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ENGINEERING THE CHIRAL ANOMALY : THE QUANTUM HALL EFFECT

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Abstract

Some practical aspects of the chiral anomaly present in quantum electrodynamics are discussed in connection with the fractional Hall effect. The possibility that certain instabilities may develop is pointed out.

The purpose of this brief note is to expand on the notion that the chiral anomaly of quantum elecrodynamics [1,2] has important consequences for the theory of the quantum Hall effect[3,4].

Let us suppose that one has an insulating material subject to an electromagnetic field $(\underline{E}, \underline{B})$.After functionally integrating over electronic degrees of freedom,one obtains an energy of polarization $\mathcal{U}(\underline{E}, \underline{B})$ in the 'local field' limit,which in turn yields an effective lagrangian density(in gaussian units)

$$\mathcal{L} = \frac{1}{8\pi}\left(\underline{E}^2 - \underline{B}^2\right) - \mathcal{U}(\underline{E}, \underline{B}) \tag{1}$$

For example, the computation of $\mathcal{U}(\underline{E},\underline{B})$ for the vacuum, (treated as an insulator) is discussed in treatises in quantum electrodynamics[5,6].

The polarization and magnetization of an insulator follow from the computation of $\mathcal{U}(\underline{E},\underline{B})$by using

$$\underline{P} = -\frac{\partial \mathcal{U}}{\partial \underline{E}} \tag{2a}$$

$$\underline{M} = -\frac{\partial \mathcal{U}}{\partial \underline{B}} \tag{2b}$$

Thus the original Maxwell displacement field $\underline{D}$ and magnetic intensity $\underline{H}$, i.e.

$$\underline{D} = \underline{E} + 4\pi \underline{P} \tag{3a}$$

$$\underline{H} = \underline{B} - 4\pi \underline{M} \tag{3b}$$

here appear in the effective quantum electrodynamics lagrangian, eq.(1), as

$$\underline{D} = 4\pi \frac{\partial \mathcal{L}}{\partial \underline{E}} \tag{4a}$$

$$\underline{H} = -4\pi \frac{\partial \mathcal{L}}{\partial \underline{B}} \tag{4b}$$

Since $\mathcal{U}(\underline{E},\underline{B})$ can in principle have terms higher order than quadratic in $\underline{E}$and $\underline{B}$,when $\mathcal{L}$ is treated as an effective quantum lagrangian,photon photon interactions are adequately described, i.e. the induced currents in the insulator are given by the possibly non linear equations of state

$$\rho = -div\underline{P} \tag{5a}$$

$$\underline{J} = \frac{\partial \underline{P}}{\partial t} + c\ curl\underline{M} \tag{5b}$$

The point here of importance is that there are chiral terms in the insulator energy whose influence on photon dynamics is rather subtle. [3,4].Suppose that there is a chiral contribution to the polarization energy of the form

$$\mathcal{U}_5 = -\frac{g}{c}\left(\underline{E}\cdot\underline{B}\right) \tag{6}$$

where g is a pseudoscalar coupling strength. In virtue of eqs.(2) and (6), there will be chiral polarization and magnetization in the insulator which obey

$$\underline{P}^* = \frac{g}{c}\underline{B} \tag{7a}$$

$$\underline{M}^* = \frac{g}{c}\underline{E} \tag{7b}$$

However the chiral charge density and current vanish in the bulk insulator

$$\rho^* = 0 \tag{8a}$$

$$J^* = 0 \qquad (8b)$$

where eq.(8a) follows from eqs.(5a),(7a) and the absence of magnetic charge $div\underline{B} = 0$,while eq.(8b) follows from eqs.(5b),(7)and Faraday's Law $curl\underline{E} + \frac{1}{c}\frac{\partial \underline{B}}{\partial t} = 0$.

Since $\rho^* = 0$ and $\underline{J}^* = 0$ when chiral energies are included, it would at first appear that photon interactions cannot at all be modified by chirality. However, this is really not the case. To start, one may note that charge conservation dictates that an insulator without any initial net charge, cannot then develop a net charge when disturbed by an electromagnetic field. Thus, a bulk charge density in an insulator, as given by eq.(5a), must (at the 'boundary surface' of the insulator) develop surface charge density known to be described by the polarization vector via

$$\sigma = (\underline{n} \cdot \underline{P}) \qquad (9)$$

where $\underline{n}$ is a unit vector normal to the boundary surface. Similar considerations for the magnetization yield a 'surface current' on the boundary known to be

$$\underline{K} = c\,(\underline{n} \wedge \underline{M}) \qquad (10)$$

Let us now return to the question of chiral contributions to the current and charge densities. Eqs.(7a) and (9) imply a surface charge density of

$$\sigma^* = \frac{g}{c}\,(\underline{n} \cdot \underline{B}) \qquad (11)$$

while the surface current density exhibits the Hall effect in the form of a boundary current contribution via eqs.(7b)and (10),i.e.

$$\underline{K}^* = g\,(\underline{n} \wedge \underline{E}) \qquad (12)$$

Thus the coupling strength g now appears as a boundary surface Hall conductance, and the 'chiral' energy contribution indeed becomes experimentally observable.

Let us now consider the electrical engineering implications of the chiral energy terms in quantum electrodynamics.For a MOSFET device,an earthed metal becomes a capacitor plate, with an oxide insulator, and finally a 'gate region' semi-conductor at voltage V becomes the second capacitor plate. The thermodynamic law for the full capacitor charge Q reads

$$dF = -SdT - QdV - \mathcal{M}d\Phi, \qquad (13)$$

where Φ is the magnetic flux through the plates and $\mathcal{M}$ is the magnetic moment per unit area of the capacitor plates. The chiral energy term in eq.(6) now reads (for engineering purposes) as

$$\mathcal{U}_5 = -\frac{g}{c}V\Phi \qquad (14)$$

while the 'polarization' charge in eq.(11) reads

$$Q^* = \frac{g}{c}\Phi \qquad (15)$$

Thus the charge to flux ratio in quantum units

$$\nu = \left(\frac{Q^*}{e}\right) / \left(\frac{\Phi}{\phi_0}\right), \tag{16}$$

where $\phi_0 = \frac{2\pi\hbar c}{e}$ yields in the usually defined quantum units

$$g = \frac{e^2}{2\pi\hbar}\nu \tag{17}$$

It is interesting to ask whether $\mathcal{U}_5$, i.e. the chiral energy, can induce a phase transition in the thermal equation of state for the capacitor charge $Q(V, \Phi, T)$. A one-photon loop computation of the chiral -Casimir effect indicates an 'instability' in the free energy for sufficiently strong coupling strength g which in fact may already have been observed[7]. Such an instability can be stabilized by photon-photon interactions implicit in $\mathcal{U}(\underline{\mathrm{E}}, \underline{\mathrm{B}})$, although multiphoton loops have not yet been computed in detail by us. The intriguing possibility exists in principle, that the vacuum can be forced into a phase transition by a Hall conductance strength g on boundary surfaces, i.e. even in an Abelian gauge theory (such as quantum electrodynamics) a chiral phase transition can be topologically induced in a finite volume by appropriate conditions imposed on the boundary surfaces.

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