ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

LNF-85/14(R)
14 Maggio 1985

M. Greco: HIGHER ORDER E.M. RADIATIVE CORRECTIONS TO $e^+e^- \rightarrow \mu^+\mu^-$ AROUND THE Z_0

Servizio Documentazione
dei Laboratori Nazionali di Frascati
Cas. Postale 13 - Frascati (Roma)

INFN - Laboratori Nazionali di Frascati
Servizio Documentazione

LNF-85/14(R)
14 Maggio 1985

M. Greco: HIGHER ORDER E.M. RADIATIVE CORRECTIONS TO $e^+e^- \to \mu^+\mu^-$ AROUND THE Z_0

ABSTRACT

Known results for e.m. radiative effects at LEP/SLC energies are reviewed. The relevance of higher order terms for precision measurements of the Z_0 mass and width is discussed.

Precision tests of the electroweak theory at LEP/SLC energies require an accuracy at the level of 1% and therefore demand a correct treatment of e.m. radiative corrections(1). Indead first order corrections, for example, reduce the Z peak cross section by more than 50%, or shift the zero in the μ forward-backward asymmetry of about ($\pm$ 360) MeV, for an energy resolution of $(10^{-1}-10^{-2})$. It is crucial therefore that high order corrections are properly taken into account if the Z mass and width have to be measured to an accuracy of the order of 50 MeV.

Previous studies of these effects have been presented in the past few years(2,3). In this note we briefly update the theoretical results for the reaction $e^+e^- \to \mu^+\mu^-$. A more complete discussion, also for other processes of immediate experimental interest, will be presented elsewhere(4).

Our considerations apply to a typical experiment which satisfy the following requirement:

(i) The muons should be back-to-back within a certain accolinearity angle J, of a few degrees ($J \leqslant 5°$). The energy resolution $\Delta\omega$ depends upon J (see eq.(1)).

(ii) An electromagnetic calorimeter of finite and small angular resolution δ is centered along the muon direction. In principle it will not discriminate between a charged particle and the accompanying collinear photons.

Then, using (i) and (ii), one would be sure that all but a fraction $\Delta \equiv \Delta\omega/E$ of the beam energy ($\sqrt{s} = 2E$) is taken by the muons and the accompanying hard protons. For small J and δ fully analytic expressions can be used, neglecting hard photon effects of order $(\frac{\alpha}{\pi}\Delta, \frac{\alpha}{\pi}\delta)$. On the contrary all double logarithmic terms of the form $\frac{\alpha}{\pi}\ln(\frac{s}{m_e^2})\ln(\Delta, \Gamma/M)$, $\frac{\alpha}{\pi}\ln\delta^2\ln\Delta$, where M and Γ are the mass and the width of the Z or simple logs as, $\frac{\alpha}{\pi}\ln(\frac{s}{m_e^2})$, $\frac{\alpha}{\pi}\ln(\Delta, \Gamma/M, \delta^2)$ can be resummed to all orders, using known results on the exponentiation of the infrared and mass singularities.

For a given accollinearity angle J, the maximum energy k_{max} taken by undetected soft photons, which defines the energy resolution $\Delta\omega$, is given by

$$k_{max} = \Delta\omega = \frac{\sqrt{s}}{1+\cos J}\left\{-(1-\cos J) + 2\left[(1-\cos J)\frac{1}{2} - \frac{m_\mu^2}{s}(1+\cos J)\right]^{1/2}\right\} . \tag{1}$$

Then for J = 1°, 3° and 5° one obtains $\Delta = \Delta\omega/E = (1.7)\%$, $(5.1)\%$ and $(8.3)\%$ respectively.

We define, as usual, $\beta_{e,\mu} = \frac{2\alpha}{\pi}\left[\ln(\frac{s}{m_{e,\mu}^2}) - 1\right]$ and $\beta_{int} = \frac{4\alpha}{\pi}\ln(\mathrm{tg}\,\theta/2)$, where θ is the c.m. scattering angle. Then first order radiative corrections to the Born cross sections, which include vertex, vacuum polarization and box diagrams and soft photon bremsstrahlung give, using the notation of ref.(2),

$$\begin{aligned}\frac{d\sigma}{d\Omega} = {} & \left(\frac{d\sigma_{QED}}{d\Omega}\right)_0\left[1 + (\beta_e + \beta_\mu + 2\beta_{int})\ln\Delta + C_F^{QED}\right] + \\ & + \sum_{i=V,A,VA}\left(\frac{d\sigma_{int,i}}{d\Omega}\right)\cdot\left\{1 + (\beta_\mu + \beta_{int})\ln\Delta + \right. \\ & \left. + \frac{1}{\cos\delta_R}\mathrm{Re}\left[e^{i\delta_R}(\beta_e\ln\bar{E} + \beta_{int}\ln\bar{I})\right] + C_F^{int,i}\right\} + \\ & + \left(\frac{d\sigma_{res}}{d\Omega}\right)_0\left\{1 + \beta_\mu\ln\Delta + \beta_e\ln|\bar{E}| + 2\beta_{int}\ln|\bar{I}| - \beta_e\delta(s,\Delta\omega)\mathrm{ctg}\,\delta_R + C_F^{res}\right\},\end{aligned} \tag{2}$$

where

$$(M^2 - iM\Gamma - s)^{-1} \equiv e^{i\delta_R} \sin \delta_R (M\Gamma)^{-1} ,$$

$$\bar{E} = \frac{\Delta}{1 + \frac{\Delta s}{M\Gamma} e^{i\delta_R} \sin \delta_R} , \qquad \bar{I} = \frac{\Delta}{\Delta + \frac{M\Gamma}{s} e^{-i\delta_R}/\sin \delta_R} \tag{3}$$

and

$$C_F^{QED} = \frac{3}{4}(\beta_e + \beta_\mu) + \delta_{VP}^{tot} + \frac{2\alpha}{\pi}\left(\frac{\pi^2}{3} - \frac{1}{2}\right) + X_V ,$$

$$C_F^{int,i} = \frac{3}{4}(\beta_e + \beta_\mu) + \frac{1}{2}\delta_{VP}^{tot} + \frac{2\alpha}{\pi}\left(\frac{\pi^2}{3} - \frac{1}{2}\right) + X_i \qquad (i=V,A,VA) , \tag{4}$$

$$C_F^{res} = \frac{3}{4}(\beta_e + \beta_\mu) + \frac{2\alpha}{\pi}\left(\frac{\pi^2}{3} - \frac{1}{2}\right) + f(Y_V,Y_S) .$$

For the sake of simplicity in the above equation we have not reported the explicit form of $\delta(s,\Delta\omega)$, X_i and $f(Y_V,Y_A)$ They can be found in ref.(2). Furthermore δ_{VP}^{tot} refers to the full vacuum polarization correction, which includes all leptons and hadrons.

In calorimetric type measurements, where collinear hard radiation ($k \gtrsim \Delta\omega$) from the final particles is detected within a small cone of half opening angle δ, one has to add to the r.h.s. of each of eqs.(2) the following correction factor[5-7]

$$\delta_j^{coll} = d\sigma_j^0 \frac{4\alpha}{\pi}\left[\left(\ln\frac{E}{\Delta\omega} - \frac{3}{4}\right)\ln\left(\frac{E\delta}{m_\mu}\right) - \frac{1}{2}\ln\left(\frac{E}{\Delta\omega}\right) + \frac{1}{2}\left(\frac{9}{4} - \frac{\pi^2}{3}\right)\right] . \tag{5}$$

Then, in agreement with the Kinoshita-Lee-Nauenberg theorem on the mass singularities[8], the m_μ-dependence disappears after adding eqs.(5) to (2) and the overall correction factor to the Born cross sections can be simply obtained from (2) by the substitution

$$\beta_\mu\left(\ln\Delta + \frac{3}{4}\right) \rightarrow \frac{2\alpha}{\pi}\left\{\ln\frac{4}{\delta^2}\left(\ln\Delta + \frac{3}{4}\right) + \left(\frac{3}{2} - \frac{\pi^2}{3}\right)\right\} . \tag{6}$$

From the known results on the exponentiation of soft and collinear divergences[9] one obtains then the final result

$$\begin{aligned}\frac{d\sigma}{d\Omega} = {} & \left(\frac{d\sigma_{QED}}{d\Omega}\right)_0 \left\{\Delta^{\beta_e+2\beta_{int}+\beta_\delta} + \bar{C}_F^{QED}\right\} + \sum_{i=V,A,VA}\left(\frac{d\sigma_{int,i}}{d\Omega}\right)_0 \cdot \\ & \cdot \left\{\Delta^{\beta_{int}+\beta_\delta}\frac{1}{\cos\delta_R}\,\mathrm{Re}\left[e^{i\delta_R}\,\bar{E}^{\beta_e}\,\bar{I}^{\beta_{int}}\right] + \bar{C}_F^{int,i}\right\} + \\ & + \left(\frac{d\sigma_{res}}{d\Omega}\right)_0 \left\{\Delta^{\beta_\delta}|\bar{E}|^{\beta_e}|\bar{I}|^{2\beta_{int}}\left[1-\beta_e\delta(s,\Delta\omega)\mathrm{ctg}\delta_R\right] + \bar{C}_F^{res}\right\} ,\end{aligned} \tag{7}$$

where $\beta_\delta = \frac{2\alpha}{\pi}\ln(4/\delta^2)$ and $\bar{C}_F^{(j)} = C_F^{(j)}(\beta_\mu \to \beta_\delta) + \frac{2\alpha}{\pi}(\frac{3}{2} - \frac{\pi^2}{3})$. The contribution of the muon loop to the vacuum polarization factor δ_{VP}^{tot} is kept unchanged.

The effect of higher order turns on the total cross section is shown in Figs. 1 for $\delta \sim 1°$ and $\Delta = 0.1$ and 0.01 respectively, and compared with the Born terms and first order corrections. Both the size and the positiоn of the peak are cleraly affected. Resonable changes in δ do not alter appreciably the result.

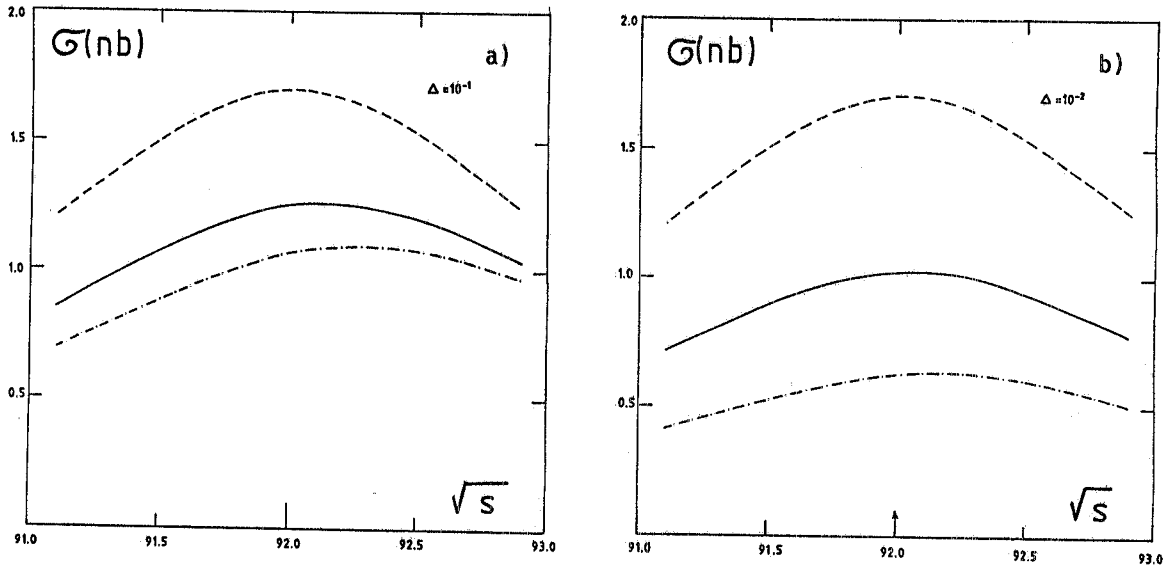

FIG. 1 - a) Cross section integrated on the scattering angle θ, for $\Delta = 10^{-1}$ and $\delta \simeq 1°$. The electroweak parameters are M = 92 GeV, Γ = 2.9 GeV and $\sin^2\theta_W = 0.23$. Dashed curve: Born cross section; dot-dashed curve: first order correction; full curve: all orders correction. b) Same as a) for $\Delta = 10^{-2}$.

In Fig. 2 the forward-backward asymmetry is also shown for the same choice of the parameters. Notice the shift of the zero, which strongly depends on the value of Δ. Clearly a precise determination of the value of M is quite sensitive to the energy resolution of experiments.

To conclude, precise measurements of the mass and the width of the neutral weak boson with LEP/SLC experiments depend crucially on a correct treatment of e.m. radiative effects. To this aim higher order terms compulsorily have to be taken into account.

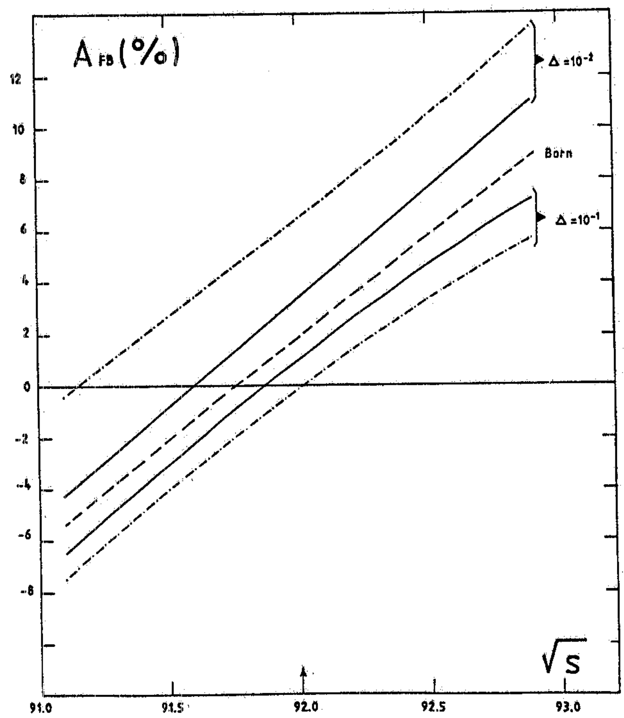

FIG. 2 - Integrated forward-backward asymmetry for $\sqrt{s} \sim M$. The notation is the same of Fig. 1a.

REFERENCES

(1) - G.Altarelli, LEP Jamboree, CERN, March 19, 1985.

(2) - M.Greco, G.Pancheri-Srivastava and Y.Srivastava, Nuclear Phys. B171, 118 (1980) (E: B197, 543 (1982)).

(3) - F.A.Berends, R.Kleiss and S.Jadach, Nuclear Phys. B202, 63 (1982); M.Böhm and W. Hollik, Nuclear Phys. B204, 45 (1982).

(4) - M.Greco et al., to be published on Rivista del Nuovo Cimento.

(5) - G.Sterman and S.Weinberg, Phys. Rev. Letters 39, 1436 (1977).

(6) - G.Curci and M.Greco, Phys. Letters 79B, 406 (1978).

(7) - M.Caffo, R.Gatto and E.Remiddi, Nuclear Phys. B252, 378 (1985).

(8) - T.Kinoshita, J. Math. Phys. 3, 650 (1962); T.D.Lee and M.Nauenberg, Phys. Rev. 133, 1549 (1964).

(9) - See for example ref.(6) and references therein.