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Kinetics of Soft-Graviton Emission in Dilute Gases.

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Summary. – The notion, due to Handel, that soft-graviton emission in macroscopic bodies has measurable consequences is discussed for dilute gases. The Feynman amplitudes for graviton emission are briefly indicated as they would enter into condensed-matter transitions for dense fluids.

Introduction. In a previous note (1) it was shown that if a gravitational wave of frequency ω is propagated in a fluid of viscosity $\eta(\zeta)$, then the inverse mean length with which the wave is damped is given by

$$l^{-1}(\omega) = (16\pi G/c^3)\,\mathrm{Re}\,\eta(\omega + io^+)\,. \tag{1}$$

It is interesting to consider the physical meaning of eq. (1) from the view-point of Feynman amplitudes for graviton absorption, *i.e.* $l(\omega)$ is the mean free path of a graviton in a fluid medium. It is well known that viscosity $\eta(\zeta)$ is determined by fluctations in the stress tensor within the fluid. It is the stress fluctuations which absorb the graviton in a mean free path $l(\omega)$, as in eq. (1).

The work which follows was inspired by an unpublished manuscript by HANDEL (2). Its purpose is to show the manner in which quantum gravity renormalizes the stress fluctuations in a fluid, and thus produces low-frequency noise sources in the viscous transport of fluid mass. For this purpose we shall use the « physical kinetics » of a dilute

(1) A. WIDOM, G. MEGALOUDIS, T. D. CLARK and R. PRANCE: *J. Phys. A*, **14**, L213 (1981).
(2) P. HANDEL: University of Missouri Preprint.

gas, and the inelastic cross-sections for soft-graviton emission as derived by WEINBERG (3). The analogy between shoft-poton emission as an electrical-noise source and soft-graviton emission as a mechanical-noise source has been discussed by SRIVASTAVA (4).

Soft-graviton emission in a gas. For gas particles of mass m colliding at a relative velocity v with a cross-section $d\sigma$, one finds the following result (3) for the probability of emitting a graviton at frequency ω in a bandwidth $d\omega$. Let d^2h be the inverse mean free path for a given gas particle to scatter into a solid angle $d\Omega$, while emitting a graviton into the band width $d\omega$, *i.e.* with n particles per unit volume

$$\left(\frac{d^2h}{d\Omega\, d\omega}\right)=\left(\frac{2}{5\pi}\right)\left(\frac{Gm^2}{\hbar c}\right)\left(\frac{v}{c}\right)^4\sin^2\theta\left(\frac{d\sigma}{d\Omega}\right)\left(\frac{n}{\omega}\right). \tag{2}$$

The viscosity mean free path for the collision is known to be

$$\Lambda^{-1}=n\int\sin^2\theta\, d\sigma\,, \tag{3}$$

hence, by integrating eq. (3) over solid angles one arrives at a «$(1/\omega)$-noise» result for soft gravitons similar to a previously discussed electronic soft-photon result (5), *i.e.* for graviton production on collision

$$dh=(2/5\pi)(Gm^2/\hbar c)(v/c)^4\Lambda^{-1}(d\omega/\omega)\,. \tag{4}$$

The soft-graviton production rate per gas particle follows from eq. (4) as an average over relative velocity

$$d\Gamma(\omega)=\langle v\, dh(v,\omega)\rangle\,, \tag{5}$$

so the central result of this section is that

$$d\Gamma(\omega)=(2/5\pi)(Gm^2/\hbar c)\langle(v/c)^5(c/\Lambda)\rangle(d\omega/\omega)\,. \tag{6}$$

In the next part the velocity average on the right-hand side of eq. (6) will be directly related to the viscosity of the gas

$$\eta=(mn)\nu\,, \tag{7}$$

where ν is the usual «kinematic viscosity».

Viscosity. In gas kinetic theory, it is known that the kinematic viscosity ν is determined by (6)

$$\langle(v^5/\Lambda)\rangle=(40/\nu)(k_B T/m)^3\,, \tag{8}$$

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(3) S. WEINBERG: *Gravitation and Cosmology* (Wiley & Son, New York, N.Y., 1972).
(4) Y. SRIVASTAVA: *Radiation and Noise*, Latin-American School of Physics, Cali, Colombia, 1982 (World Scientific, 1983).
(5) A. WIDOM, G. PANCHERI, Y. SRIVASTAVA, G. MEGALOUDIS, T. D. CLARK, H. PRANCE and R. J. PRANCE: *Phys. Rev. B*, **26**, 1475 (1982); **27**, 3412 (1983).
(6) *Physical Kinetics* (Landau-Lifshitz Course of Theoretical Physics, Vol. **10**), edited by E. M. LIFSHITZ and L. P. PITAEVSKI (Pergamon Press, London, 1981).

so that the production rate of gravitons $d\Gamma(\omega)$ per gas particle is determined by v, independently of the detailed nature of the two-body elastic-scattering cross-section $d\sigma$, *i.e.* eqs. (6) and (8) imply

$$d\Gamma(\omega) = (8/5\pi)(Gm^2/\hbar c)(k_B T/mc^2)^3 (c^2/v)(d\omega/\omega) . \tag{9}$$

Macroscopic-graviton production. For a macroscopic mass M, the quantum gravity coupling strength

$$\alpha_G = (GM^2/\hbar c) \tag{10}$$

is decidedly *not* small. Thus, the graviton production rate of a macroscopic sample of gas, *i.e.*

$$d\dot{n}(\omega) = \gamma(d\omega/\omega) , \tag{11}$$

as a « $(1/\omega)$-noise » process has a surprisingly high value (for N gas particles $M = Nm$),

$$\gamma = (8/5\pi)\,\alpha_G(k_B T/mc^2)^3 (c^2/Nv) . \tag{12}$$

The key factor holding down the gravitational radiation losses is that the stress energy in the random motions of atoms is small on the scale of the rest mass energy of the atoms. In the case of a gas, this ratio of is order $(k_B T/mc^2)$ and the manner in which this enters γ is given in eq. (12); (c^2/N) is easily made of order unity, and as previously stated α_G is *not* small.

Conclusion. If one considers the motions of macroscopic quantities of fluids, then the low-frequency noise in viscous transport due to quantum gravitational-wave emission can be appreciable. This basic idea of Handel is worthy of note. The above simple kinetic theory is meant only to verify (by orders of magnitude) that quantum gravity has macroscopic mechanical engineering implications.

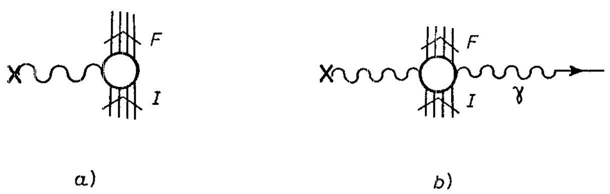

Fig. 1. – In *a*) is shown an external strain applied to a fluid yielding the stress tensor matrix element $\langle F(\text{out})|T_{\mu\nu}(x)|I(\text{in})\rangle$ entering into the Kubo formula for viscosity. In *b*) is shown the same process with an outgoing « spatial strain » as an emitted gravitation, yielding the matrix element in eq. (13). Multigraviton emission follows from *b*) by the usual Bloch-Nordsieck theory of « infra-red catastrophies » [4,5].

The Feynman diagrams in quantum gravity which enter the low-frequency noise in the mechanical engineering stress are shown in fig. 1, along with a physical description of the process. Quantum field-theoretical dense-matter evaluations proceed via the stress tensor commutator matrix element

$$-i\hbar K^{FI}_{\mu\nu\sigma\tau}(x, y) = \langle F(\text{out})|[T_{\mu\nu}(x), T_{\sigma\tau}(y)]|I(\text{in})\rangle , \tag{13}$$

which enter into mechanical engineering in a manner similar to the way in which current commutators enter into electrical-engineering results for the case of photons [7].

Evaluations are reserved for future publications, but here we only note that the short space-time interval divergences of quantum gravity [8] simply will not enter into mechanical-engineering considerations, *i.e.* a comparison between eq. (1) and eq. (3) shows that we are merely discussing the Kubo formula for viscosity. Although quantum gravity renormalizes stress correlations, only macroscopic length scales are of engineering importance in the final results. Viscosity is (of course) a finite quantity.

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(7) A. WIDOM: *J. Low Temp. Phys.*, **37**, 449 (1979); *Phys. Rev. B*, **21**, 5166 (1980).
(8) R. P. FEYNMAN: *Acta Phys. Polon.*, **24**, 697 (1963).