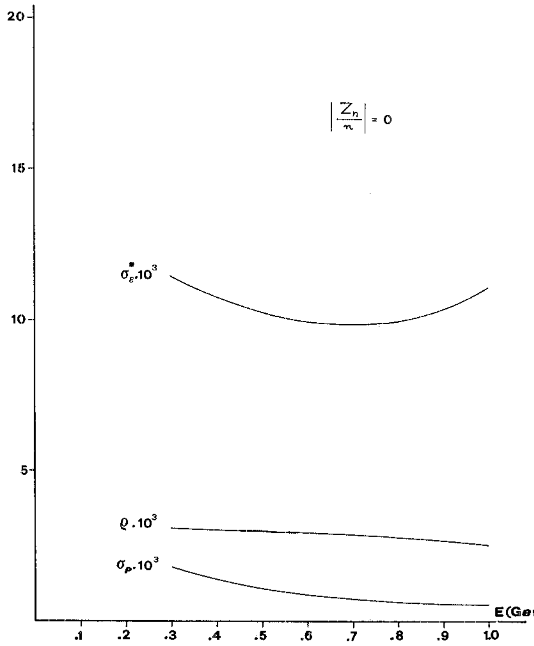

Fig. 7. ρ, σ_p and σ_ϵ^* for $|Z_n/n| = 0$.

in fig. 6. The behaviour of ρ, σ_p (the energy spread of the electron beam) and σ_ϵ^* (the effective energy spread which takes inhomogeneous broadening into account) is shown in figs. 7–10 for $|Z_n/n| = 0$, 0.1 Ω, 1 Ω, 10 Ω.

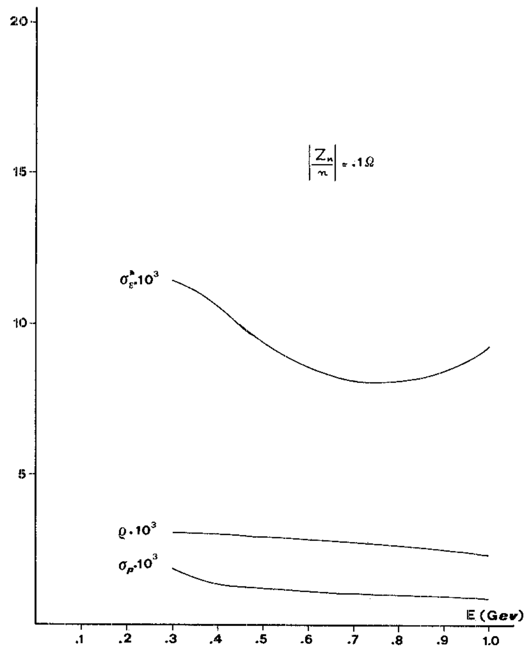

Fig. 8. ρ, σ_p and σ_ϵ^* for $|Z_n/n| = 0.1$ Ω.

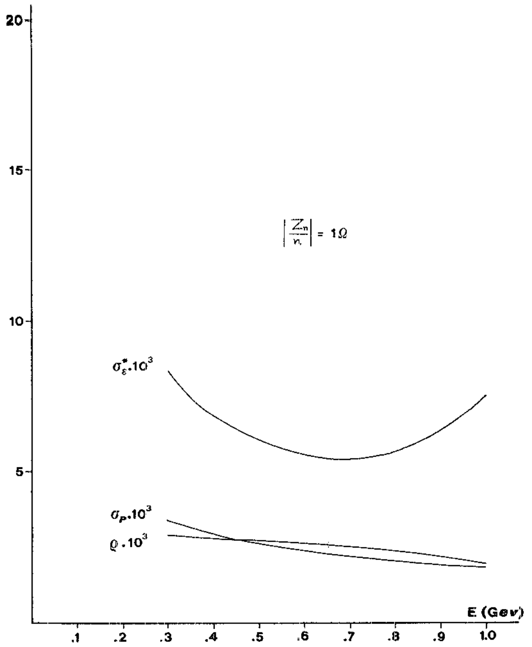

Fig. 9. ρ, σ_p and σ_ε^* for $|Z_n/n| = 1\ \Omega$.

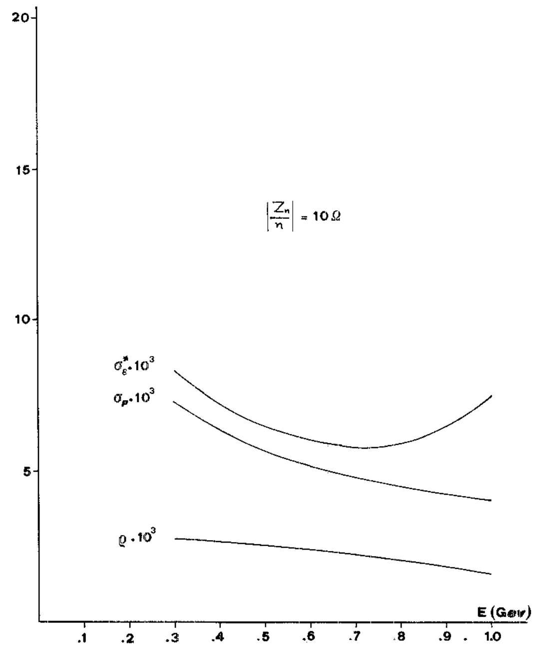

Fig. 10. ρ, σ_p and σ_ε^* for $|Z_n/n| = 10\ \Omega$.

From these plots some conclusions can be drawn:

a) the difference between ρ and σ_p does not depend strongly on beam energy;

b) for large coupling impedances ($|Z_n/n| > 1\ \Omega$) condition (22) cannot be matched since the intrinsic energy spread in the beam is larger than ρ;

c) for $|Z/_n/n| < 1\ \Omega$ the contribution of the inhomogeneous broadening is much larger than the intrinsic energy spread, leading to $\sigma_\varepsilon^* > \rho$. This is due, however, to the rather low value of the horizontal beta function in the undulator ($\beta_x \sim 1.3$ m). This value must be increased by a factor larger than 10 to get $\sigma_\varepsilon^* \approx \sigma_p$, and this can easily be done by designing the undulator insertion in a different way. As a drawback, the larger value of β_x in the undulator will decrease the value of ρ, but only like $\beta_x^{-1/6}$, while σ_ε^* scales like β_x^{-1}.

5. Conclusions

The effect of intra-beam scattering and bunch lengthening on the performance of a storage ring for the generation of high frequency coherent radiation appears to be critical in low energy (< 0.5 GeV) and low emittance storage rings. A possible way to avoid the increase in emittance and energy spread induced by intra-beam scattering is the use of wigglers at other straight sections in the ring to increase damping. The insertion of wigglers, however, could make the operation of the storage ring more troublesome, and the system must be carefully designed, in order to avoid increase in energy spread (due to the wigglers) and in emittance (due to mismatch between the optical functions of the wigglers and those of the storage ring).

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