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S. Tazzari: ELECTRON STORAGE RINGS FOR THE PRODUCTION OF SYNCHROTRON RADIATION

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ELECTRON STORAGE RINGS FOR THE PRODUCTION OF SYNCHROTRON RADIATION

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1.- INTRODUCTION

In the lecture an introductory overview is given on storage ring sources of synchrotron radiation, in particular sources of rather hard X-rays ($h \gtrsim 1$ KeV). Requirements set by the users, properties of the devices used to produce the radiation, and finally design criteria to be followed to optimize a storage ring for the particular purpose are discussed.

2. - THE PHYSICS AND THE EXPERIMENTAL REQUIREMENTS

The study of the interaction of X-rays with matter goes back to the beginning of the century and has resulted in an amazingly large number of applications and findings of primary importance. Interesting reviews can be found in Refs.(1) and (2) from which I will widely quote.

The basic processes characterizing the interaction of X-ray photons with matter are absorption and scattering: on the first is based for example radiography, on the second the study of crystals and molecules. The importance radiographic techniques have acquired in everyday life needs not be stressed. As an example from the past in the field of crystallography, it suffices to quote the unravelling of the structure of the DNA molecule, which "will stand out as one of the greatest scientific achievements of this century".

Up to a few years ago, when the new sources of X-rays became available from electron accelerators and storage rings, X-rays were only produced by means of X-ray tubes. In the conventional X-ray tube the radia

tion is produced by bremsstrahlung of electrons as they strike the anode and by the deexcitation of electronic core levels in the atoms of the anode excited by incident electrons. Both processes are very inefficient, and more then 99 per cent of the electron energy is converted into heat. The dissipation of this heat sets practical limitations on the X-ray intensity that can be obtained from X-ray tubes. The spectrum consists of a continuous background with lines superimposed at fixed energies, and the angular distribution of emitted photons is approximately isotropic. Polarization is only partial and function of energy for the continuous part of the spectrum(3).

Synchrotron radiation suffers from none of these problems: in principle any intensity can be produced either in a continuous spectrum or in sharp tunable lines, it has excellent directional properties, well defined polarization and, in addition, it can be made to have a very fast time structure.

The availability on many order of magnitudes more intense, tunable sources (see also Fig.4) opens up entirely new scenarios of possibilities of interest to a multitude of disciplines and applications. This is well illustrated in Fig. 1 where fields of activity and experiment

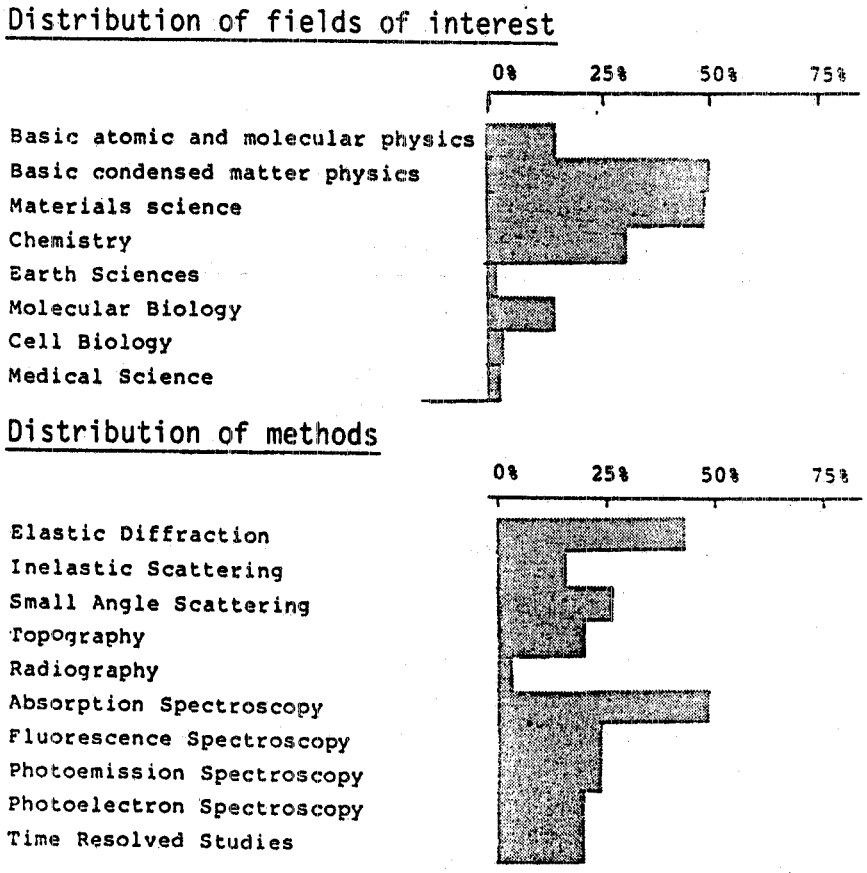


FIG. 1 - (From Ref.(2)).

al methods are listed together with their relative frequency of occurence at existing SR sources. It is sufficient here to mention that an extremely wide spectrum of problems, all of importance, ranging from energy technology to semiconductor device production, from the basics of cathalisis and corrosion to the properties of metals and alloys, from the detailed structure of protein molecules to preventive diagnostics of heart diseases (Fig. 2) can be studied.

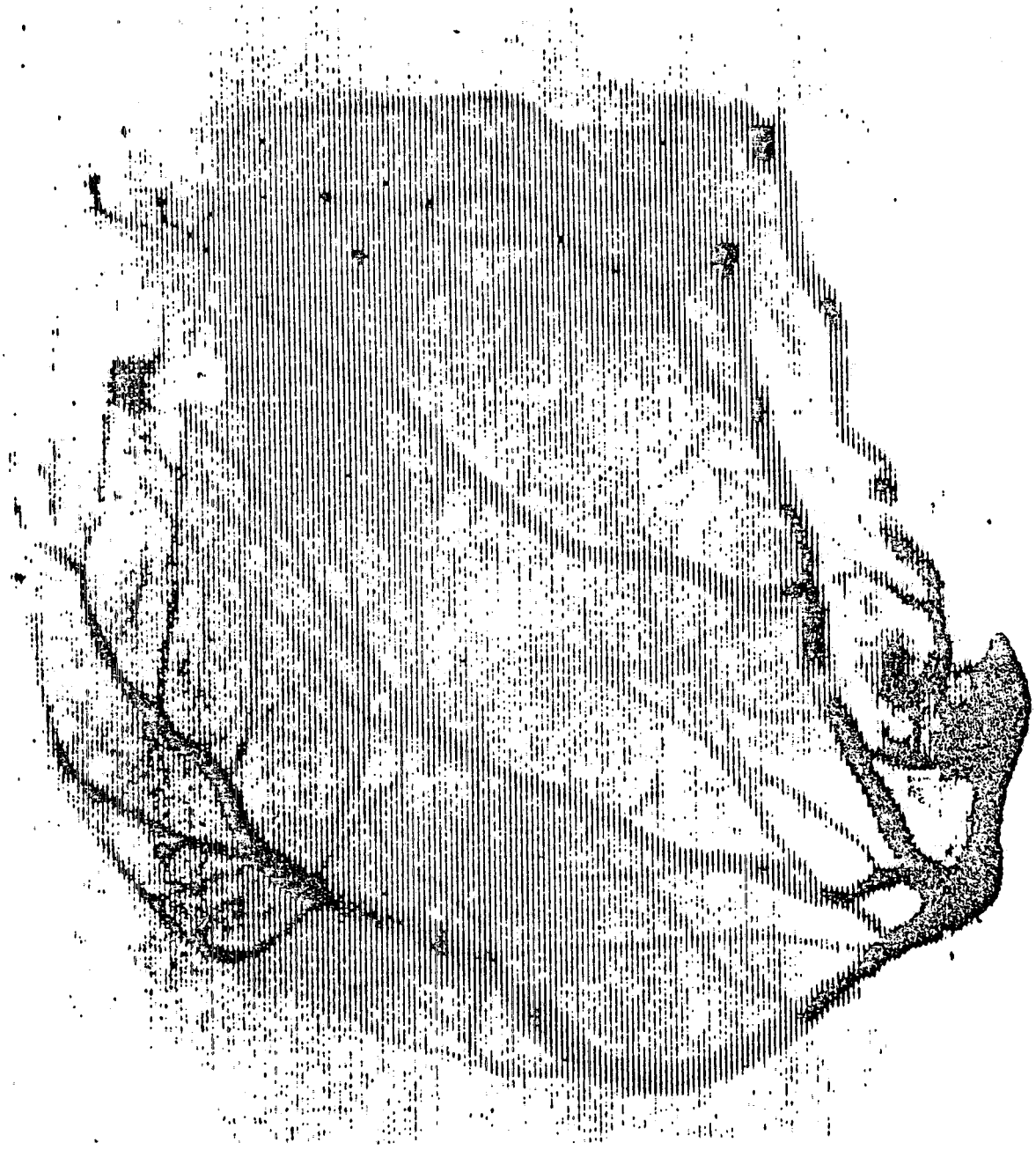


FIG. 2 - Synchrotron X-ray of the arteries in a pig's heart with a 20 mg/l concentration of iodine (From Ref.(2)).

The basic requirements of an experimenter can be summarized, with some degree of oversimplification, in just a few points:

a) The largest possible number of the desired energy photons should reach the sample so that samples containing fewer and fewer atoms (more dilute solutions, monoatomic surface layers, very small biological samples) can be studied.

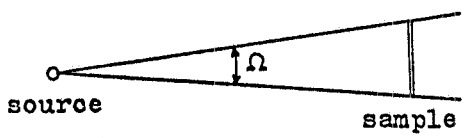
b) The photons reaching the sample should have the smallest possible divergence.

For unfocused beams this simply translates into the requirement of a high brightness, Φ_Ω, (see Fig. 3)

$$\Phi_\Omega = \frac{d^3n}{dt\ d\Omega\ (\Delta\lambda/\lambda)}$$

where $(\Delta\lambda/\lambda)$ is the desired energy bandwidth.

A. UNFOCUSED BEAM



Requirements:

a) a large number of photons should reach the sample
b) the beam should have a small divergence (diffraction experiments)

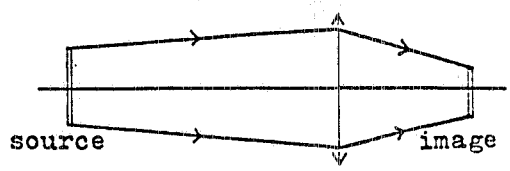
BRIGHTNESS

$$\Phi_\Omega = \frac{d^4n}{dt\ d\Omega\ (d\lambda/\lambda)}$$

n - number of photons;
t - time;
λ - wavelength;
Ω - solid angle

$$\left[\Phi_\Omega\right] = \frac{\text{photons/s}}{(\text{mrad})^2\ 0.1\%\ \text{band width}}$$

B. FOCUSED BEAM



Requirements:

a) a large number of photons should reach the sample (detector)
b) the image of the source should be small
c) the divergence of the beam reaching the sample (detectoor) should be small

BRILLIANCE

$$B = \frac{d^4n}{dt\ d\Omega\ ds\ (d\lambda/\lambda)}$$

s - source size;

$$B = \frac{\text{photons/s}}{(\text{mrad})^2(\text{mm})^2\ 0.1\%\ \text{band width}}$$

FIG. 3

If the beam is focused unto the sample, the smaller the source dimensions and divergence, the smaller the spot size and divergence of the photon beam at the sample. The figure of merit is now brilliance B (see Fig. 3)

$$B = \frac{d^4 n}{dt\, d\Omega\, dS(\Delta\lambda/\lambda)} .$$

B is the most important figure of merit for hard X-ray sources. The increase in brilliance obtained over the last decades from improving their design is dramatic and is shown in Fig. 4.

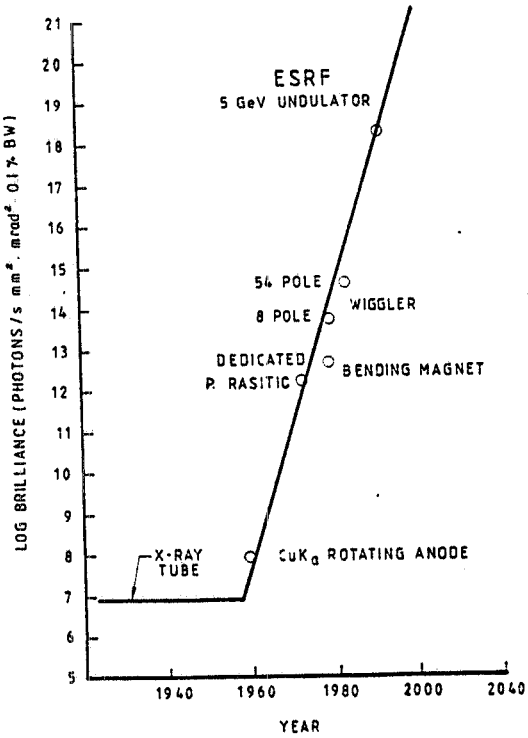


FIG. 4

3.- THE PRODUCTION OF SYNCHROTRON RADIATION

An electron beam traveling in a curved path will lose energy by radiation, the synchrotron radiation. In a storage ring the energy lost in a turn is restored by the RF system, so that the average electron beam energy, E_0, is a constant.

On storage rings built for HEP, synchrotron radiation generated in the bending magnets is a somewhat unwanted feature: the fast increase of radiated power with energy ($P \propto E^4/\varrho$ where ϱ is the bending radius) makes it expensive to move to very high energies (think of LEP with its 2 km of RF cavity).

High power and intensity are to the contrary welcome to SR users, who first set up their beam lines, looking at bending magnets, on practically all HEP rings.

The geometry of a bending magnet source is illustrated in Fig.5a. The spectrum is continuous and a "critical energy"

$$\varepsilon_c = \frac{3}{4\pi}\, \lambda_{ce}\, m_0 c^2\, \frac{\gamma^3}{\varrho_0} \qquad (\lambda_{ce}\text{: electron Compton wavelength})$$

is usually defined that divides the power spectrum into two equal halves (more details can be found in Refs.(4) and (5)).

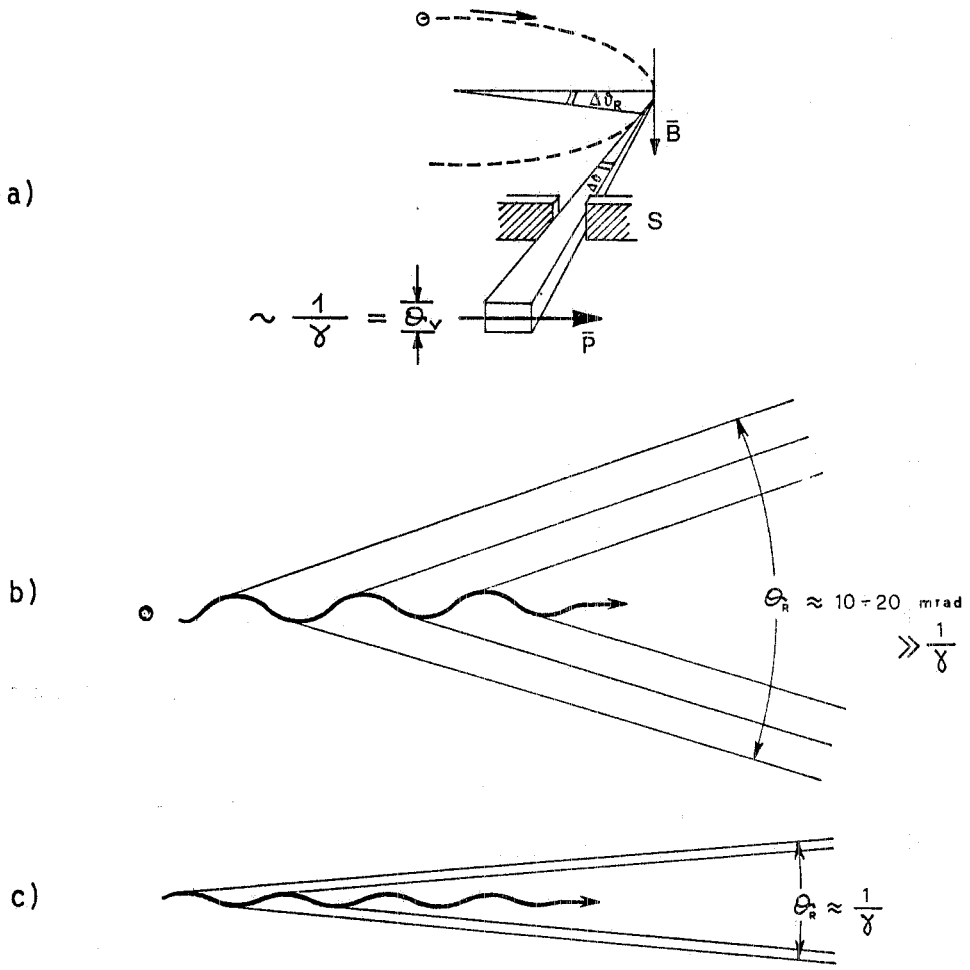


FIG. 5 - a) Bending magnet source; b) Wiggler source; c) Modulator source.

Since the vertical angular aperture of the emerging radiation fan is much smaller than its radial aperture (θ_V being of the order of $1/\gamma$) the number of photons per unit time and per unit horizontal aperture angle is often integrated over the vertical distribution, giving the "flux", Φ,

$$\Phi = \frac{d^2n}{d\theta_R\, dt} \quad .$$

The horizontal aperture is defined by slits, because the fan radiated by any given magnet is usually much larger than needed by any beam line.

Because of the natural vertical collimation and the small effective source size these sources are intrinsecally "brilliant" compared to X-ray tubes.

Note that on a given machine ϱ is a fixed quantity so that the critical energy of the radiation can only be changed by changing the operating energy. Also, increasing the experiment acceptance, θ_R, only increases the flux to the sample, without affecting the brilliance.

A way to improve on this is to use an "insertion device", i.e. a special magnet to be placed on a straight section of the main lattice. The main lattice should of course ideally "see" the device as a strainght section.

A sequence of alternating magnetic fields, a "wiggler" magnet, producing an orbit (usually lying in the horizontal plane) that starts and ends tangent to the main orbit is an obvious solution. It has the advantage that the local radius of curvature, ϱ_W, is now, to within reasonable limits, an independent variable so that the critical energy can be changed by changing the field (i.e. ϱ_W) even when the storage ring energy is kept fixed.

The horizontal radiation aperture angle from one "pole" is now:

$$\theta_M = \frac{K}{\gamma} \quad , \qquad K = \frac{eB_W\lambda_W}{2\pi m_0 c}$$

and depends on the wiggler magnetic field B_W, and spatial period λ_W. Depending on the value of K, θ_W can be much larger or of the order of (or indeed smaller than) the "natural" aperture of the radiation.

The magnet is called a "wiggler" if $K \gg 1$, otherwise it is an "undulator". Besides freeing the value of ϱ_W (usually higher fields than in the bends are used, to produce higher energy X-ray beams) (A single period very high field wiggler is often called a "wavelength shifter"), wigglers give higher flux than bending magnets, for the same overall power, because of the compressed fan and because several "poles" can radiate into the same fan angle (see Fig. 5b). For the same source size the brilliance is now also increased by (approximately) the number of poles N. For more details see Ref.(6).

When $K \lesssim 1$, the aperture of the radiation fan due to the wiggler in the trajectory becomes comparable with the natural aperture of the radiation (Fig. 5c); the magnet becomes an "undulator". Interference effects between the radiation beams from different poles set in. The spectrum is modified and discrete lines appear at wavelengths (in the forward direction)

$$\lambda_i = \frac{\lambda_u}{i\gamma^2}\left(1 + \frac{K^2}{2}\right) , \qquad (i = 1,3,5,\ldots) \qquad (1)$$

where λ_u is the spatial period of the magnetic structure and i the harmonic number (only odd harmonics appear in the forward direction).

The angular aperture of the radiation (from a single electron) is narrowed down, because of interference, to

$$\theta_u \approx 1/(\gamma\sqrt{N}) \tag{2}$$

so that, for large N, the brilliance can become extremely high.

4.- EFFECTIVE SOURCE SIZE

The radiation is produced by an electron beam that has a finite size in the four dimensional (transverse) phase space (x,x',z,z'). For a gaussian beam the envelope is an ellipsoid with standard deviations σ_x, σ'_x, σ_z, σ'_z.

In a storage ring the product $\varepsilon_x = \sigma_x \cdot \sigma'_x$, the betatron radial emittance, is a constant of the motion determined by the lattice; ε_z is proportional to ε_x through the coupling constant K of horizontal to vertical oscillations. The value of K can be controlled by means of the lattice tunes (or of coupling elements such as tilted quadrupoles) and its ultimate value depends on machine errors and imperfections; it is usually much less than 1 (0.1-0.01).

One has, at a point where α_x, α_z and the dispersion vanish:

$$\varepsilon_x = \sigma_x \cdot \sigma'_x = \sigma_x \frac{\sigma_x}{\beta_x} = \frac{\sigma_x^2}{\beta_x}, \qquad \varepsilon_z = K\varepsilon_x = \sigma_z \sigma'_z = \sigma_z \frac{\sigma_z}{\beta_z} = \frac{\sigma_z^2}{\beta_z}. \tag{3}$$

The values of σ_x, σ'_x, σ_z, σ'_z, are of course functions of the position along the machine. From (3) it is however clear that small beam sizes and divergences need small emittances. It is also seen that by properly choosing the β functions at the radiation source point, one can balance (within limits) beam dimensions against divergence or vice versa. In the vertical plane, by keeping the coupling small, beam sizes smaller than in horizontal can be achieved. K values as small as 0.01 have been obtained on existing machines.

Because of diffraction effects, a radiation beam of wavelength λ_R also has a finite intrinsic emittance, the same in both planes, given by

$$\varepsilon_R = \lambda_R = \sigma_R \cdot \sigma'_R . \tag{4}$$

Since the actual source dimensions are, in the end, the r.m.s. combination of the electron beam dimension, σ_B, and the diffraction values, σ_R (for gaussian beams):

$$\sigma_{effective} = (\sigma_B^2 + \sigma_R^2) \tag{5}$$

in all four dimensions. The beam emittance has to be compared with the radiation one, and should ideally be the smallest to achieve the highest brilliance in the beam.

When dealing with X-rays of the order of 10 KeV the corresponding wavelength is $\lambda_R \approx 1$ Å $= 10^{-10}$ M: this sets the scale for the ideally desirable beam emittance. It is found that achieving $\varepsilon_x = 10^{-10}$ m is very difficult (although ideally not impossible) even for a perfect lattice and in the zero current limit.

Note that, when discussing undulators where interference effects became important we have implicitely assumed that the beam angular spread would be smaller than $\sigma_R' = 1/(\gamma\sqrt{N})$. If not, interference will tend to be washed out.

Since $\varepsilon_B \gtrsim \varepsilon_R$ while ideally one would like to have

$$\sigma_B' = (\varepsilon_B/\beta)^{1/2} \lesssim \sigma_R' \tag{6}$$

a rather large β should be provided at places where one wants to put undulators.

On the contrary, for a wiggler magnet or a bending magnet source the beam angular spread plays hardly any role because it is usually much smaller than the aperture of the fan of radiation produced by the curvature in the trajectory. The objective is then to have $\sigma_B \leq \sigma_R$ and this calls for low values of β.

5.- LOW EMITTANCE LATTICES

The horizontal emittance in a storage ring is given by the expression

$$\varepsilon_x = C\, E^2 \frac{\langle H/\varrho^3 \rangle}{\langle 1/\varrho^2 \rangle} \tag{7}$$

where $\langle\ \rangle$ means the average around the storage ring (in practice in the magnets because $\varrho \to \infty$ elsewhere). H is called the lattice invar-

iant and is given by

$$H = \gamma D_x^2 + 2\alpha D_x D_x' + \beta_x D_x'^2 \tag{8}$$

where D_x is the horizontal dispersion function. Note that for an isomagnetic lattice ($\varrho = \varrho_0$ = constant) eq. (7) becomes

$$\varepsilon_x = C_q(\gamma^2 \langle H \rangle)/(J_x \varrho_0) \ , \qquad C_q = 3.84\text{x}10^{-13}\ \text{m} \tag{9}$$

(J_x is the damping partition number, usually ~ 1 and E has been replaced by the dimensionless γ).

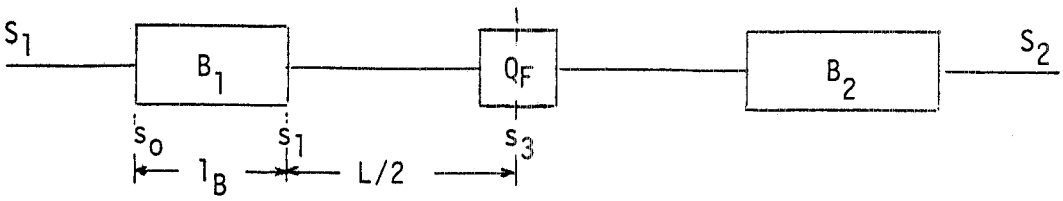
From eq.(9) it is clear that to make ε_x small one should make $\langle H \rangle$ small. (Since $\langle H \rangle$ is proportional to ϱ_0, see e.g. eq.(13), the bending radius drops out of eq.(9)).

From eq.(8), one sees that to make $\langle H \rangle$ small, D_x (but also D_x', α_x, β_x) should be small in the magnets. The physics is that dispersion should be small at places where photons are radiated because the energy change associated with the emission of a photon causes the onset of betatron oscillations (proportional to D_x) that contribute to the equilibrium of ε_x[10].

We will see in the following that minima for H can be found for various types of lattices, involving proper choices for D_x, β_x and their derivatives in the bending magnet.

A very simple lattice, that has low dispersion in the bending magnet and therefore can be made to have a low emittance is the so called "double focusing achromat" (a Chasman-Green lattice from the names of the physicists who first proposed it).

Its basic "cell is shown below



B_1, B_2 are bending magnets and Q_F is a focusing quadrupole. Some additional focusing is of course needed on either side of the basic cell. The dispersion D_x is zero in straight sections S_1 and S_2, so that the basic cell is an "achromat": note that the cell is symmetric around s_3, so that in fact the basic lattice building block is the first half-cell. Zero dispersion in S_1, S_2 is also useful because RF,

injection and insertion devices are best located in dispersion free straights.

The dispersion D_x obeys the equation

$$D_x''(s) + g(s)D_x(s) = \frac{1}{\varrho(s)} \tag{10}$$

so that, when one assumes $D_x(s_o) = D_x'(s_o) = 0$, and considers a uniform field magnet ($n = 0$, $\varrho = \varrho_o = \text{const}$) it is easy to find that

$$D_x(s_1) = \varrho_o(1 - \cos\varphi_B) \ , \qquad \varphi_B = \frac{l_B}{\varrho_o} \tag{11}$$

$$D_x'(s_1) = \sin\varphi_B \ .$$

The symmetry condition $D_x'(s_3) = 0$, determines (for a given L/2) the strength of quadrupole Q_F that makes the cell achromatic.

As mentioned above, quadrupole doublets (or triplets) can be inserted in S_1 and S_2 to shape the β functions. Since D_x is excited only by the bending field, these quadrupoles do not affect the dispersion.

The emittance of a lattice built of such cells can be easily computed[7]. It is found that when β_x is made to have a minimum value

$$\beta_{x_{min}}(s^*) = \frac{\sqrt{3}}{B\ \sqrt{5}}\ l_B \tag{12}$$

at a point $s^* = s_o + 0.375\ l_B$. The invariant $\langle H \rangle$ has a minimum

$$\langle H \rangle_{min} = \varrho_o \frac{\varphi_B^3}{4\ \sqrt{15}} \tag{13}$$

and the emittance has a minimum

$$\varepsilon_{x_{min}} = C_q \gamma^2\ \varphi_B^3/(4\ \sqrt{15}) \tag{14}$$

in practical units

$$\varepsilon_{x_{min}} = 2.48\text{x}10^{-14}\ \gamma^2\ \varphi_B^3\ \ (\text{m}\cdot\text{rad}) \ . \tag{15}$$

Note that by decreasing φ_B, i.e. by increasing the number of cells, the emittance can be made to decrease very fast at any given energy. However the required value of $\beta_{x_{min}}$ also decreases fast and soon becomes difficult to achieve (or gives unacceptably high values

of β_x at the magnet ends).

For instance to achieve an emittance $\varepsilon_{x_{min}} = 10^{-9}$ m·rad at 5 GeV with a radius ϱ_0 = 20 m, one would need approximately 42 cells (84 bending magnets, each ~1.5 m long) and a value $\beta_{x_{min}}$ = 0.15 m. Strong focusing is required to obtain such low β's at very many places (84) around the machine, and a large β oscillations, giving large chromaticities (see par. 6) are obtained.

It is shown in Ref. (7c) that to end up with a reasonably well behaved lattice the emittance has to be increased by a factor of two to three over the ideal value given by eq.(15).

Also note that lowering the emittance tends to increase the machine circumference. Eventually very low values of φ_B make it even difficult to extract the radiation from the machine because of the interference of beam ports with the lattice elements.

As an example, a variant of such a lattice designed for the European Radiation Facility(8) is shown in Fig. 6.

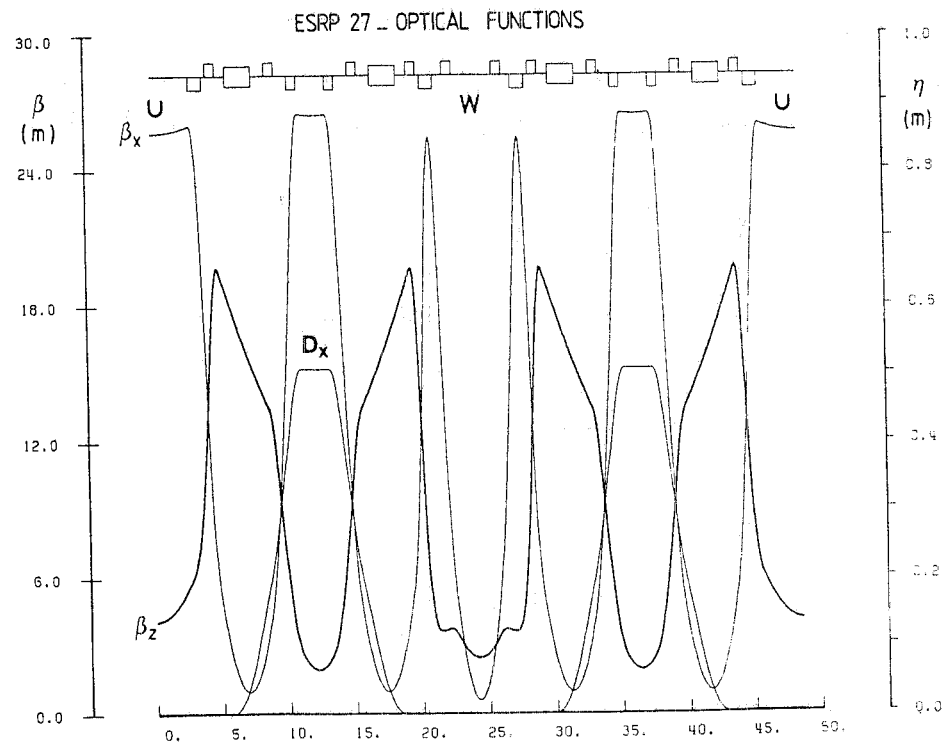


FIG. 6 - Double focusing achromat lattice for the ESRF: 1/16 of the ring.

It has 32 cells and gives an emittance of ~7×10^{-9} m·rad, about three times as large as predicted from eq. (15), but with $\beta_{x_{min}}$

of the order of 1 m.

The horizontal beam size in the bending magnet has a minimum of less than 100 μm. Notice that many ($\sim$ 30) long straight sections are provided for insertion devices, and that in those marked U (undulator) the β functions tend to be rather high (especially β_x) while in those marked W (wiggler), β's tend to be low, for the reasons explained in par. 4.

The dependence of emittance on the product $\gamma^2 \varphi_B^3$ exibited by eq. (14) is not peculiar to the Chasman-Green lattice, but is a rather general property.

For instance the emittance of the regular FODO lattices used in all large accelerators is[14]:

$$\varepsilon_x = 4 \frac{C_q}{J_\varepsilon} F(\mu_c)\gamma^2 \varphi_B^3 = 7.68\text{x}10^{-13} F(\mu_c)\gamma^2\theta^3 \ . \qquad (16)$$

$F(\mu_c)$ is a function of the betatron phase advance per cell, μ_c, and is given by:

$$F(\mu_c) = \left[1 - \frac{3}{4}\sin^2\frac{\mu_c}{2} + \frac{1}{60}\sin^4\frac{\mu_c}{2}\right] \Big/ \left(\sin^2\frac{c}{2}\sin\mu_c\right). \qquad (17)$$

It has a minimum at around μ_c = 3 /4, where its value is $F(3\pi/4)$ = = 0.62. The minimum is however rather flat in the range $100° \leqslant \mu_c \leqslant$ $\leqslant 160°$. In this range of phase advances

$$\varepsilon_{x_{min}} \approx 5\text{x}10^{-13}\gamma^2 \varphi_B^3 \quad (\text{m}\cdot\text{rad}) \ . \qquad (18)$$

Notice that the coefficient is about 20 times larger than that in eq. (15) meaning that for the same minimum emittance one needs about 3 times as many bending magnets.

In practice however it is easier to come close to the minimum emittance with a FODO lattice than with a Chasman-Green type and the factor on the number of magnets is generally less than or close to 2.

When designing a machine with a very large number of straights while trying to keep the circumference reasonable, not many FODO cells are allowed in between straights, and it is rather hard, especially if you are accustomed to HE colliding beam storage ring lattices, to even recognise the FODO structure in the lattice of Fig. 7, also proposed as an alternative for the ESRF.

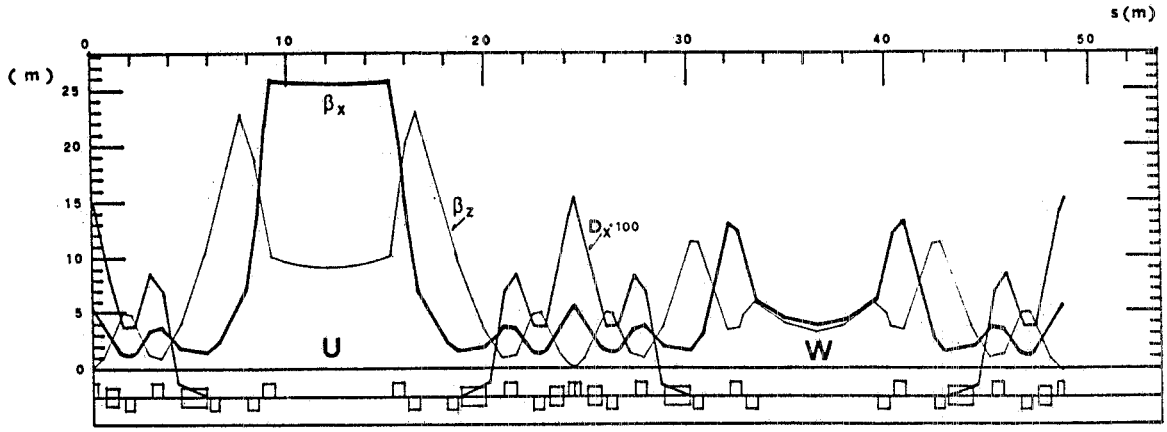


FIG. 7 - A FODO type lattice for the ESRF.

However its basic properties, and notably the regular behaviour and the low values of β_x, β_z in the "arcs" are recognizeable.

Note that $\mu_c = 3\pi/4$ implies very strong focusing, and that chromaticity problems may be expected here too. Very strong focusing is of course inherent in the concept of all low emittance lattices.

As a side remark (but more can be found in Ref.(8) and its bibliography) note that low emittance lattices, because they usually have low D_x, also have very low momentum compaction factor, α_c. Recall that the linear part of α_c is given by

$$\alpha_c^{(o)} = \frac{1}{L} \oint \frac{D_x(s)}{\varrho(s)} ds \ . \tag{19}$$

For the ESRF, $\alpha_c^{(o)} = 3 \times 10^{-4}$. Nonlinear terms may therefore become important and have to be watched.

The low value of $\alpha_c^{(o)}$ of course also affects bunch length (very short bunches can be obtained, at least for vanishing current).

6.- SOME OF THE PROBLEMS TO BE SOLVED IN CONNECTION WITH LOW EMITTANCE LATTICES

6.1.- Chromaticity correction

One of the foremost problems is the correction of the lattice chromaticity (see for instance Ref.(9)).

The natural (linear) chromaticities are found, from the linearized equations of motion, to be:

$$\frac{\Delta Q_{x,z}}{(\Delta p/p)} = \xi_{T_{x,z}} = \frac{1}{4} \oint \beta_{x,z}(s)\ K_{x,z}(s)\ ds \tag{20}$$

where $\Delta Q_{x,z}$ is the tune shift suffered by an off momentum particle, and $K(s) = G(s)/(B_0 \varrho_0)$ is the normalized quadrupole gradient.

The physics is very simple: given the quadrupole gradients, K(s), that give the right Q for an on-momentum particle, a particle with higher energy will be less strongly focused (and vice versa). A negative value of ξ_T will in general be obtained.

The strong focusing required to obtain a low emittance implies strong gradients and, often, also rather large values of $\beta(s)$, so that ξ_T tends to become large. In the lattices we have been considering a value of $\xi_T \sim 50$-100 is not unusual meaning that, unless it is corrected, a particle only a few standard deviations away from the average energy will travel through strong resonances and be lost.

Keep in mind that eq.(20) is only the linear part of the chromaticity: second and higher order effects may become important when ξ_T is large.

The way to correct the linear chromaticity is to add extra momentum dependent focussing. A sextupolar field placed where the dispersion is non zero will do just that. More precisely its correction term is:

$$\xi_S = \frac{1}{4} \oint \beta(s)\ D_x(s)\ K_s(s) \tag{21}$$

where $K_s(s)$ is the normalized sextupolar gradient. It has to be chosen in such a way that ξ_S has the opposite sign with respect to ξ_T, in both planes

$$K_s(s) = \frac{G_s}{B_0 \varrho_0}\ , \qquad G_s = \frac{1}{2}\frac{B''}{a^2}\ . \tag{22}$$

Note that ξ_S has to be large to compensate for the large ξ_T, and that D_x is usually everywhere small. This means that strong sextupoles are needed.

The sextupolar fields introduce strong nonlinearities in the equations of motion and the linear approximation is no longer sufficient. To second order in $(\Delta p/p)$ changes of the β-functions and of the dispersion with momentum have to be taken into account.

Non chromatic and higher order effects, such as a Q shift depending on the betatron oscillation amplitude are also found. These can be corrected by sextupoles in dispersion-free regions of the machine.

Finally, resonances are excited by the non linear terms (e.g. the third order excited by sextupolar fields).

Unless all these effects are properly corrected the net result is that the phase space volume inside which particles are stable, the dynamic aperture, tends to become vanishingly small.

Injection then becomes very difficult or impossible, and lifetimes become insufficient.

In order to obtain reasonably large dynamics apertures, complicated arrangements of separately powered sextupoles have to be found for each lattice.

For instance the ESRP lattice shown in Fig. 6 requires five separate sextupole families (rather than the two necessary to correct the linear chromaticity only) and the lattice of Fig. 7, about seven families. The arrangement of sextupoles and the limits of the dynamic aperture are studied by tracking many particles around the ring (of course in a computer simulation) for many hundreds of turns.

The dynamic aperture of the ESRP lattice (ESRP-27), and of a "detuned" version having twice the emittance (ESRP-30), is shown in Fig. 8. Note that it shrinks for off-momentum particles ($\Delta p/p \neq 0$). Fig.9 shows two examples of phase-space plots, obtained by tracking particles with large initial amplitudes of oscillation (near the dynamic aperture limits). The position in phase space of the particle is recorded turn by turn, at a given azimuth along the ring (in the example: the center of a long straight section).

Note how high order resonances distort the shape of the phase space trajectory, which would be a perfect ellipse if the lattice were linear.

The problem of optimizing the chromaticity correction is central to the design of low emittance rings, and is still attacked largerly by trial and error since a complete theory is lacking.

6.2.- Lifetimes

The lifetime of the beam in a storage ring is determined by several effects: radial and energy oscillations[10], bremsstrahlung[8] and scattering on the residual gas atoms, beam-beam bremsstrahlung, intrabeam scattering, etc.

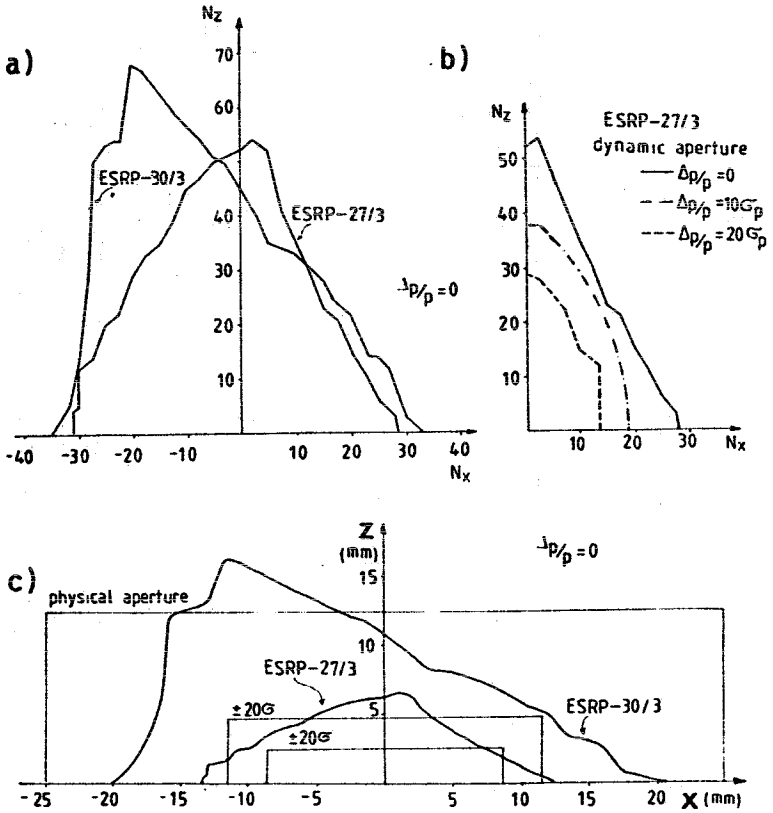


FIG. 8 - Dynamic apertures: ESRF lattices 27/3 and 30/3.

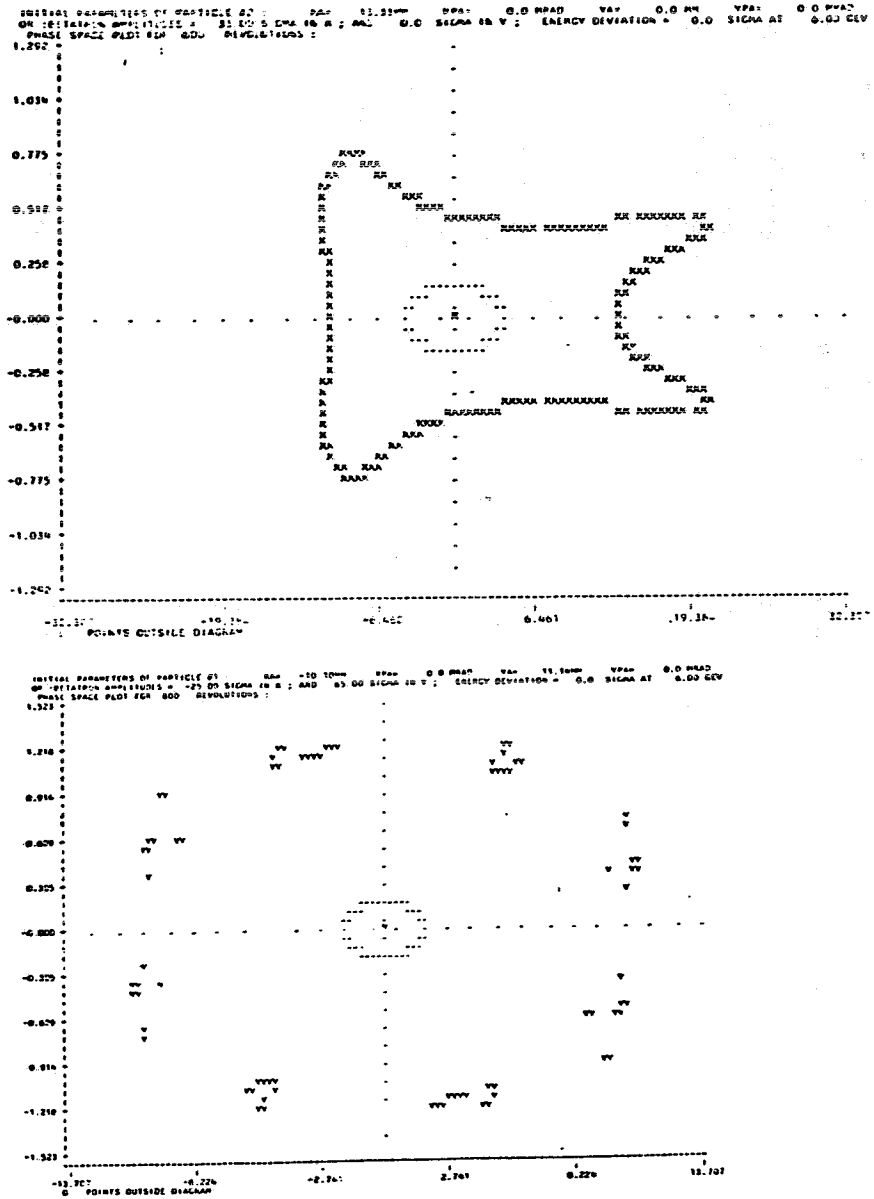


FIG. 9 - Tracking with sextupoles: ESRF lattice 27/3.

One is used in high energy colliding beam facilities to consider bremsstrahlung as the main mechanism leading to the loss of particles.

However for low emittance rings, when the dynamic aperture and the vacuum chamber aperture are much smaller than usual, scattering on the residual gas and intrabeam scattering have to be considered and may become dominant.

Let us consider scattering: a particle is lost when it is scattered through and angle greater than that limiting the dynamic aperture at the position where the scattering event takes place, or such that the ensuring oscillation amplitude will take it across the boundary determined by the physical chamber aperture. Let $(d\sigma/d\Omega)_i$ be the corresponding differential cross-section for an atom of species i and let n_i be the density of scattering atoms of species i in the residual gas.

The scattering lifetime, τ_s, is then given by

$$\frac{1}{\tau_s} = C \sum_i n_i \int_\Omega \left(\frac{d\sigma}{d\Omega} \right)_i d\Omega \tag{23}$$

where $d\Omega$ is the solid angle over which the particle is lost.

If the machine acceptance is limited by the dynamic aperture, represented in real space by an ellipse having half axes $N_x\sigma_x$ and $N_z\sigma_z$ (with N_x, N_z integers) it can be shown[11] that:

$$\frac{1}{\tau_s} = \frac{4cr_e^2K}{\gamma^2} \frac{\pi}{2} \left(\frac{\bar{\beta}_x}{N_x^2\varepsilon_z^2} + \frac{\bar{\beta}_z}{N_z^2\varepsilon_z^2} \right) \Sigma n_i p_i z_i^2 \tag{24}$$

where $\bar{\beta}_x$, $\bar{\beta}_z$ are averages over the machine and z_i is the atomic number of the scattering atom. Note that when ε_x, ε_z, N_x, N_z become small and, possibly, $\bar{\beta}_x$, $\bar{\beta}_z$ large, τ_s decreases rapidly. Since n_i is proportional to the residual gas pressure, great care has to be taken in the design of low emittance SR, to have low residual gas pressures (in the range of 10^{-9} torr), large dynamic apertures, and the lowest possible values of $\bar{\beta}_{x,z}$, z_i.

Intrabeam scattering causes both a loss of particles (in longitudinal phase space) and a diffusion process in the six dimensional phase space (see A. Piwinski's lectures, in these Proceedings). Diffusion in competititon with damping will lead to new equilibrium beam dimensions and the emittance may blow up compared to the single particle value. In its simplest form it was discovered quite same time ago in AdA, the first e^+e^- storage ring built at Frascati[12,13].

The loss rate is determined by the equation

$$\frac{dn_b}{dt} = -\alpha n_b^2 \tag{25}$$

where

$$\alpha = \frac{4\pi r_e^2 c}{\gamma^2 \varepsilon_p} \left\langle \frac{J(\varepsilon_p, p_\perp)}{8\pi^{3/2}\sigma_x\sigma_y\sigma_z} \right\rangle \tag{26}$$

n_b is the number of particles in a bunch, ε_p the momentum acceptance, $p_\perp = \gamma\sigma_x'$, and $J(\varepsilon_p, p_\perp)$ a complicated function of ε_p and $p_\perp$. The brackets indicate the average over the ring. Given the lattice, α has to be computed numerically. Again note that when σ_x, σ_y, σ_z are small and σ_p (determined by the dynamic acceptance) is also small, $\tau_T = 1/\alpha$ decreases very rapidly.

At low energies the radiation damping time becomes long and diffusion processes start to blow the beam up. This tends to increase the lifetime but at the expense of emittance. As an example lifetimes emittance and energy spread computed for the ESRF are shown in Fig. 10 as functions of the operating energy at a current of 4 mA/bunch.

Intrabeam scattering sets a limit on the emittances and beam dimensions obtainable at low energies for a given current. At high energy it limits the current that can be stored in a bunch with accetable lifetime.

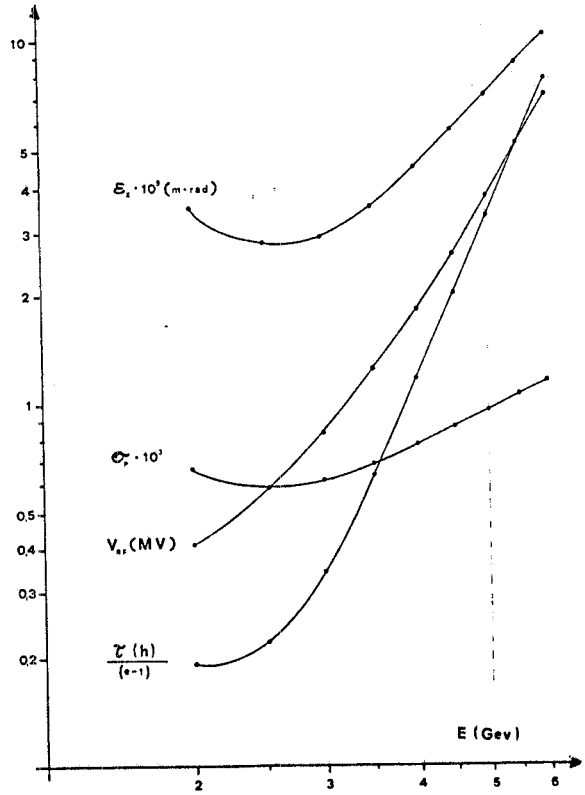


FIG. 10 - ESRF: Touschek lifetime, emittance, and momentum spread versus operating energy.

6.3.- Effects of wigglers and undulators

Wigglers, and to a lesser extent undulators, besides providing the desired radiation beams also produce side effects that have to be con

sidered.

It can be shown[15] that end fields of a wiggler magnet produce a net focusing in the vertical plane. The fact that wigglers are usually located at places where β_z is low tends to diminish the effect on the lattice, but proper corrections have still to be provided.

More important are usually the effects on the momentum spread, the damping time and the emittance.

The additional radiation produced in a wiggler magnet will tend to increase the quantum fluctuations and lower the damping time. Depending on the value of the dispersion function at the wiggler location (but be careful: the self-generated dispersion has to be taken into account!) one or the other effect will tend to dominate and the beam emittance may either increase or decrease.

If the dispersion is zero (or small) a wiggler will actually decrease the beam emittance. As an example the effect of a 72 m overall wiggler length on the emittance of the ESRF is given in Fig. 11a as

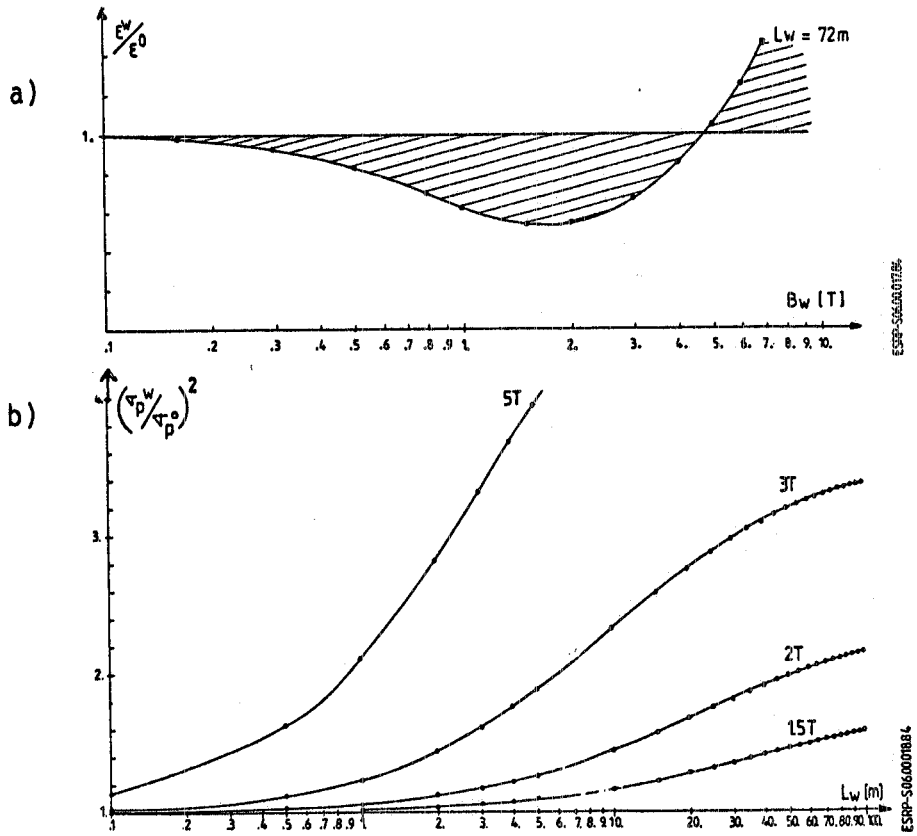


FIG. 11 - a) ESRF: effect of wigglers on emittance. ε_0 is the unperturbed value ; b) ESRF: effect of wigglers on momentum spread.

a function of the wiggler peak magnetic field. The lattice dispersion is zero and the emittance decreases up to the point where the dispersion generated by the wiggler itself takes over.

To the contrary the momentum spread is always increased. Fig. 11b shows the behaviour of σ_p for the ESRF, at various wiggler peak fields, as a function of the overall magnet length. This effect can become a problem when the momentum acceptance is limited by nonlinearities.

Undulators, having lower field and shorter periods, produce lesser effects. However lifetime problems may arise when very small gaps are required[8] to obtain the desired field values.

Last, nonlinearities in the wiggler/undulator fields may also be a problem, and, for machines designed to have very many of these devices, their effect has to be assessed in detail.

6.4.-

As a further point it should be mentioned that the problem of keeping many tenths of beam lines alligned to very high precision over long periods of time and long distances is one of the main worries of the designer. Alignment tolerances and closed orbit detection and correction require state of the art techniques. Also, the stability of the ground in the presence of microseismic activity due to man-produced noise (such as traffic) and natural causes (wind, seisms, waves in the ocean) has to be carefully assessed.

7.- THE EUROPEAN SYNCHROTRON RADIATION FACILITY

A European facility dedicated to the production of hard X-rays has been the subject of much study in the past five or six years. Feasibility studies were started as far back in 1979 under the auspices of the European Science Foundation[16]. The latest work, funded by a number of European Governements and hosted by CERN, was carried out by the ESRP[8]. It is assumed to provide enough information for the construction to be launched in the near future. Its main parameters are listed in Table I, and a general view of the lattice is shown in Fig. 12.

TABLE I - ESRF 1984 (Lattice 27)

Electron energy	5 GeV
Electron current	100 mA (multibunch) Possibility to increase to 200-300 mA
Number of straights for insertion devices	30
Circumference	770 m
Critical wavelength:	
Bending magnets	0.9 Å
Multipole wigglers	from 0.5 Å to several Å
Wavelength shifters	from 0.5 Å to 0.2 Å
Minimum wavelength for the fundamental from undul.	0.86 Å
Mode of operation	multibunch with a possibility of single bunch operation
Emittance	$\varepsilon_x = \pi \times 7 \cdot 10^{-9}$ rad·m
Straight sections length	6 m
Full length for undulators:	6 m
β = 25.8 m , σ_x = 0.43 mm , σ'_x = 0.017 mrad	
Full length for multipole wigglers	3 m
β = 0.57 m , σ_x = 0.064 mm , σ'_x = 0.113 mrad	
Great flexibility for replacement of an undulator by a multipole wiggler and vice versa.	
Maximum number of wiggler/undulators	28-30
Injector	5 GeV, 10 Hz Synchrotron
Preinjector	microtron (e^-) or linac (e^+)

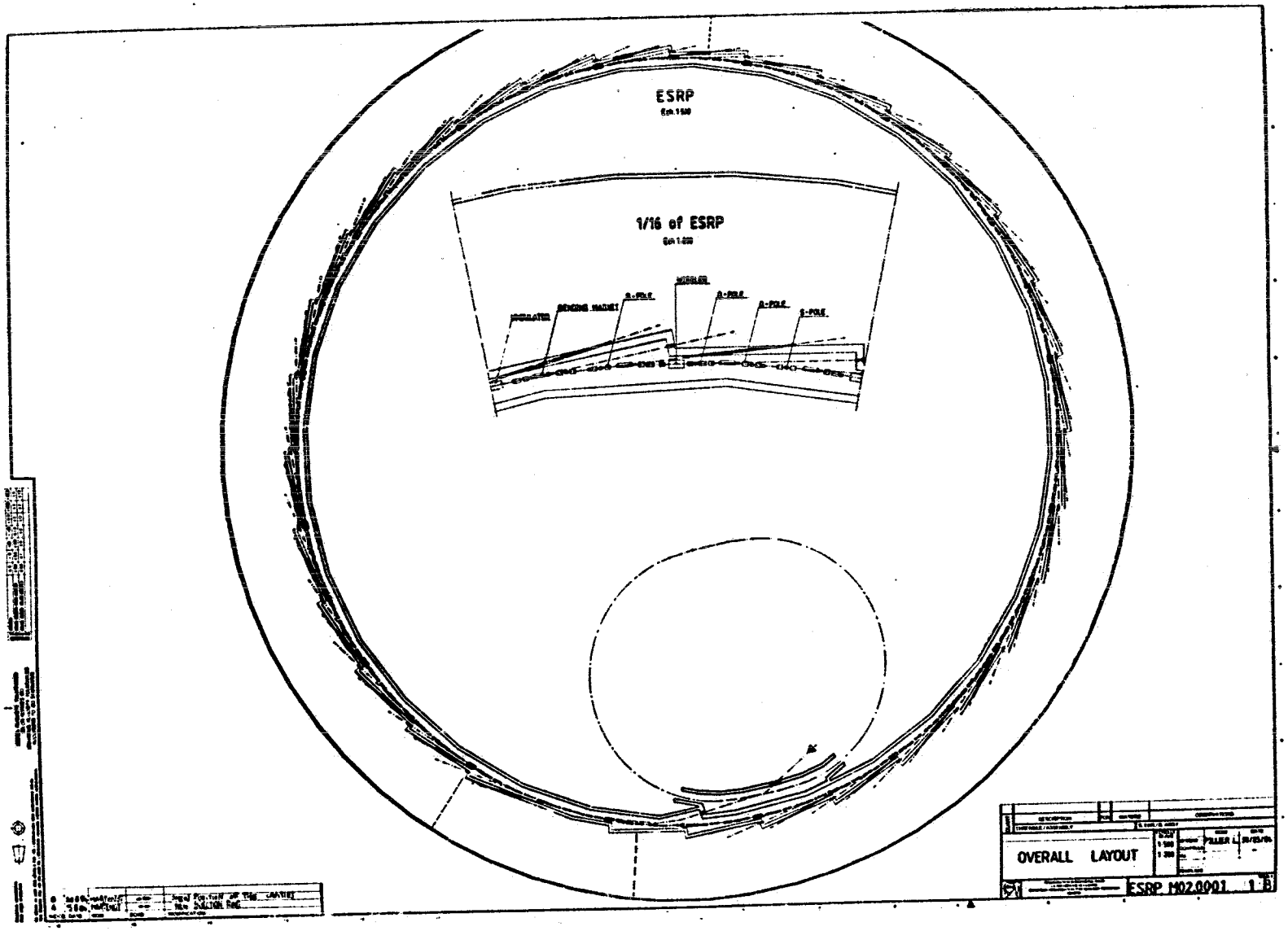


FIG. 12 - A general view of the ESRF.

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