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F. Palumbo:

QUANTIZATION OF GAUGE THEORIES ON A TORUS

## QUANTIZATION OF GAUGE THEORIES ON A TORUS

F. Palumbo  
INFN - Laboratori Nazionali di Frascati, Frascati, Italy

### ABSTRACT

Field theories on a torus are characterized by the existence of zero-momentum modes. In the case of gauge theories these modes make more complicated the solution of the Gauss constraint. This difficulty is overcome by using a gauge in which the constraint on zero-momentum modes decouples from the constraints on the other modes. The resulting unregularized Hamiltonian is shown to be unbounded from below both in the abelian and nonabelian case. The only known regularization which can prevent this instability is to give a mass to the gauge field.

### 1.- INTRODUCTION

A specific feature of field theories on a torus is the existence of zero-momentum modes ( $z_{mm}$ ). These modes can play a special role in the thermodynamic limit. While the equations involving other modes go smoothly into the continuum when the volume of the torus goes to infinity, the equations involving  $z_{mm}$  must be treated separately to investigate whether the measure can develop a singularity at zero-momentum. A well known example is Bose-Einstein condensation in statistical mechanics. Another example is provided by abelian gauge theories where due to  $z_{mm}$  an energy scale

can be introduced<sup>(1)</sup> through radiative corrections.

In non abelian gauge theories zmm can be expected to be even more important due to the more complicated infrared structure. This point has been fully appreciated by some authors, who have emphasized the possible role of classical zmm<sup>(\*)</sup> (torons) in determining the structure of the vacuum<sup>(2,3)</sup>. Lüscher<sup>(3)</sup> has also proposed an effective Hamiltonian in the Fock space generated by zmm .

The purpose of this paper is to perform the canonical quantization of gauge theories on a torus taking zmm into proper account. zmm enter the Gauss constraint and make more difficult its solution. This difficulty is overcome here by introducing a gauge where the constraint for zmm decouples from the constraint on other modes. The resulting Hamiltonian is unbounded from below when the gauge fields are coupled to fermionic matter both in the abelian and nonabelian case. This instability is not present if the coupling is with bosonic matter only, but cannot be prevented if in addition to bosonic matter there is fermionic matter, so that it is relevant to Supersymmetric theories.

This vacuum instability is an infrared problem, and we know that gauge theories must be regularized in the infrared. As a consequence of the present result, however, the only acceptable regularization on a torus is to give a mass to the gauge field<sup>(4)</sup> (to replace the gauge multiplet by a massive vector multiplet in Supersymmetric theories).

The instability takes place through the coupling of zmm to the volume average of the fermion current. If such an average vanishes, the Hamiltonian is bounded from below, so that there is no such a problem in the presence of color confinement. The problem remains, however, in perturbation theory.

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(\*) We call them classical because they are defined by  $H_i = E_i = 0$ .

The same instability related to zmm affects the theory of a spinor and a massless scalar with Yukawa coupling, but in this case it can be removed by adding a quartic scalar self-interaction, which is also necessary to make the theory renormalizable. What makes gauge theories peculiar with respect to zmm is that there is no interaction term which can regularize them.

The paper is written in the following way. In Sec. 2 we perform the quantization in the abelian case and show that the Hamiltonian is not bounded from below. In Sec. 3 we perform the quantization in the non abelian case, in Sec. 4 we prove the vacuum instability, and in Sec. 5 we summarize our results and conclusions. In the Appendix we discuss a difficulty in the Lüscher<sup>(3)</sup> approach due to the use of the Coulomb gauge.

## 2.- THE ABELIAN CASE

The Hamiltonian density of an abelian gauge field interacting with a fermion field is<sup>(+)</sup>

$$\mathcal{H} = \frac{1}{2} E^2 + \frac{1}{2} H^2 + g A_k j_k + \mathcal{H}_F \quad (1)$$

where  $j_k$  and  $\mathcal{H}_F$  are the current and Hamiltonian density of the fermion field,

$$H_i = \frac{1}{2} \epsilon_{ijk} \partial_j A_k \quad (2)$$

and  $E_k$  is the momentum conjugate to the gauge field  $A_k$

$$\{A_h(\vec{x}), E_k(\vec{y})\} = \delta_{hk} \delta^3(\vec{x} - \vec{y}). \quad (3)$$

We first consider the theory at the classical level and therefore the l.h.s. of eq.(3) is the Poisson bracket of  $A_h$  and  $E_k$ . These variables are subject to the Gauss constraint

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(+) Repeated indices should always be understood summed over.

$$\Phi = \partial_k E_k + g j_0 = 0 . \quad (4)$$

Since we perform the quantization on a torus we can expand  $A_h$  and  $E_k$  in Fourier series

$$A_k = \frac{1}{L^{3/2}} \left[ Q_k + \sum_{\vec{n} \neq 0} A_{k\vec{n}} e^{i \frac{2\pi}{L} \vec{n} \cdot \vec{x}} \right] \stackrel{\text{def}}{=} \frac{1}{L^{3/2}} Q_k + \bar{A}_k , \quad (5)$$

$$E_k = \frac{1}{L^{3/2}} \left[ P_k + \sum_{\vec{n} \neq 0} E_{k\vec{n}} e^{i \frac{2\pi}{L} \vec{n} \cdot \vec{x}} \right] \stackrel{\text{def}}{=} \frac{1}{L^{3/2}} P_k + \bar{E}_k .$$

The Fourier coefficients must satisfy the reality conditions

$$A_{k\vec{n}} = A_{k, -\vec{n}}^* , \quad E_{k\vec{n}} = E_{k, -\vec{n}}^* .$$

It follows from eq.(3) that the zmm  $Q_k$  and  $P_k$  satisfy canonical Poisson brackets.

Introducing also for the current density a decomposition analogous to (5)

$$j_k = \frac{1}{L^{3/2}} I_k + \bar{j}_k \quad (6)$$

we have for the Hamiltonian

$$H = \frac{1}{2} P^2 + g Q_k I_k + \int d^3x \left[ \frac{1}{2} \bar{E}_k \bar{E}_k + \frac{1}{2} H^2 + g \bar{j}_h \bar{A}_k + \mathcal{H}_F \right] . \quad (7)$$

The quantization is now achieved by eliminating the redundant variables by a gauge fixing and by replacing the remaining variables by quantum operators. We do not need to specify the gauge fixing for the present purposes. Let us only emphasize that it cannot act on  $Q_k$  which is gauge invariant because  $\Phi$  commutes with it.

It is now obvious that the Hamiltonian (7) is unbounded from be-

low. This can be checked by taking its expectation value in the state

$$\Psi = (\pi Q_0^2)^{-\frac{1}{4}} \exp \left[ -\frac{(Q_k - \bar{Q})^2}{2Q_0^2} \right] \chi \quad (8)$$

where  $\chi$  is a state functional which does not depend on  $Q_k$ , such that

$$\langle \chi | I_k | \chi \rangle = \delta_{k3} I, \quad I \neq 0.$$

Therefore for  $\bar{Q} \rightarrow \infty$

$$\langle \Psi | H | \Psi \rangle \rightarrow g I \bar{Q}.$$

One might object that such a result is not peculiar of gauge theories. The Hamiltonian of a spinor and a massless scalar with a Yukawa coupling, for instance, exhibits the same feature related to zmm. In this latter case, however, the instability can be avoided by adding to the Hamiltonian a selfinteraction  $\varphi^4$ , which is also necessary to make the theory renormalizable. What makes gauge theories peculiar w.r. to zmm is that there is no term analogous to  $\varphi^4$  which can prevent the infrared instability. The only way to do it is to regularize by giving to the gauge field a mass to be put equal to zero at the end.

If the gauge field is coupled to a scalar field rather than to a fermion field, a term  $g^2 \varphi^* \varphi A^2$  is present in the Hamiltonian, which prevents the infrared instability. Addition of such a coupling to the coupling with a fermion field, however, cannot obviously avoid it. The present result is therefore relevant to Supersymmetric gauge theories on a torus. The only way we can see to regularize these theories is to replace the gauge multiplet by a massive vector multiplet, whose mass is set equal to zero at the end of the calculations.

### 3.- QUANTIZATION OF NONABELIAN GAUGE THEORIES

In the non abelian case the Hamiltonian is still given by eq.(1) but eqs.(2) and (4) defining the magnetic strength  $H_i$  and the Gauss constraint  $\Phi$  must be replaced by

$$H_i^a(A) = \frac{1}{2} \epsilon_{ijk} \left[ \partial_j A_k^a - \partial_k A_j^a + gf^{abc} A_j^b A_k^c \right], \quad (9)$$

$$\Phi^a = \mathcal{D}_k^{ab}(A) E_k^b + gj_0^a = 0, \quad (10)$$

where  $f^{abc}$  are the structure constants of the color group and  $\mathcal{D}_k$  the covariant derivative in the adjoint representation

$$\mathcal{D}_k^{ab}(A) = \delta^{ab} \partial_k + gf^{abc} A_k^c. \quad (11)$$

Due to the nonlinearity of  $H_i$  and  $\Phi$  the zmm are coupled to the other modes and we must explicitly define a gauge fixing to perform the quantization. In order to do it we need the following definitions

$$A_{i\vec{n}} = \begin{cases} B_{i\vec{n}} & \text{for } n_3 \neq 0, \\ C_{i\vec{n}} & \text{for } n_2 \neq 0, n_3 = 0, \\ D_{i\vec{n}} & \text{for } n_1 \neq 0, n_2 = n_3 = 0. \end{cases} \quad (12)$$

We will use the obvious notations

$$B_i = \frac{1}{L^{3/2}} \sum_{\vec{n}=0} B_{i\vec{n}} \exp i \frac{2\pi}{L} \vec{n} \cdot \vec{x}$$

and so on. We fix the gauge for non zero-momentum modes by requiring

$$B_3 = C_2 = D_1 = 0. \quad (13)$$

We call this gauge the gauge  $A_3 \sim 0$ , because  $A_{3\vec{n}}$  in the continuum limit

is zero almost everywhere in momentum space, i.e. everywhere but on the surface  $p_3=0$ .

We define the variables  $F_i, G_i$  and  $H_i$  conjugate<sup>(o)</sup> to  $B_i, C_i$  and  $D_i$

$$E_{i\vec{n}} = \begin{cases} F_{i\vec{n}} & \text{for } n_3 \neq 0, \\ G_{i\vec{n}} & \text{for } n_2 \neq 0, n_3 = 0 \\ H_{i\vec{n}} & \text{for } n_1 \neq 0, n_2 = n_3 = 0. \end{cases} \quad (14)$$

The variables  $F_3, G_2$  and  $H_1$  conjugate to  $B_3, C_2$  and  $D_1$  are obviously not independent.

Integrating  $\Phi$  over the volume of the torus and using the gauge fixing we get

$$f^{abc} Q_k^c P_k^b + R^a = 0, \quad (15)$$

where

$$R^a = \int d^3x \left\{ j_0^a + f^{abc} \left[ D_2^c H_2^b + D_3^c H_3^b + C_1^c G_1^b + C_3^c G_3^b + B_1^c F_1^b + B_2^c F_2^b \right] \right\}. \quad (16)$$

We see that  $R$  contains only independent variables, so that the volume average of  $\Phi$  is a constraint on  $z_{mm}$ , decoupled from the constraints on the other modes. Such a decoupling does not occur in the Coulomb gauge, as explained in the Appendix, and it is the advantage of the gauge  $A_3 \sim 0$ .

Before solving this constraint in order to select the independent variables of zero-momentum, we show how to solve for  $H_1, G_2$  and  $F_3$ . We find  $H_1$  by integrating  $\Phi$  over  $x_2$  and  $x_3$

$$H_1 = - \mathcal{D}_1^{-1} (L^{-3/2} Q) R_1 \quad (17)$$

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(o) There will be no occasion of confusion between this latter component of  $\vec{E}_i$  and the magnetic strength  $H_i$ .

where

$$\begin{aligned}
 R_1^a = & gf^{abc}(Q_2^c H_2^b + Q_3^c H_3^b + D_2^c H_2^b + D_3^c H_3^b) + \\
 & + \frac{1}{L} gf^{abc} \int dx_2 (C_1^c G_1^b + C_3^c G_3^b) + \\
 & + \frac{1}{L^2} \int dx_2 dx_3 \left[ gj_0^a + gf^{abc} (B_1^c F_1^b + B_2^c F_2^b) \right].
 \end{aligned} \tag{18}$$

We next determine  $G_2$  by integrating  $\Phi$  over  $x_3$

$$G_2 = - \mathcal{D}_2^{-1} (L^{-3/2} Q + D) R_2 \tag{19}$$

where

$$\begin{aligned}
 R_2^a = & gf^{abc} (Q_1^c G_1^b + Q_3^c G_3^b + D_3^c G_3^b + C_1^c G_1^b + C_3^c G_3^b + \\
 & + (Q_k^c + D_k^c + C_k^c) H_k^b + \frac{1}{L} \int dx_3 \left[ gj_0^a + \right. \\
 & \left. + gf^{abc} (B_1^c F_1^b + B_2^c F_2^b) \right].
 \end{aligned} \tag{20}$$

Finally we get  $F_3$  directly from  $\Phi$

$$F_3 = \mathcal{D}_3^{-1} (C + D) R_3 \tag{21}$$

where

$$\begin{aligned}
 R_3^a = & \partial_1 F_1^a + \partial_2 F_2^a + \partial_k (G_k^a + H_k^a) + gf^{abc} \left[ \overline{A}_1^c F_1^b + \right. \\
 & \left. + \overline{A}_2^c F_2^b + \overline{A}_k^c (G_k^b + H_k^b) \right] + gj_0^a.
 \end{aligned} \tag{22}$$

It is perhaps worth while noticing that the fields appearing in the argument of  $\mathcal{D}_k^{-1}$  do not depend on  $x_k$ .

We will solve the constraint (15) for the zmm only for the SU(2) color group. In this case it is convenient to introduce new variables through the polar representation<sup>(5)</sup>

$$Q_i^a = f_{in}(\theta) \lambda_n h_n^a(\varphi) \quad (23)$$

where  $f$  and  $h$  are orthogonal matrices, which implies that the sum over  $n$  extends from 1 to 3. The new variables are three angles  $\theta_t$  which parametrize  $f$ , three angles  $\varphi_a$  which parametrize  $h$  and the three  $\lambda_n$ 's. Let us denote them altogether by  $q_s$ ,  $s = 1, \dots, 9$ .

To perform the change of variables we must express the old momenta  $P_i^a$  in terms of the new momenta  $p_s$

$$P_i^a = M_{si}^{-1a} p_s \quad (24)$$

where

$$M_{ni}^{-1a} = \frac{\partial \lambda_n}{\partial Q_i^a}; \quad M_{bi}^{-1a} = \frac{\partial \varphi_b}{\partial Q_i^a}, \quad M_{ti}^{-1a} = \frac{\partial \theta_t}{\partial Q_i^a}. \quad (25)$$

We can eliminate in  $M_{bi}^{-1a}$  and  $M_{ti}^{-1a}$  the derivatives of angles in favor of the derivatives of matrix elements

$$M_{bi}^{-1a} = \lambda_n S_{nm}^k f_{in} h_m^a \eta_{pb}^{-1} h_k^p, \quad (26)$$

$$M_{ti}^{-1a} = \lambda_m S_{nm}^k f_{in} h_m^a \xi_{bt}^{-1} f_{bk},$$

where

$$S_{ij}^k = \varepsilon_{ijk} \frac{1}{\lambda_j^2 - \lambda_i^2}, \quad \text{no summation on } i, j, \quad (27)$$

$$\eta_{an} = -\frac{1}{2} \varepsilon_{nbc} h_{kb} \frac{\partial h_k^c}{\partial \varphi_a}, \quad (28)$$

$$\xi_{t1} = -\frac{1}{2} \varepsilon_{1bc} f_{bm} \frac{\partial f_{cm}}{\partial \theta_t}.$$

This enables us to express the constraint and the Hamiltonian in terms of the new variables. The constraint for  $z_{mm}$  becomes

$$\varepsilon^{abc} Q_k^c P_k^b + R^a = \eta^{-1ab} p_b + R^a = 0 \quad (29)$$

showing that the angles  $\varphi_a$  are pure gauge variables. We choose for these angles the gauge fixing

$$h_n^a = \delta_n^a . \quad (30)$$

We next perform the transformation for the different pieces of the Hamiltonian. We first rewrite each piece in terms of zmm and other modes. We have for the electric part

$$\int d^3x \frac{1}{2} E_k^a E_k^a = \frac{1}{2} p_k^a p_k^a + \frac{1}{2} \int d^3x \bar{E}_k^a \bar{E}_k^a . \quad (31)$$

The second term on the r.h.s. can obviously be rewritten in terms of independent variables by using eqs.(24) to (28). For the sake of brevity we will write explicitly only the first term which, taking into account the constraint (29) becomes<sup>(5)</sup>

$$\begin{aligned} \frac{1}{2} p_i^a p_i^a &= \frac{1}{2} p_{\lambda_n}^2 + \sum_{n < m} \frac{1}{(\lambda_n^2 - \lambda_m^2)^2} \left[ (\lambda_n p_{\lambda_n} - \lambda_m p_{\lambda_m}) (\lambda_n^2 - \lambda_m^2) + \right. \\ &\left. + \frac{1}{4} (\lambda_m^2 + \lambda_n^2) \varepsilon_{kmn}^2 (l_k^2 + L_k^2) + \lambda_m \lambda_n \varepsilon_{kmn}^2 l_k L_k \right] \end{aligned} \quad (32)$$

where

$$l_k = f_{mk} \xi_{mt}^{-1} p_{\theta_t} , \quad (33)$$

$$L_k = -h_{km} R^m .$$

In order to transform the magnetic energy it is convenient to use the following expression of the magnetic strength

$$H_i^a(A) = H_i^a(\bar{A}) + g \varepsilon^{abc} \varepsilon_{ilm} Q_l^b (\bar{A}_m^c + \frac{1}{2} Q_m^c)$$

in which the zmm are separated from the other modes. The magnetic energy

becomes

$$\int d^3x \frac{1}{2} H^2(A) = W + \int d^3x \left[ H^2(\bar{A}) + g \varepsilon^{abc} \varepsilon_{ilm} Q_1^b \cdot \right. \\ \left. \cdot \left( \bar{A}_m^c + \frac{1}{2} L^{-3/2} Q_m^c \right) H_i^a(\bar{A}) + g^2 \varepsilon^{abc} \varepsilon_{ilm} \varepsilon^{ade} \varepsilon_{ijk} Q_1^b Q_1^c Q_1^d Q_1^e \right] \quad (34)$$

where we have isolated for later convenience the term

$$W = \frac{1}{4} g^2 \varepsilon^{abc} \varepsilon_{ilm} \varepsilon^{ade} \varepsilon_{ijk} Q_1^b Q_1^c Q_1^d Q_1^e = \\ = \frac{1}{2} g^2 (\lambda_1^2 \lambda_2^2 + \lambda_1^2 \lambda_3^2 + \lambda_2^2 \lambda_3^2) \quad (35)$$

Only for this term will we need the explicit expression in terms of the new variables.

Finally the interaction energy with a current is

$$\int g J_k^a A_k^a = g Q_k^a I_k^a + \int d^3x g \bar{J}_k^a A_k^a = g \lambda_n f_{kn} I_k^n + \int d^3x \bar{J}_k^a A_k^a \quad (36)$$

#### 4.- VACUUM INSTABILITY IN THE NONABELIAN CASE

The main difference concerning vacuum instability w.r. to the abelian case is the presence of higher powers of the  $Q_k^a$  in the pure gauge part of the Hamiltonian. Since the coupling to the current remains linear in the  $Q_k^a$ , we must show that the higher powers do not avoid the instability. In order to do it we use the trial state functional

$$\Psi = \frac{\sqrt{\bar{\lambda}}}{\pi^{3/4} \lambda_0^2} \exp. \left( -\bar{\lambda} \frac{\lambda_1^2 + \lambda_2^2}{2\lambda_0^3} \right) \exp \left[ -\frac{(\lambda_3 - \bar{\lambda})^2}{2\lambda_0^2} \right] F(\lambda_1, \lambda_2, \lambda_3) \chi \quad (37)$$

In the above equation F is a correlation function which prevents a divergence in  $\frac{1}{2} p_k^a p_k^a$  when  $\lambda_i = \lambda_j$ , for instance

$$F = \prod_{i < j} \left\{ 1 - \exp \left[ - \frac{(\lambda_i^2 - \lambda_j^2)^2}{\lambda_0^2} \right] \right\},$$

while  $\chi$  is such that

$$\langle \chi | I_k^a | \chi \rangle = \delta^{a3} \delta_{k3}^I. \quad (38)$$

As a consequence for  $\bar{\lambda} \rightarrow \infty$

$$\langle \Psi | \int d^3x g_{jk}^a A_k^a | \Psi \rangle \rightarrow C_1 \bar{\lambda}, \quad C_1 \text{ a constant}, \quad (39)$$

showing that the coupling to the current behaves as in the abelian case.

Let us now consider the new terms. It is immediate to check that those containing only  $z_{mm}$  do not grow faster than  $\bar{\lambda}$

$$\langle \Psi | \frac{1}{2} P_{kk}^a P_{kk}^a | \Psi \rangle \rightarrow C_2 \bar{\lambda}, \quad \langle \Psi | W | \Psi \rangle \rightarrow C_3 \bar{\lambda}. \quad (40)$$

In order to estimate the terms involving also the other modes we perform the transformation

$$\bar{A} \rightarrow \frac{1}{\sqrt{\bar{\lambda}}} \bar{A}, \quad \bar{E} \rightarrow \sqrt{\bar{\lambda}} \bar{E} \quad (41)$$

which should be understood to hold only for the independent components. Therefore  $B_3$ ,  $C_2$  and  $D_1$  remain zero, while  $F_3$ ,  $G_2$  and  $H_1$  are still given by eqs.(17), (19) and (21) but with the independent components rescaled according to (41). It can be checked from these equations that also the dependent components of  $\bar{E}_k$  do not grow faster than  $\sqrt{\bar{\lambda}}$  as the independent ones. Using the rescaled variables we find that there is no term in the Hamiltonian which grows faster than  $\bar{\lambda}$

$$\langle \Psi | H | \Psi \rangle \rightarrow (K + C_1) \bar{\lambda}, \quad K \text{ a constant}. \quad (42)$$

Since we can make  $K+C_1 < 0$  by choosing  $I$  in eq.(38) large enough and  $gI < 0$ , the Hamiltonian is not bounded from below.

The same considerations made in the abelian case concerning the coupling to bosonic matter and Susy apply obviously also to the nonabelian case.

## 5.- SUMMARY AND CONCLUSION

We have performed the quantization of gauge theories on a torus taking the zmm into proper account. This result has been achieved in the gauge  $A_3 \sim 0$ , in which the constraint on zmm decouples from the others.

The formulation of the quantum theory obtained in this way is rather cumbersome, and probably it is not the most suitable one for practical calculations. The complications are due to the necessity to take into account terms defined in regions of the phase space which are of vanishing measure in the continuum limit. They cannot for this reason be neglected. On the one hand, as mentioned in the introduction, terms defined at one single point of phase space, the zmm, can introduce an energy scale in the abelian theory through radiative corrections. On the other hand, as we have seen, they make the Hamiltonian unbounded from below both in the abelian and the nonabelian case.

This is an infrared problem, and we already know that gauge theories must be regularized in the infrared. The present result, however, implies that among the known regularizations only that of giving a mass to the gauge field can make the Hamiltonian bounded from below. As already mentioned this regularization allows the introduction of an energy scale into the abelian theory by a mechanism which is likely to work also in the nonabelian case.

Now there is an important difference in this connection between the abelian and the nonabelian theory. The first one is unitary for all the values of the mass of the gauge field while the second one becomes unitary only in the limit of vanishing gauge field mass<sup>(4)</sup>.

We should also note that the unboundedness is due to the existence of an average value of the fermion current over the quantization volume. If there is color confinement such an expectation value must be zero. This situation is best illustrated by the Schwinger model<sup>(6)</sup> where the same infrared instability would occur, but we know, because the model has been solved, that the current average vanishes. Therefore in the non-abelian case the present instability might be an obstacle in the way of perturbation theory just because related to another nonperturbative feature, namely color confinement.

It is perhaps worth while noticing that the Hamiltonian of gauge theories on a lattice, due to the compactness of the gauge variables, is bounded from below. This raises a problem in the way of identifying the thermodynamic limit of gauge theories on a lattice with continuum gauge theories.

A final remark concerns Susy gauge theories. These theories should be regularized in a supersymmetric way, by replacing the gauge multiplet by a massive vector multiplet. Among other things, this fact is relevant to the Witten index<sup>(7)</sup>. The present results show that it cannot be evaluated by quantizing the degrees of freedom of the classical theory which are presumably associated to low lying excitations, without taking into account the problem of the infrared regularization.

I am very grateful to Prof. C. Becchi for many discussions of the content of the present paper.

APPENDIX

In order to solve the Gauss constraint in the Coulomb gauge it is convenient to introduce its Fourier transform

$$\begin{aligned} \Phi_{\vec{n}}^a = & i \frac{2\pi}{L} n_k E_{k\vec{n}}^a + gf^{abc} L^{-3/2} \left[ \sum_{\vec{m} \neq 0} A_{k\vec{m}}^b E_{k, \vec{n}-\vec{m}}^b + \right. \\ & \left. + Q_k^c E_{k\vec{n}}^b + A_{k\vec{n}}^c P_k^b \right] + gj_{0\vec{n}}^a = 0, \quad \vec{n} \neq 0 \end{aligned} \quad (A1)$$

$$\Phi_0^a = gf^{abc} L^{-3/2} (Q_k^c P_k^b + \sum_{\vec{m}=0} A_{k\vec{m}}^c E_{k, -\vec{m}}^b) + gj_{00}^a. \quad (A2)$$

In the Coulomb gauge the longitudinal components of  $E_k^a$  are dependent variables and, unlike the gauge  $A_3 \sim 0$ , such dependent variables appear also in  $\Phi_0^a$ . As a consequence the constraint for  $z_{mm}$  does not decouple from the equations for the other modes, and the  $P_k^a$  appearing in eq.(A1) should be regarded as functions of the longitudinal components of  $E_k^a$  as determined by eq.(A2). Lüscher<sup>(3)</sup> has suggested to solve  $\Phi_{\vec{n}}$  only for  $\vec{n} \neq 0$ , leaving a residual invariance of the Hamiltonian w.r. to spatially constant gauge transformations and the corresponding generator  $\Phi_0^a$  as a constraint on the states. In his actual calculations, however, he does not take into account the term in the sum in eq.(A2).

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