ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

LNF-85/37

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Estratto da:
N.I.M. A237, p. 343(1985)

Servizio Documentazione
dei Laboratori Nazionali di Frascati
Cas. Postale 13 - Frascati (Roma)

Nuclear Instruments and Methods in Physics Research A237 (1985) 343–352
North-Holland, Amsterdam

STATE OF ART OF STORAGE RINGS FOR FEL OPERATION

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Electron storage rings are the most promising sources for free electron lasers at short wavelengths. The main properties of such devices are recalled, and the problems related to their use as FEL sources are discussed.

1. Introduction

The use of storage rings as electron sources for free electron lasers in the visible and ultraviolet region of the electromagnetic spectrum appears very attractive for the following reasons:

a) the energy of the storage ring can be easily tuned, thus providing a comfortable way to change the wavelength of the laser over a wide range;
b) the FEL wavelength ranges, in principle, from the infrared to the soft X-ray region, so that it may provide a unique source of coherent radiation, with good efficiency, at wavelengths where no other lasers exist;
c) the stored current can be large, since it is the result of an accumulation process;
d) the density of the electrons depends on the focusing properties of the ring lattice, and it can be made rather high.

The major drawback of the use of a storage ring for FEL operation is, of course, its cost and complexity; this is the reason why all the experiments performed or under way until now are made on storage rings built for other purposes: ACO [1] at Orsay, VEPP-3 [2] at Novosibirsk and Adone [3] at Frascati have been designed as electron/positron storage rings for high-energy physics, while the VUV [4] ring at NSLS in Brookhaven is a synchrotron radiation source.

The first ring designed for dedicated free electron laser operation has been approved and will be built at Stanford University.

2. Electron storage rings

An electron of energy $E = m_0c^2\gamma$ ($\gamma = 1/\sqrt{1-\beta^2}$; $\beta = v/c$) follows a circular trajectory of radius ρ in a constant magnetic field B if

$$E(\text{GeV}) = 0.3B(\text{T})\rho(\text{m}). \quad (1)$$

The ideal orbit of a storage ring is therefore a "race-track" trajectory where bending magnets are connected by straight sections. Calling s the coordinate along this ideal orbit and x and z the coordinates in the plane of the orbit and its perpendicular, the position of each electron can be represented by these three coordinates (see fig. 1).

The motion of an electron around the ideal orbit is stable if some focusing is provided: all the existing storage rings are designed with a "strong focusing, separated function" lattice, where the magnets provide the bending, while quadrupoles create an arrangement of focusing and defocusing lenses. For a particle whose energy satisfies eq. (1) the equations of motion are [5]

$$\frac{d^2x(s)}{ds^2} = K_x(s)x(s), \qquad \frac{d^2z(s)}{ds^2} = K_z(s)z(s), \quad (2)$$

and the solutions are the so-called betatron oscillations

$$x(s) = A_x\sqrt{\beta_x(s)}\cos(\varphi_x(s) - \vartheta_x), \qquad z(s) = A_x\sqrt{\beta_z(s)}\cos(\varphi_z(s) - \vartheta_z), \quad (3)$$

where $\beta_x(s)$ and $\beta_z(s)$ are the betatron functions of the

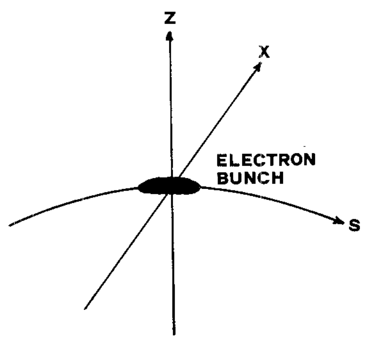

Fig. 1. Reference frame for the motion of an electron in a storage ring.

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(North-Holland Physics Publishing Division)

lattice and

$$\varphi(s)=\int_0^s \mathrm{d}s/\beta(s). \tag{4}$$

The constants A_x, A_z, ϑ_x, ϑ_z are related to the initial displacement and angular deviation of each electron from the central trajectory.

The trajectory of a particle with $A_x = 0$ (no horizontal betatron amplitude), whose energy differs from the "synchronous" energy E given by eq. (1) by an energy difference δE, is displaced from the ideal orbit by an amount

$$x(s)=\eta(s)\delta E/E. \tag{5}$$

The function $\eta(s)$ is called the "dispersion" and it is a property of the magnetic structure of the lattice: together with $\beta_x(s)$ and $\beta_z(s)$ it describes the motion of the particles in the ring. In most storage rings, where the magnets bend the electrons in the horizontal plane only, the dispersion has a finite value only in this plane, while no relation exists between the vertical displacement and the energy of the particles.

Storage rings can almost always be divided into a given number of cells: this means that the magnetic elements (bending magnets, quadrupoles and straight sections) are arranged in a sequence which repeats itself a certain number of times. This number is called the periodicity of the machine, and the functions $\beta_x(s)$, $\beta_z(s)$ and $\eta(s)$ are the same in each cell. The integrals

$$Q_x=\int_0^L \varphi_x(s)\mathrm{d}s=\int_0^L \frac{\mathrm{d}s}{\beta_x(s)},$$
$$Q_z=\int_0^L \varphi_z(s)\mathrm{d}s=\int_0^L \frac{\mathrm{d}s}{\beta_z(s)}, \tag{6}$$

(where L is the total length of the ring) are called the betatron wavenumbers of the ring and their physical meaning is the number of pseudo-oscillations (eq. (3)) performed by the particles at each revolution.

Electrons traversing a constant magnetic field B radiate a fraction of their energy in the form of photons, whose number and energy fluctuate statistically. The wavelength of these photons is distributed according to a universal function $F(\xi)$ of the variable $\xi=\lambda/\lambda_c$ where λ_c is called the "critical wavelength"

$$\lambda_c(\text{Å})=\frac{18.6}{B(\text{T})E^2(\text{GeV})}. \tag{7}$$

The function $F(\xi)$ gives the number of photons per mA of stored current, per mrad of trajectory and per unit frequency bandwidth, and it is shown in fig. 2. The average energy loss by radiation for an electron in one turn of the storage ring is

$$U_0(\text{GeV})=8.85\times10^{-5}\frac{E^4(\text{GeV})}{\rho(\text{m})}. \tag{8}$$

However, the emission of photons from an electron traversing a magnetic field is stochastic and fluctuations around the energy loss (eq. (8)) must always be taken into account.

The energy lost by emission of synchrotron radiation needs to be restored in some way, otherwise the stored electrons would drift towards the inside of the ring and hit the vacuum chamber. RF cavities with an oscillating electric field

$$V(t)=V_0\cos(h\omega_0 t+\chi) \tag{9}$$

are used to fulfill this requirement (h is the ratio between the RF frequency and the revolution frequency

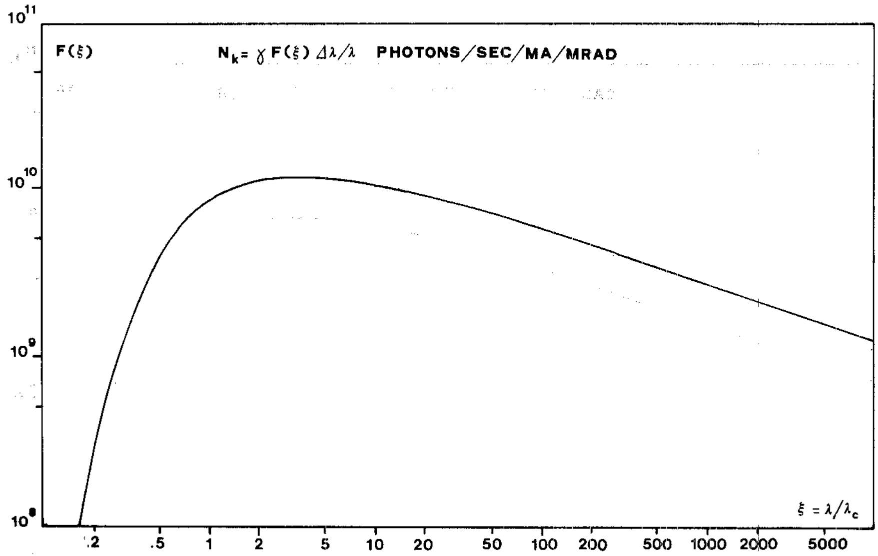

Fig. 2. Spectral distribution of the synchrotron radiation from a bending magnet.

$\omega_0/2\pi$). The ideal "synchronous" electron will pass through the cavity when

$$eV(t) = U_0, \tag{10}$$

so that its energy will be restored at each passage through the cavity; however, the energy loss by radiation is not always exactly U_0 because of the above mentioned fluctuations, and the energy balance (10) will in this case not be fulfilled. We will show now that the stored particles perform stable oscillations in energy and longitudinal position with respect to the synchronous electron.

Consider, for example, a particle with an initial energy slightly larger than the synchronous energy E. Since the dispersion η of a storage ring is in most cases always positive, the particle will follow a longer path in the bending magnets: the ratio between the lengthening of the trajectory and the relative energy displacement is called the momentum compaction

$$\alpha_c = \frac{\delta L/L}{\delta E/E}. \tag{11}$$

Since the velocity of sufficiently high energy electrons differs from the speed of light by a negligible amount, the particle will arrive at the RF cavity with a certain delay and, if the phase of the electric field is correct (namely if the slope of the field is negative), it will gain less energy from the cavity than the synchronous particle, and therefore the motion in the time–energy space will be stable. The particles will oscillate around the equilibrium condition (10) with a frequency called the synchrotron frequency ν_s

$$\nu_s = \frac{1}{2\pi}\sqrt{\frac{\alpha_c e\dot{V}_0}{T_0 E}}, \tag{12}$$

where $\dot{V}_0$ is the derivative of the electric field at the equilibrium phase defined by eq. (10), and T_0 the time needed by the synchronous electron to perform one full revolution. The delay in time of a particle multiplied by the speed of light c is the distance in azimuth s (see fig. 1) from the synchronous particle, and this explains why the electrons in a storage ring are collected in "bunches" around the h stable points where the phase of the cavity satisfies condition (10).

Expression (8) for the energy loss per turn shows that the radiated power increases with the energy of the electrons. This means that an electron with a positive energy deviation radiates more than the synchronous one, and therefore the energy oscillations are damped. The damping time constant for energy oscillations is given by

$$\tau(s) \approx \frac{T_0 E}{U_0} = 4.8\times10^{-4}\frac{R\rho}{E^3(2+\alpha_c R/\rho)}, \tag{13}$$

where R is the average radius of the ring, i.e. its total length divided by 2π. Betatron oscillations are also damped, even if the effect is mainly due to the interaction with the electric field in the RF cavity. For the vertical plane we have

$$\tau_z(s) = 4.8\times10^{-4}R\rho/E^3, \tag{14}$$

and for the horizontal one

$$\tau_x(s) = 4.8\times10^{-4}\frac{R\rho}{E^3(1-\alpha_c R/\rho)}. \tag{15}$$

The stochastic emission of photons in the bending magnetic field is the source of synchrotron oscillations. It excites also the betatron oscillations through the following mechanism. Imagine a synchronous electron emitting a photon at a point in the machine where the dispersion is not vanishing: since the photon is emitted in an extremely narrow cone (its aperture is $\sim 1/\gamma$) around the direction of motion of the electron, the particle does not change its direction, but its energy is decreased by the energy of the emitted photon; the orbit corresponding to the new energy is given by eq. (5), so that the electron is displaced by $\eta\delta E/E$ from the orbit, and will perform betatron oscillations around it.

It is easy to understand now that the size of the bunch in the horizontal and longitudinal directions is determined by an equilibrium between the stochastic excitation due to the emission of synchrotron radiation and the damping described above. The particles in the beam will exhibit Gaussian distributions in the three directions (see fig. 1). Moreover, the energy distribution, also resulting from the same equilibrium, will also be Gaussian. The relative energy spread is commonly used

$$\sigma_p = \sigma_E/E = 8.6\times10^{-4}E(\text{GeV})/\sqrt{\rho}. \tag{16}$$

The bunch length will have an rms spread given by

$$\sigma_s = \frac{c\sigma_p\alpha_c}{2\pi\nu_s}, \tag{17}$$

while in the horizontal direction

$$\sigma_x = \sigma_p\sqrt{\frac{2M\beta_x(s)}{1+\varepsilon^2} + \eta^2(s)}, \tag{18}$$

where β_x is the horizontal betatron function introduced by eq. (3); to explain the meaning of M let us introduce the functions

$$\alpha_{x,z}(s) = -\tfrac{1}{2}\frac{d\beta_{x,z}(s)}{ds}, \tag{19}$$

and

$$\gamma_{x,z}(s) = \frac{1+\alpha_{x,z}^2(s)}{\beta_{x,z}(s)}. \tag{20}$$

$\alpha_{x,z}(s)$, $\beta_{x,z}(s)$ and $\gamma_{x,z}(s)$ are called the Twiss functions of the storage ring. M is a very important characteristic of the machine, whose physical meaning will be clear in the following, and is given by

$$M = \frac{1}{2\pi\rho}\int_{\text{magnets}} \left(\gamma_x\eta^2 + 2\alpha_x\eta\eta' + \beta_x\eta'^2\right)\mathrm{d}s. \tag{21}$$

In a perfect machine with the bending only in the horizontal plane there is no reason why particles should oscillate in the vertical plane. However, alignment errors for the bending magnets and tilts in the quadrupole fields introduce some coupling between horizontal and vertical betatron oscillations. This coupling is represented by the variable ε which appears in eq. (18)

$$\varepsilon_{\min} < \varepsilon < 1. \tag{22}$$

$\varepsilon_{\min}$ gives information about the magnitude of tilt errors in the machine, and the maximum value $\varepsilon = 1$ is reached when the fractional parts of Q_x and Q_z are the same. The vertical beam size depends on ε and it is given by

$$\sigma_z(s) = \sigma_p\sqrt{\frac{2M\beta_z(s)\varepsilon^2}{1+\varepsilon^2}}\,. \tag{23}$$

Let us consider now the angles between the trajectories of the electrons and the ideal orbit: they also have Gaussian distributions with standard deviations

$$\sigma_{x'} = \sigma_p\sqrt{\frac{2M\gamma_x(s)}{1+\varepsilon^2} + \eta'^2(s)}\,, \tag{24}$$

$$\sigma_{z'} = \sigma_p\sqrt{\frac{2M\gamma_z(s)\varepsilon^2}{1+\varepsilon^2}}\,. \tag{25}$$

The second term under the square root in eqs. (18) and (24) represents the contribution of synchrotron oscillations to the beam size and divergence. If we take into account only the betatron contribution, we can define the natural emittance

$$\varepsilon_0 = 2M\sigma_p^2, \tag{26}$$

which coincides with the product $\sigma_x\sigma_x'$ for $\varepsilon = 0$ and $\alpha_x = 0$. Where α_x is not vanishing, ε_0 (multiplied by π) is the area of the ellipse occupied by the beam in the phase space x, x' and it is a constant in the whole machine. This constant, the "emittance" of the storage ring, is one of the most important parameters of the ring, which defines the "quality" of the beam for FEL operation.

The last subject of this brief summary of the properties of an electron storage ring is injection. The particles to be stored are usually accelerated in a separate accelerator, called the injector. By means of a magnetic channel the accelerated electrons are guided towards a straight section of the ring, where they enter the vacuum chamber, parallel to the central orbit, through a septum magnet which gives the final deflection to the injected particles, without stray fields on the axis of the straight section (see fig. 3).

The injected beam will perform betatron oscillations around the central orbit, and these oscillations will be damped with the damping constant (eq. (13)). This damping time is usually much longer than the revolution time, so that it is easy to imagine that the particles, after a certain number of revolutions, will hit the septum again. This can be avoided by the use of a kicker, namely a pulsed dipole field placed one fourth of betatron wavelength downstream the injection point, giving an angular kick equal and opposite in sign to the angle of the trajectory of the injected particles. The kicker will be switched off after a revolution time, so that no residual betatron oscillation will remain.

This is a possible scheme for injection, if a single pulse from the injector is sufficient to store all the desired electrons into the ring. Of course, this is not the case for most situations, so that both the injected beam and a stored beam must be considered. For this purpose, a second kicker, placed one fourth of betatron wavelength upstream the injection point, can be used: the stored beam will be kicked upwards and will pass near the septum without hitting it, and will be deflected

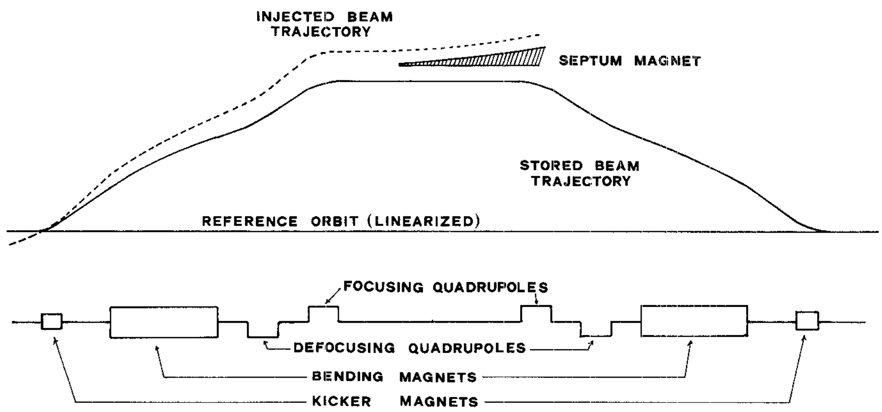

Fig. 3. Schematic layout of injection into a storage ring.

again by the second kicker, so that its orbit will follow an "injection bump" restricted to the space between the two kickers. The injected beam will enter the storage ring in coincidence with the passage of the stored one, and will "see" only the second kicker, being deflected by an angle slightly smaller than what would be necessary to correct its orbit completely (see fig. 3). A residual betatron oscillation will remain for the injected beam, which will merge into the stored one after a few damping times: after this has happened, it will be possible to inject another pulse from the injector, so that the injection rate will be determined mainly by the damping constants (13), (14) and (15) of the ring. This is only one of the reasons why it is often convenient to choose the injection energy at the same value of the operation energy, avoiding ramping the energy of the ring from injection to stored beam operation.

3. A free electron laser in a storage ring

The wavelength of the first harmonic of the radiation coming on axis from a transverse undulator is

$$\lambda = \frac{\lambda_0}{2\gamma^2}(1+K^2), \tag{27}$$

where λ_0 is the period of the undulator and the parameter K is given by

$$K = 66.0\, B_0(\mathrm{T})\lambda_0(\mathrm{m}), \tag{28}$$

B_0 being the maximum field in the undulator. Neglecting K^2 (which is usually of the order or less than 1 for permanent magnet undulators) in eq. (27) one can see that with a minimum undulator period of the order of 1 cm, and with the energy of the electrons ranging from ~ 0.1 to ~ 2 GeV the wavelength of a storage ring FEL can range, in principle, from the near infrared to the soft X-ray region.

To examine in some detail what are the requirements on the electron beam quality, let us consider the gain of the system, namely the relative increase per passage in the intensity of the laser field, which is given by [6]

$$G = 32\sqrt{2}\pi^2\sqrt{\lambda^3\lambda_0}\,\frac{K^2}{(1+K^2)^{3/2}}\,\frac{I_p N_w^3}{I_A \Sigma} f(x), \tag{29}$$

where I_p is the peak current in the bunch, $I_A = 1.7\times10^4$ A, Σ is the transverse cross section of the electron and photon beams, N_w is the number of periods of the undulator and $f(x)$ is an odd function

$$f(x) = (\cos x - 1 + \tfrac{1}{2}x\sin x)/x^3, \tag{30}$$

$$x = 4\pi N_w \frac{\gamma_0 - \gamma_R}{\gamma_R}, \tag{31}$$

where γ_0 is the relativistic factor of the electrons and γ_R is the resonant energy, which satisfies eq. (27). Formula (26) is valid under the following conditions

$$\sigma_p \ll \frac{1}{2N_w}, \tag{32}$$

$$\sigma_{x',z'} \ll \frac{1}{\sqrt{2N_w}\,\gamma}. \tag{33}$$

In the visible region of the electromagnetic spectrum, the transmission of commercially available mirrors is of the order of 10^{-3}, so that a gain of this order will be sufficient to obtain a self-sustained laser action in an optical cavity with mirrors at each side of the undulator. In the ultraviolet wavelength range mirrors have a reflectivity of the order of 50%, and the requirements on the electron beam quality, to reach such a gain, become much more demanding.

A single-pass device, consisting in an undulator installed on a bypass of a storage ring, has recently been proposed for the production of coherent radiation in the VUV and soft X-ray wavelength ranges. In this case it is not correct to speak of a gain, since the device does not amplify an existing electromagnetic field, but the coherent radiation is created in a single passage of the electron beam: the pulses are separated by the time needed to restore the equilibrium properties of the electron beam after each passage through the undulator. A storage ring is needed, instead of a single pass accelerator, because of the required beam quality. The efficiency of the energy transfer from the electron to the photon beam is measured by the parameter

$$\rho = \left[\frac{K\gamma_0^2\Omega_p\lambda_0}{8\pi c\gamma_R^2}\right]^{2/3}, \tag{34}$$

with

$$\Omega_p = \left[\frac{4\pi r_e n_0 c^2}{\langle\gamma_0\rangle^3}\right]^{1/2}, \tag{35}$$

with r_e the classical electron radius, n_0 the density of the electron bunch and $\langle\gamma_0\rangle$ the average initial electron energy. The condition for the growth of the exponential instability leading to the production of the coherent photon beam is

$$\sigma_\varepsilon^* < \rho, \tag{36}$$

where σ_ε^* is an "effective" energy spread given by the quadratic combination of the "natural" energy spread of the electron bunch σ_p and the inhomogeneous broadening coming from the emittance of the electron beam

$$\sigma_\varepsilon = \tfrac{1}{2}\left(\frac{2\pi}{\lambda_0}\right)^2 \frac{K^2}{1+K^2}\varepsilon_z\beta_z + \frac{\gamma^2}{1+K^2}\left(\frac{\varepsilon_x}{\beta_x} + \frac{\varepsilon_z}{\beta_z}\right), \tag{37}$$

and the number of undulator periods should be at least

$$N_w \approx 1/\rho. \tag{38}$$

From eqs. (29) and (38) we can easily recognize that one of the most important requirements for a storage ring for FEL operation is the possibility of inserting long undulators in straight sections free from any obstruction of machine components, such as quadrupoles, sextupoles etc.: this is probably the main disadvantage of using existing storage rings, mainly designed for high-energy physics, as FEL sources. This requirement is not easily fulfilled, from the point of view of machine optics, in a periodic structure, so that long "insertions" into the ring must be designed to optimize the performance. The first requirement to an insertion of this kind (of course, after the free length needed to accomodate the undulator) is that the dispersion function $\eta(s)$ must vanish over the length of the undulator, for two reasons: the first is that a non-zero dispersion will increase the horizontal size of the beam, thus leading to a reduction in gain, and the second is that a magnetic field (from a wiggler or an undulator) combined with the dispersion increases the emittance of the beam.

Beam size and divergence should be as small as possible, since a high electron density allows for large gain (see eqs. (29), (34), (35)), while a small divergence limits the inhomogeneous broadening (see eqs. (33), (37)). From eqs. (18), (23)–(25) one sees that the value of the β function in the undulator plays an opposite role for the beam size and the beam divergence. Since in an insertion it is possible to choose the value of the β function, an optimization can be made, and it depends on the particular configuration of the FEL scheme. The behaviour of the β function in a straight section, whose center is a symmetry point for the machine, is given by

$$\beta(s) = \beta_0 + s^2/\beta_0, \tag{39}$$

where β_0 is the value of $\beta(s)$ at the straight section midpoint: the beam size increases from the center to the end of the undulator and, if the undulator is long and β_0 too small, there will be a measurable difference between the beam sizes at the center and the end of the straight section, while the divergence will remain constant.

Another important constraint to the design of a FEL storage ring is the limit to the free space for the beam given by the undulator gap, which is determined by the undulator period and the desired magnetic field. This limit becomes severe when the FEL wavelength is in VUV region or smaller, since in this case only the permanent magnet solution for the undulator is possible. However, in this case a special design of the straight section vacuum chamber, with movable walls, will help much, separating the aperture requirement for injection of the beam into the storage ring from the operation of the laser with the beam already stored and stabilized.

It is clear that the best way to meet all these requirements is the design of a very low emittance storage ring, with long insertions with vanishing dispersion for the undulator. This is a common feature of storage rings designed for the use as synchrotron radiation sources, while the optimization of the luminosity in electron–positron storage rings for high-energy physics needs a large emittance to overcome the beam–beam limit. Storage rings such as ACO at Orsay, VEPP-3 at

Table 1
Parameters of FEL experiments on operating storage rings. The data refer to the measured experimental conditions.

	ACO-1	ACO-2	ACO-3	ADONE	VEPP-3	NSLS-VUV
Type of experiment	Amplification	Amplification	Oscillation above threshold	Amplification	Oscillation below threshold	Amplification
Maximum energy (GeV)	0.54	0.54	0.54	1.5	3.5	0.7
Experiment energy (GeV)	0.15	0.24	0.166	0.62	0.35	0.35
FEL wavelength (Å)	4880	4880	6476	5145	6300	3500
Undulator type	Superconducting	Permanent	Permanent with optical klystron	Normal conducting	Permanent with optical klystron	Permanent
Period length (cm)	4.0	7.8	7.8	11.6	6.5	6.5
Number of periods	23	17	2×7	20	2×4.5	39
Maximum field (kG)	4.0	3.1	3.1	4.5	7.0	5.5
Average current/bunch (mA)	12	15	8–50	18.5	25	–
rms bunch length (cm)	8.7	15.6	15–30	13	–	–
Horizontal rms beam size (mm)	0.26	–	0.3–0.5	2.5	0.2	–
Vertical rms beam size (mm)	0.15	–	0.3–0.5	0.2	0.1	–
Energy spread	6.5×10^{-4}	–	$0.9–1.3\times10^{-4}$	–	4×10^{-4}	–
Laser waist (mm)	0.64	0.8	–	1.0	0.35	–
Measured peak gain	3×10^{-4}	1.5×10^{-4}	–	2×10^{-4}	1.5×10^{-2}	–
Ref.	[17]	[18]	[1]	[19]	[2]	[4]

Novosibirsk and ADONE in Frascati have been used for FEL experiments, but the theoretical gain calculated for these machines is always low, mainly because of the large emittance of the beam and the small straight section length available for the undulator. The situation is better for the VUV ring of the National Synchrotron Light Source in Brookhaven, where an FEL experiment is also under way. Table 1 shows the parameters of these accelerators, which are relevant to FEL operation.

4. The design of low emittance storage rings

As previously mentioned, the requirement of low emittance is the most important in the design of storage rings for synchrotron radiation, together with the condition of vanishing dispersion in the straight sections where insertion devices have to be installed. The best solution for this problem is an optical structure made of achromatic bends (the Chasman–Green arrangement): each bend begins with vanishing dispersion ($\eta_0 = 0$, $\eta_0' = 0$) at the beginning of a magnet. For magnets of the sector type with parallel pole faces, the dispersion at its end is

$$\eta_1 = \rho(1 - \cos \vartheta_m), \tag{40}$$

$$\eta_1' = \sin \vartheta_m, \tag{41}$$

where ρ is the bending radius and ϑ_m the bending angle of the magnet. Assume now that after the magnet there is a straight section of length L: since the dispersion is the trajectory of a particle with $\delta E/E = 1$ (see eq. (5)), the dispersion at the end of the straight section will be

$$\eta_2 = \eta_1 + L\eta_1' = \rho(1 - \cos \vartheta_m) + L \sin \vartheta_m, \tag{42}$$

$$\eta_2' = \eta_1' = \sin \vartheta_m. \tag{43}$$

After the straight section it is possible to place a horizontally focusing quadrupole with focusing constant K_q and half-length L_q. The transfer matrix of the half-quadrupole is

$$\begin{bmatrix} \eta_3 \\ \eta_3' \end{bmatrix} = \begin{bmatrix} \cos K_q L_q & \dfrac{\sin K_q L_q}{K_q} \\ -K_q \sin K_q L_q & \cos K_q L_q \end{bmatrix} \cdot \begin{bmatrix} \eta_2 \\ \eta_2' \end{bmatrix}, \tag{44}$$

If now we choose L_q and K_q so that $\eta_3' = 0$, the dispersion will have zero slope at the center of the quadrupole and, if on the other side there is the same magnetic arrangement (namely if the elements are placed symmetrically with respect to the center of the quadrupole), the dispersion and its derivative will vanish at the end of the second magnet, making the whole "cell" achromatic (see fig. 4). Since the dispersion is "excited" only by bending magnets, it is clear that the insertions for wigglers and undulators must care only for the behaviour of the β functions, since the dispersion vanishes everywhere outside the achromatic bends.

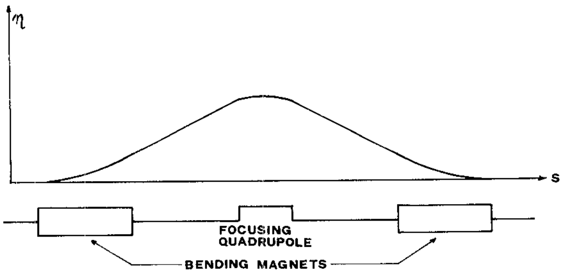

Fig. 4. Schematic behaviour of the dispersion function η in a "Chasman–Green" achromatic bend.

The emittance of the ring is related to the dispersion by eq. (21). It is clear that the achromatic bend solution provides low η and η' in the bending magnets, even if it is possible to imagine other arrangements providing lower emittance (however, the achromatic bend is the only way to get vanishing dispersion everywhere outside). The horizontal β function has to be shaped properly to minimize the emittance: this can be calculated analytically to obtain the minimum emittance

$$\varepsilon_{min} = 9.5 \times 10^{-8}\, E^2 \vartheta_m^3 \left(1 - \frac{11}{210}\vartheta_m^2 + \cdots\right), \tag{45}$$

where the energy of the ring is in GeV and the emittance in m · rad. From eq. (45) one can see that, for a given energy, the minimum emittance depends only on the number of achromatic bends.

The design of an optical structure with the minimum emittance (45) is rather easy: computer programs which optimize the magnetic lattice of a storage ring are commonly available in many accelerator laboratories. However, the optical solution which satisfies eq. (45) exhibits, in most cases, sharp minima of β_x inside the bending magnets, and this has two major drawbacks:

a) matching of the β functions of the achromatic bends with those of the insertion becomes quite troublesome and quadrupole doublets or triplets are necessary at each side of the undulator;
b) β functions reach large values in the quadrupoles placed aside the bending magnets, and this is a source of strong "chromatic" effects [8]: many machine parameters, in particular the betatron wavenumbers, depend strongly on the energy of the particles, and in many cases nonlinear corrections (mainly with sextupole fields) are needed to avoid overlapping of resonances or the excitation of instabilities like the head–tail effect [9].

The behaviour of a storage ring with nonlinear elements is one of the most interesting problems in the accelerator field, not only for low emittance machines for FEL operation or synchrotron radiation sources, but also for large high-energy colliding beams rings, such as LEP. Simulations with numerical codes are extensively used to define the dependence of betatron wavenumbers and optical functions on the energy of the particles, and the maximum amplitude of stable betatron oscillations.

In many cases it turns out that the limitations introduced by these nonlinear corrections are too severe for the optical solutions providing the minimum emittance (eq. (45)). A compromise has to be found in these cases, which gives, depending on the particular structure, an increase in emittance by a factor 2–3 with respect to eq. (45).

The criteria to follow in the optimization of the lattice with nonlinear corrections are mainly related to the dependence of betatron wavenumbers and β functions on the energy of the particles and on the amplitude of betatron oscillations, and to the stability of large amplitude oscillations. Fig. 5 shows, as an example, the dependence of the horizontal betatron wavenumber Q_x on the energy displacement $\delta E/E$ of the electron with sextupolar corrections on an off. It can be seen that, without the correction, the tune spread corresponding to a given energy acceptance δE_{max} is very large, leading to resonance overlapping and beam instability. With the corrections on, the derivative of Q_x with respect to the energy vanishes for the synchronous particle; however, the residual spread induced by the higher order terms in the chromaticity correction should not exceed a maximum limit which depends, of course, on the particular magnetic structure of the ring.

A second source of betatron frequency spread introduced by the chromaticity correction is the dependence of the tunes on the amplitude of betatron oscillations. This effect may be corrected by means of octupole fields, but it is normally preferred, in the design of low emittance storage rings, to avoid the introduction of high order nonlinear terms: as it has been shown in the preceding section, large amplitude oscillations occur at injection, so that, in order to reach a good injection efficiency, it is necessary to limit the Q-spread on amplitude.

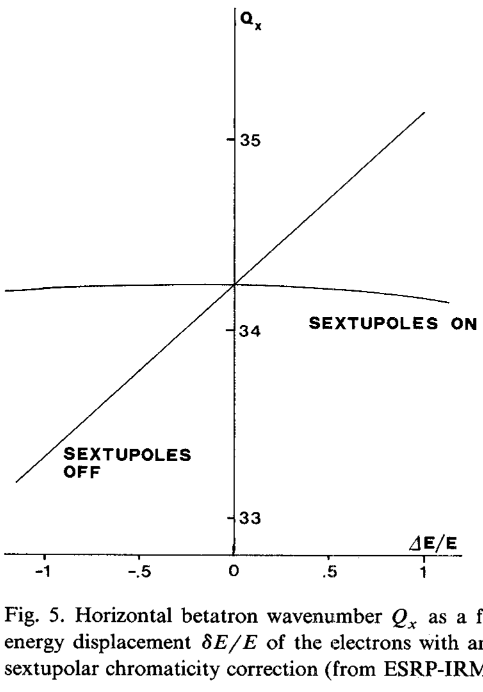

Fig. 5. Horizontal betatron wavenumber Q_x as a function of energy displacement $\delta E/E$ of the electrons with and without sextupolar chromaticity correction (from ESRP-IRM-26-84).

Another important parameter to be kept in mind is the stability of large amplitude betatron oscillations. Fig. 6 shows a typical representation in phase space of the behaviour of an electron in a storage ring with sextupolar chromaticity correction [10]. Each point describes the position x and the angle x' of an electron in its successive passages in a given point of the storage ring; the trajectory (which should be an ellipse with nonlinear corrections switched off) is distorted by the sextupolar fields, and its shape becomes similar to a triangle when the fractional part of the horizontal betatron frequency approaches a third of an integer. The amplitude of such a trajectory depends on the initial conditions of the betatron motion and it may become unstable: it is customary, therefore, to divide the phase space of the initial conditions into a stable and an unstable region. The border between them is called the "dynamic aperture", and it should be large enough to accomodate at least 6–7 standard deviations in the beam sizes and to fulfill injection requirements. The connection between the nonlinear terms introduced by the chromaticity correction and the alignment and current errors in the quadrupoles and bending magnets must also be considered, since low emittance structures are very sensitive to this kind of inaccuracies. Computer codes [11] have been written for this specific purpose, and they are of great help in the design of correction schemes.

Another important subject to take into account in the design of a storage ring for FEL operation is the power delivered by the laser: it has been shown [12] that an electron beam passing at each turn through a FEL cavity will increase its energy spread until an equilibrium is reached between the excitation in the cavity and the damping provided by the emission of synchrotron radiation in the storage ring itself. This equilibrium is reached when the power from the laser has a maximum value

$$P_L \cong \frac{U_0 I_{av}}{2N_w}, \tag{46}$$

where U_0 is the energy emitted by the electrons as synchrotron radiation in the ring, I_{av} is the average electron current and N_w the number of periods in the undulator. It may be therefore convenient to use wigglers to increase U_0 by putting them in a vanishing dispersion section of the storage ring to avoid any increase in emittance. It should be kept in mind that a

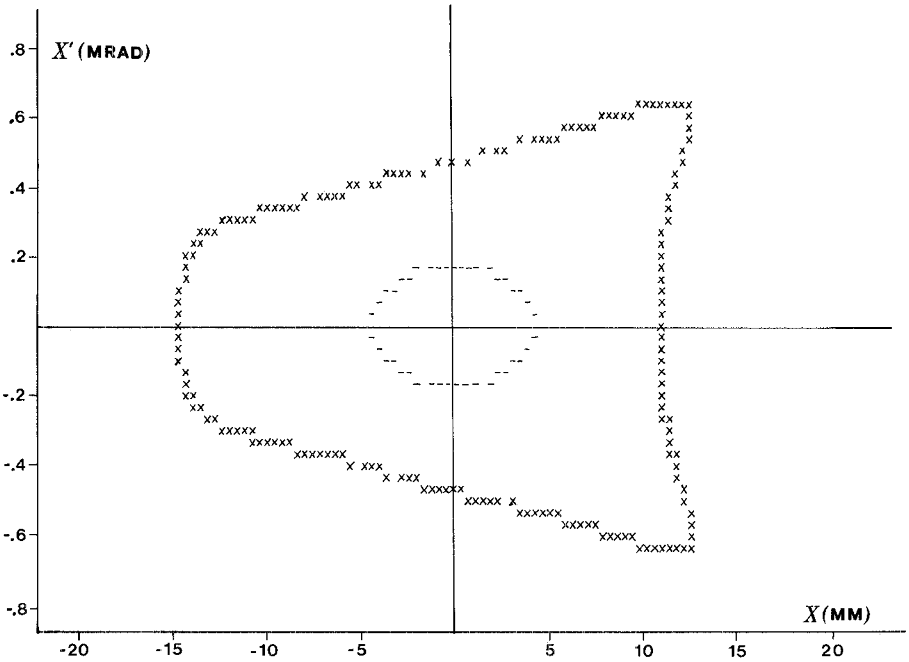

Fig. 6. Phase space trajectory for a particle in a storage ring with sextupolar chromaticity correction (from ref. [10]).

high-field wiggler for production of synchrotron radiation represents a rather strong perturbation to the optical structure of the ring, and it has therefore to be carefully matched.

5. Limitations to the performance of a FEL storage ring

It has been shown in section 3 that the emittance of the electron beam is one of the most important quality factors of a storage ring for FEL operation. Let us briefly describe the physical phenomena which may limit the possibility of realizing very small emittances with the design criteria described above.

Particles in a storage ring perform betatron and synchrotron oscillations (see section 2) in the plane perpendicular to the reference trajectory: each electron has therefore a transverse momentum in a reference frame moving with the bunch center of mass, and can collide with other electrons. In the elastic scattering a fraction of the initial energy can be transferred to the longitudinal direction, and this fraction must be multiplied by the relativistic factor γ in the transformation to the laboratory frame. Since γ is of the order of 10^3 for the energy range of interest for FEL operation, the small energy associated to the betatron and synchrotron oscillations can be transformed in a non negligible fraction of the electron total energy. If this fraction is larger than the energy acceptance of the ring (which may be a limitation due to the rf system or to the nonlinear terms of the chromaticity correction, or also to the limited aperture of the vacuum chamber) both the colliding electrons will be lost and a decay rate in the stored current will be observed; if otherwise the energy deviation is smaller than the energy acceptance, this kind of process, called "intra-beam scattering" or "multiple Touschek effect", will be a source of noise in the energy distribution of the beam, exactly as the emission of synchrotron radiation.

A good way to counteract the additional energy spread coming from intra-beam scattering is the insertion of damping wigglers into the ring, since the damping constants are essentially proportional to the total energy radiated in the ring (see eq. (13)). Wigglers could be, however, a source of additional energy spread, if their field is larger than the field in the bending magnets [13].

The increase in energy spread due to intra-beam scattering leads to a corresponding increase in the transverse and longitudinal dimensions and in the divergence of the beam (see eqs. (17), (18), (23)). The effect may be dramatic for high current, low emittance storage rings operating at low energy [14]: however, in this energy range it compensates by some amount the decay rate caused by large energy transfers, which would otherwise determine a short lifetime of the stored current.

Another possible reason for an increase of the electron beam emittance is the trapping of ions of the residual gas in the vacuum chamber [15]. Under some particular conditions (bunch current, residual pressure, beam energy, bunch spacing) the ions may "see" a periodic field created by the passage of the electron bunches, which can make a potential well where they can be trapped. This leads to an increase in the local pressure and therefore in the scattering rate. The effect is an increase in beam size and divergence and in betatron wavenumber spread. For this reason it may be necessary to operate FEL storage rings with positrons instead of electrons, as it has been proposed, for example, for the European Synchrotron Radiation Facility [16], where the requirement for a small emittance is crucial.

Another source of beam quality degradation is the well known bunch lengthening effect, which has been observed in all the existing storage rings in the energy range below 1 GeV. It is caused by the interaction of the beam with the broad band longitudinal coupling impedance of the ring, and it is associated to an increase in bunch length and energy spread. The dependence of the bunch length on the stored current is given (in the bunch lengthening regime) by

$$\sigma_{s1} = R\left\{\frac{\alpha_c I e}{\sqrt{2\pi}\, E\nu_s^2 T_0^2}\left|\frac{Z_n}{n}\right|\right\}^{1/3}, \tag{47}$$

where I is the current per bunch and $|Z_n/n|$ the effective longitudinal coupling impedance of the ring. As can be seen from eq. (12) the product $E\nu_s^2$ does not depend on the energy of the electrons, and so, in the lengthening regime, the bunch length becomes independent of energy and scales like $I^{1/3}$. Of course, σ_{s1} cannot be smaller than the radiation bunch length σ_s given by eq. (17), and therefore bunch lengthening has a threshold at the current for which eqs. (17) and (47) give the same value. The energy spread in the bunch lengthening regime is increased, with respect to the radiation energy spread (eq. (16)) by the same factor as the bunch length.

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