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VECTOR BOSON PRODUCTION AT COLLIDERS: A THEORETICAL REAPPRAISAL

G. ALTARELLI

CERN, Geneva, Switzerland and Dipartimento di Fisica, Università "La Sapienza", Roma, Italy
INFN, Sezioue di Roma, Italy

R.K. ELLIS*

INFN, Sezione di Roma, Italy

M. GRECO and G. MARTINELLI

INFN, Laboratori Nazionali di Frascati, Italy

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We study the production of vector bosons in hadron-hadron collisions via the Drell-Yan mechanism in QCD. Our treatment of the transverse momentum and rapidity distributions of the produced bosons takes all available theoretical principles and results into account in a systematic way. The resulting q_T distribution reduces to the perturbative limit for large q_T, includes the summation of soft gluons and reproduces the known results for the total cross section. A full numerical analysis of W and Z cross sections at collider and tevatron energies is made.

1. Introduction

The production of W and Z bosons at the CERN $p\bar{p}$ collider [1,2] tests the Drell-Yan mechanism [3] in a completely new energy regime. The total cross section for vector boson production σ and the rapidity differential cross section $d\sigma/dy$ are predicted by the QCD-improved parton model [4] as an expansion in the strong coupling constant α_s. The corrections of order α_s to these cross sections have been calculated and found to be important [5,6]. They increase the naive parton model prediction by an energy and rapidity dependent factor commonly referred to as the "K-factor". At fixed target energies the $O(\alpha_s)$ corrections are dangerously big and resummation techniques must be invoked [7] in an attempt to control the perturbation series. At collider energies their size is reduced because the coupling constant is smaller and, for the production of weak intermediate bosons, they lead to a

* Address after 1 April 1984: FERMILAB, Batavia, IL 60510, USA

correction of about 30%. The total cross section is therefore more reliably predicted by perturbation theory at these energies with a smaller theoretical error on the overall normalization.

The prediction of the boson transverse momentum distribution is more subtle, since all order effects need to be taken into account. Renormalization group-improved perturbation theory is valid when the transverse momentum q_T is of the same order as the vector boson mass Q. The large q_T tail of the transverse momentum distribution was one of the early predictions of the QCD-improved parton model [8, 9]. As q_T becomes less than Q, such that $\Lambda \ll q_T \ll Q$, a new scale is present in the problem and large terms of order

$$\frac{1}{q_T^2}\alpha_s^n(q_T^2)\ln^m(Q^2/q_T^2), \qquad m \leqslant 2n-1,$$

occur, forcing the consideration of all orders in n. These terms are characteristic of a theory with massless vector gluons. Fortunately, in the leading double logarithmic approximation (DLA: $m = 2n - 1$) these terms can be reliably resummed. This resummation was first attempted by Dokshitzer-Diakonov-Troyan (DDT) [10] and subsequently modified and consolidated [11]. A consistent framework for going beyond the leading double logarithmic approximation has been indicated by Collins and Soper [12, 13].

The combination of these results on the q_T distribution with the constraint on the area of the distribution provided by the integrated cross section at $O(\alpha_s)$, allows an essentially complete reconstruction of the q_T distribution to that accuracy. There is some uncertainty due to the parton intrinsic transverse momentum but it is present only in a restricted region at low q_T at collider energies. Despite these theoretical advances and several numerical analyses [14], there is to our knowledge no complete and explicit treatment in the literature, valid both in the region $q_T \sim Q$ and $q_T \ll Q$, which includes all the available information.

In this paper we re-examine the problem of the q_T distribution in Drell-Yan processes. We include in a systematic way the large amount of theoretical information accumulated in recent years. Our final expression for the q_T distribution satisfies the following requirements:

(i) At large q_T, we automatically recover the $O(\alpha_s)$ perturbative distribution coming from one-gluon emission, without the ad hoc introduction of matching procedures between hard and soft radiation.

(ii) In the region $q_T \ll Q$ the soft gluon resummation is performed at leading double logarithmic accuracy. The role of subleading terms in the summation is also discussed and evaluated.

(iii) Only terms corresponding to the emission of soft gluons, for which the exponentiation can be theoretically justified, are resummed. The proposal to exponentiate the whole first-order contribution, originally proposed by Parisi and

Petronzio [11] and more recently adopted by Halzen, Martin and Scott [15], is at variance with the all-orders analysis of Collins and Soper [12].

(iv) The integral of the q_T distribution reproduces the known results for the $O(\alpha_s)$ total cross sections (i.e. including the "K-factor").

(v) The average value of q_T^2 is also identical with the perturbative result at $O(\alpha_s)$.

(vi) All quantities are expressed in terms of precisely defined quark distribution functions at a specified scale, as for example those determined by the deep inelastic structure function F_2 at the scale Q^2 [5].

Our treatment can be augmented with higher-order terms as and when these become available. At present, it is not possible to include $O(\alpha_s^2)$ terms in an entirely consistent way because a complete calculation is lacking. The calculation of $(q\bar{q} \to B + X)$ at non-zero q_T exists at order α_s^2 [16] and was found to be numerically important at fixed target energies, but the complete two-loop calculation has not yet been performed. In the literature one can find only one correction to the double logarithmic approximation, first evaluated by Kodaira and Trentadue [17] and recently confirmed [18] using the results of ref. [16]. We include this term as a way of testing the theoretical stability of our results.

We also make a thorough numerical analysis of the W and Z boson cross sections and distributions. We pay special attention to the quantitative uncertainties due to lack of precise knowledge of the input parameters. These parameters are principally Λ_{QCD}, the parton distribution functions, the intrinsic transverse momentum and the scale of the running coupling constant in front of the $O(\alpha_s)$ terms.

Consider a proton and antiproton with momenta P_1 and P_2 respectively, which collide at total centre-of-mass energy $\sqrt{S}$ to produce a vector boson B of momentum q. In the centre-of-mass frame of the incoming hadrons, these momenta have components

$$P_1 = \tfrac{1}{2}\sqrt{S}\,(1;0,0,1),$$

$$P_2 = \tfrac{1}{2}\sqrt{S}\,(1;0,0,-1),$$

$$q = (q_0;\boldsymbol{q}_T, q_3), \qquad q^2 \equiv Q^2, \qquad q_T^2 \equiv \boldsymbol{q}_T^2. \tag{1}$$

From these variables we can define the hadronic invariants

$$T = (P_1 - q)^2, \qquad U = (P_2 - q)^2, \tag{2}$$

and the rapidity y:

$$y = \tfrac{1}{2}\ln\frac{q_0 + q_3}{q_0 - q_3} = \tfrac{1}{2}\ln\frac{Q^2 - T}{Q^2 - U}\,. \tag{3}$$

According to the parton model, the production of the boson proceeds through the interaction of a parton of momentum p_1 in the proton with a parton of momentum p_2 in the antiproton. The invariant cross section is given by

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^2\boldsymbol{q}_\mathrm{T}\,\mathrm{d}y}=\sum_{i,j}\int \mathrm{d}x_1\,\mathrm{d}x_2\,f_i(x_1)f_j(x_2)\left[\frac{s}{\pi}\frac{\mathrm{d}\sigma_{ij}}{\mathrm{d}t\,\mathrm{d}u}\right]_{\substack{p_1=x_1\\ p_2=x_2P_2}}, \tag{4}$$

where f_i, f_j are the distributions of partons of type i, j in the parent hadrons and the parton cross sections are expressed in terms of the partonic variables

$$s=(p_1+p_2)^2,\qquad t=(p_1-q)^2,\qquad u=(p_2-q)^2. \tag{5}$$

The lowest-order parton cross section gives a contribution only at $\boldsymbol{q}_\mathrm{T}=0$:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^2\boldsymbol{q}_\mathrm{T}\,\mathrm{d}y}=NH(x_1^0,x_2^0)\delta^2(\boldsymbol{q}_\mathrm{T}), \tag{6}$$

where N is an overall normalization and H is the product of quark and antiquark parton densities evaluated at the points

$$x_1^0=\sqrt{\tau}\,\mathrm{e}^{y},\qquad x_2^0=\sqrt{\tau}\,\mathrm{e}^{-y},\qquad \tau=\frac{Q^2}{S}. \tag{7}$$

However, eq. (4), with perturbatively evaluated parton cross sections, is only applicable when there is a single large scale $q_\mathrm{T}\sim Q$. For this region, the cross section is well described by the emission of one [8,9] or possibly two gluons [16]. As q_T^2 becomes smaller, terms of order

$$\frac{\alpha_\mathrm{s}(q_\mathrm{T}^2)}{\pi}\frac{1}{q_\mathrm{T}^2}\ln\frac{Q^2}{q_\mathrm{T}^2},\qquad \frac{\alpha_\mathrm{s}(q_\mathrm{T}^2)}{\pi}\frac{1}{q_\mathrm{T}^2}, \tag{8}$$

become large and must be resummed if we want to have a valid perturbative prediction. The divergence at $q_\mathrm{T}^2=0$ is only apparent since for $q_\mathrm{T}=0$, the virtual diagrams intervene and cancel the divergence. We can resum the terms of eq. (8), using all the information which can be extracted from one-parton emission cross sections.

The calculation, which we describe in the next section, proceeds in five steps. None of the steps are mathematically complicated but the length of the formula makes the whole treatment rather heavy. We therefore list the steps in the manipulation of the cross sections so that the reader who is not interested in the mathematical details may skip sect. 2 and proceed to the answer given in sect. 3.

1.1 STEP I

Write the cross sections for the processes

$$q+\bar{q}\to B+g,$$
$$q+g\to B+q,$$
$$q+\bar{q}\to B, \tag{9}$$

up to and including terms of order α_s. The cross sections are written in n dimensions so that the singularities present at $q_T^2=0$ are regulated. These cross sections can be found in ref. [5].

1.2. STEP II

Insert these parton cross sections into eq. (4) to derive the corresponding hadronic cross sections. By subtraction and re-addition of the residues of the terms which are singular as $q_T^2\to 0$, we make the poles in $(n-4)$ explicit. The double poles coming from the region of soft emission cancel; the single poles are the collinear singularities which should ultimately be factored into the parton distributions.

1.3. STEP III

Separate the $O(\alpha_s)$ cross section into two pieces:

$$\frac{d\sigma}{dq_T^2\,dy}=X(q_T^2,Q^2,y)+Y(q_T^2,Q^2,y). \tag{10}$$

The terms in X contain integrable distributions which are singular at $q_T=0$. The function Y is perfectly finite as $q_T\to 0$. Up to and including $O(\alpha_s)$ the expression for X is ($C_F=\frac{4}{3}$)

$$\begin{aligned}X(q_T^2,Q^2,y)=N\Bigg\{&H(x_1^0,x_2^0)\left[\delta(q_T^2)\left(1+F(Q^2,y)\right)+S(q_T^2,Q^2,y)\right]\\&+\frac{\alpha_s}{2\pi}C_F\left(\frac{1}{(q_T^2)_+}+\delta(q_T^2)\left(\ln\left(\frac{A_T}{\mu}\right)^2-\frac{1}{\hat{\varepsilon}}\right)\right)\\&\times\left[\int_{x_1^0}^1\frac{dz}{z}H(x_1^0/z,x_2^0)P_{qq}(z)+\int_{x_2^0}^1\frac{dz}{z}H(x_1^0,x_2^0/z)P_{qq}(z)\right]\\&+\frac{\alpha_s}{2\pi}C_F\delta(q_T^2)\left[\int_{x_1^0}^1\frac{dz}{z}c_q(z)H(x_1^0/z,x_2^0)\right.\\&\left.+\int_{x_2^0}^1\frac{dz}{z}c_q(z)H(x_1^0,x_2^0/z)\right]\Bigg\}.\end{aligned} \tag{11}$$

H is the product of quark and antiquark parton densities. For simplicity, we drop the terms due to initial gluons in this section. The singularity in $1/\hat{\varepsilon}$:

$$\frac{1}{\hat{\varepsilon}} = \left(\frac{2}{4-n} + \ln(4\pi) - \gamma_{\rm E}\right) \tag{12}$$

is a manifestation of the collinear divergence. The distribution S is given by

$$S(q_{\rm T}^2, Q^2, y) = \frac{\alpha_{\rm s}}{2\pi} C_{\rm F}\left[2\left(\frac{\ln(Q^2/q_{\rm T}^2)}{q_{\rm T}^2}\right)_+ - \frac{3}{(q_{\rm T}^2)_+}\right]. \tag{13}$$

1.4. STEP IV

Perform the Fourier transform of X into impact parameter space and factor the collinear singularities into the parton distribution functions defined in deep inelastic scattering. The expression for X is now given by

$$X(b^2, Q^2, y) = N\Bigg\{H(x_1^0, x_2^0, P^2)\left[1 + F(Q^2, y) + S(b^2, Q^2, y)\right]$$
$$+ \frac{\alpha_{\rm s}}{2\pi} C_{\rm F}\left[\int_{x_1^0}^1 \frac{{\rm d}z}{z} f_{\rm q}(z) H(x_1^0/z, x_2^0) + \int_{x_2^0}^1 \frac{{\rm d}z}{z} f_{\rm q}(z) H(x_1^0, x_2^0/z)\right]\Bigg\}, \tag{14}$$

where P is a b dependent scale,

$$S(b^2, Q^2, y) = \int_0^{A_{\rm T}^2} \frac{{\rm d}k^2}{k^2} \frac{\alpha_{\rm s}(k^2)}{2\pi} C_{\rm F}(J_0(bk) - 1)\left(2\ln\frac{Q^2}{Qk^2} - 3\right), \tag{15}$$

and $A_{\rm T}^2$ is the kinematic limit for the transverse momentum squared. The resummation is performed in b space by making the replacement

$$(1 + S(b^2, Q^2, y)) \to \exp S(b^2, Q^2, y). \tag{16}$$

Thus, X may be written as

$$X(b^2, Q^2, y) = R(b^2, Q^2, y)\exp S(b^2, Q^2, y). \tag{17}$$

1.5. STEP V

The differential $q_{\rm T}$ distribution is recovered by Fourier transforming back to $q_{\rm T}$ space:

$$\frac{{\rm d}\sigma}{{\rm d}q_{\rm T}^2\,{\rm d}y} = \int \frac{{\rm d}^2\boldsymbol{b}}{4\pi} {\rm e}^{-i\boldsymbol{b}\cdot\boldsymbol{q}_{\rm T}}\left[R(b^2, Q^2, y)\exp S(b^2, Q^2, y)\right] + Y(q_{\rm T}^2, Q^2, y). \tag{18}$$

Note that upon integration over dq_T^2 the perturbative result for the total cross section $d\sigma/dy$ is obtained up to and including terms of order α_s. Similarly, the perturbative result for the average value of q_T^2 is also obtained.

2. Derivation of the q_T distribution

From ref. [5], we find that the graphs of figs. 1c and d give the following contribution to the quark–antiquark annihilation cross section in n dimensions ($n = 4 - 2\varepsilon$):

$$\frac{s}{\pi}\frac{d\sigma^{q\bar{q}}}{dt\,du} = N'\frac{\alpha_s}{2\pi}C_F\frac{1-\varepsilon}{\Gamma(1-\varepsilon)}\left(\frac{4\pi\mu^2}{q_T^2}\right)^{\varepsilon} M_1(s,t,u)\,\delta(s+t+u-Q^2), \quad (19)$$

where the matrix element squared is given by

$$M_1(s,t,u) = \frac{1}{s}\left\{(1-\varepsilon)\frac{(s-Q^2)^2}{ut} + \frac{2Q^2 s}{ut} - 2\right\}, \quad (20)$$

and N' is an overall normalization factor.

The corresponding contribution of the virtual diagrams figs. 1a and b is

$$\frac{s}{\pi}\frac{d\sigma^{q\bar{q}}}{dt\,du} = N'(1-\varepsilon)M_0(s)\left[1 + \frac{\alpha_s}{2\pi}C_F\frac{K}{\Gamma(1-\varepsilon)}\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon}\right]\delta(s+t+u-Q^2), \quad (21)$$

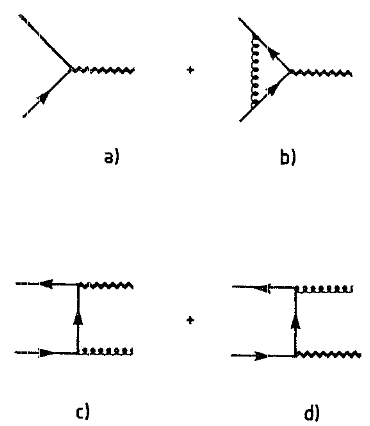

Fig. 1. Feynman diagrams for the $q\bar{q}$ annihilation process up to and including terms of order α_s. The produced vector boson is denoted by a wavy line and the gluon by a curly line.

where

$$M_0(s) = \delta(s - Q^2), \qquad K = \left(-\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} + \pi^2 - 8\right). \tag{22}$$

The substitution of these parton cross sections into eq. (4) involves integrals of the general form:

$$I = \int dx_1\, dx_2\, f(x_1, x_2)\, \delta\big(x_1 x_2 S + x_1(T - Q^2) + x_2(U - Q^2) + Q^2\big), \tag{23}$$

where the delta function present in the parton cross sections equations (19) and (21) has been expressed in hadronic variables. Performing the x_2 integration and splitting the x_1 range of integration, we find

$$I = \int_{\sqrt{\tau_+}\, e^{y}}^{1} \frac{dx_1\, f(x_1, x_2^*)}{x_1 S + U - Q^2} + \int_{\sqrt{\tau_+}\, e^{-y}}^{1} \frac{dx_2\, f(x_1^*, x_2)}{x_2 S + T - Q^2}, \tag{24}$$

where x_1^* and x_2^* are fixed to be

$$x_1^* = \frac{x_2(Q^2 - U) - Q^2}{x_2 S + T - Q^2}, \qquad x_2^* = \frac{x_1(Q^2 - T) - Q^2}{x_1 S + U - Q^2}. \tag{25}$$

The variable τ_+ is

$$\sqrt{\tau_+} = \sqrt{\frac{q_T^2}{S}} + \sqrt{\tau + \frac{q_T^2}{S}}\,. \tag{26}$$

In performing the x_1 and x_2 integrations in eq. (24), a certain proliferation of variables is inevitable. For the convenience of the reader, we have collected their definitions in table 1. To lighten the notation we shall drop the superscript on x_1^*, x_2^* in the following. We rewrite eq. (24) as

$$I = \frac{1}{S}\left\{\int_{\sqrt{\tau_+}\, e^{y}}^{1} \frac{dx_1\, f(x_1, x_2)}{x_1 - x_1^+} + \int_{\sqrt{\tau_+}\, e^{-y}}^{1} \frac{dx_2\, f(x_1, x_2)}{x_2 - x_2^+}\right\}. \tag{27}$$

Inserting the parton cross section equation (19) into eq. (23), the hadronic cross section in n dimensions is given by

$$\frac{d\sigma}{dq_T^2\, dy} = N \frac{\alpha_s}{2\pi} C_F \frac{1 - \varepsilon}{\Gamma(1 - \varepsilon)} \left(\frac{4\pi\mu^2}{q_T^2}\right)^{\varepsilon}$$

$$\times \int dx_1\, dx_2\, M_1(s, t, u)\, \delta\big(x_1 x_2 - x_1 x_2^+ - x_2 x_1^+ + x_1^0 x_2^0\big), \tag{28}$$

TABLE 1

$$\mathrm{p}(P_1)+\bar{\mathrm{p}}(P_2)\to \mathrm{B}(q)+\mathrm{X}$$

$$S=(P_1+P_2)^2;\quad T=(P_1-q)^2;\quad U=(P_2-q)^2;\quad q^2=Q^2;\quad \tau=Q^2/S$$

$$y=\tfrac{1}{2}\ln\frac{q^0+q^3}{q^0-q^3};\quad (T-Q^2)=-\sqrt{S}\left(Q^2+q_\mathrm{T}^2\right)^{1/2}\mathrm{e}^{-y};\quad (U-Q^2)=-\sqrt{S}\left(Q^2+q_\mathrm{T}^2\right)^{1/2}\mathrm{e}^{y}$$

$$x_1^0=\sqrt{\tau}\,\mathrm{e}^{y};\quad x_2^0=\sqrt{\tau}\,\mathrm{e}^{-y};\quad x_1^+=\frac{(Q^2-U)}{S};\quad x_2^+=\frac{(Q^2-T)}{S},\quad x_1^*=\frac{x_2x_1^+-\tau}{x_2-x_2^+};$$

$$x_2^*=\frac{x_1x_2^+-\tau}{x_1-x_1^+};$$

$$A_\mathrm{T}^2=\left[\frac{(S+Q^2)^2}{4S\cosh^2 y}-Q^2\right]=Q^2\frac{\left(1-x_1^{0^2}\right)\left(1-x_2^{0^2}\right)}{\left(x_1^0+x_2^0\right)^2},\quad \sqrt{\tau_\pm}=\sqrt{q_\mathrm{T}^2/S}\pm\left(\tau+q_\mathrm{T}^2/S\right)^{1/2}$$

$$s=(p_1+p_2)^2=x_1x_2S;\quad t=(p_1-q)^2=x_1(T-Q^2)+Q^2;\quad u=(p_2-q)^2=x_2(U-Q^2)+Q^2$$

and the matrix element squared expressed in hadronic variables is

$$M_1=\left\{(1-\varepsilon)\frac{(1-\tau/x_1x_2)^2}{q_\mathrm{T}^2}+\frac{2\tau}{x_1x_2q_\mathrm{T}^2}-\frac{2}{x_1x_2S}\right\}. \tag{29}$$

By addition and subtraction of the residues at $q_\mathrm{T}^2=0$, we can isolate the terms which are singular in the limit $q_\mathrm{T}^2\to 0$. We treat the x_1 and x_2 integrals resulting from the application of eq. (27) in eq. (28) separately. The x_1 integral may be written (dropping overall factors) as

$$\begin{aligned}
&\int_{\sqrt{\tau_+}\mathrm{e}^y}^1\frac{\mathrm{d}x_1}{x_1-x_1^+}H(x_1,x_2)\left\{(1-\varepsilon)\left(1-\frac{\tau}{x_1x_2}\right)^2\frac{1}{q_\mathrm{T}^2}+\frac{2\tau}{x_1x_2q_\mathrm{T}^2}-\frac{2}{x_1x_2S}\right\}\left(\frac{1}{q_\mathrm{T}^2}\right)^\varepsilon\\
&\equiv\int_{\sqrt{\tau_+}\mathrm{e}^y}^1\frac{\mathrm{d}x_1}{x_1-x_1^+}\left(\frac{1}{q_\mathrm{T}^2}\right)^{1+\varepsilon}\left\{H(x_1,x_2)\left[(1-\varepsilon)\left(1-\frac{\tau}{x_1x_2}\right)^2+\frac{2\tau}{x_1x_2}\right]\right.\\
&\qquad\left.-2H(x_1^0,x_2^0)\right\}\\
&\quad+2H(x_1^0,x_2^0)\left(\frac{1}{q_\mathrm{T}^2}\right)^{1+\varepsilon}\int_{\sqrt{\tau_+}\mathrm{e}^y}^1\frac{\mathrm{d}x_1}{x_1-x_1^+}-2\int_{\sqrt{\tau_+}\mathrm{e}^y}^1\frac{\mathrm{d}x_1}{x_1-x_1^+}\frac{H(x_1,x_2)}{x_1x_2S}\,.
\end{aligned} \tag{30}$$

The step performed in the above equation must be made before subtracting the residue of the pole at $q_\mathrm{T}^2=0$. Otherwise, one would be led to an unregulated divergence at $x_{1,2}=x_{1,2}^0$, because at $q_\mathrm{T}=0$, $x_{1,2}^+=x_{1,2}^0$, $\sqrt{\tau_+}\,\mathrm{e}^{\pm y}=x_{1,2}^0$.

A similar formula can be derived for the x_2 part of the integral. We now isolate the poles in q_T^2 by further subtraction and use of the identity

$$\int_0^{A_T^2} dq_T^2 \frac{f(q_T^2)}{(q_T^2)^{1+\varepsilon}} \equiv \int_0^{A_T^2} dq_T^2 \frac{f(q_T^2)-f(0)}{(q_T^2)^{1+\varepsilon}} + f(0)\int_0^{A_T^2} \frac{dq_T^2}{(q_T^2)^{1+\varepsilon}}, \tag{31}$$

where A_T^2 is the kinematic limit of the transverse momentum squared (cf. table 1). Eq. (31) can formally be written as

$$\frac{1}{(q_T^2)^{1+\varepsilon}} \equiv \frac{1}{(q_T^2)_+} + \left(\ln A_T^2 - \frac{1}{\varepsilon}\right)\delta(q_T^2) + O(\varepsilon). \tag{32}$$

Applying this separation to the first term in eq. (30) we obtain

$$\frac{1}{(q_T^2)_+}\int_{\sqrt{\tau_+}e^y}^1 \frac{dx_1}{x_1 - x_1^+}\left\{H(x_1,x_2)\left[\left(1-\frac{\tau}{x_1x_2}\right)^2 + \frac{2\tau}{x_1x_2}\right] - 2H(x_1^0,x_2^0)\right\}$$
$$+\delta(q_T^2)\left(\ln A_T^2 - \frac{1}{\varepsilon}\right)\int_{x_1^0}^1 \frac{dx_1}{x_1 - x_1^0}\left\{H(x_1,x_2^0)\left[(1-\varepsilon)\left(1-\frac{x_1^0}{x_1}\right)^2 + \frac{2x_1^0}{x_1}\right]\right.$$
$$\left. - 2H(x_1^0,x_2^0)\right\}. \tag{33}$$

When applying the same procedure to the second term in eq. (30), it is convenient to consider the x_1 and x_2 integrals simultaneously:

$$2H(x_1^0,x_2^0)\frac{1}{(q_T^2)^{1+\varepsilon}}\left[\int_{\sqrt{\tau_+}e^y}^1 \frac{dx_1}{x_1 - x_1^+} + \int_{\sqrt{\tau_+}e^{-y}}^1 \frac{dx_2}{x_2 - x_2^+}\right]$$
$$= 2H(x_1^0,x_2^0)\frac{1}{(q_T^2)^{1+\varepsilon}}\ln\frac{(1-x_1^+)(1-x_2^+)S}{q_T^2}. \tag{34}$$

By use of eq. (32) and the identity

$$\frac{1}{(q_T^2)^{1+\varepsilon}}\ln q_T^2 = \left(\frac{\ln q_T^2}{q_T^2}\right)_+ + \left(\tfrac{1}{2}\ln^2 A_T^2 - \frac{1}{\varepsilon^2}\right)\delta(q_T^2) + O(\varepsilon), \tag{35}$$

eq. (34) may be written as

$$2H(x_1^0,x_2^0)\left\{\frac{1}{(q_T^2)_+}\ln[(1-x_1^+)(1-x_2^+)S] - \left(\frac{\ln q_T^2}{q_T^2}\right)_+\right\}$$
$$+2H(x_1^0,x_2^0)\delta(q_T^2)\left\{\left(\ln A_T^2 - \frac{1}{\varepsilon}\right)\ln[(1-x_1^0)(1-x_2^0)S] + \frac{1}{\varepsilon^2} - \tfrac{1}{2}\ln^2 A_T^2\right\}. \tag{36}$$

The third term in eq. (30) is completely finite as $q_T \to 0$ and requires no subtraction. Three further manipulations complete step II:

(i) Addition of the contributions from the x_2 part of the integral where not already included.

(ii) Addition of the virtual contribution of eq. (21). After dropping the lowest-order term and the same overall factor as in the above, the virtual contribution is

$$H(x_1^0, x_2^0)\delta(q_T^2)\left(-\frac{2}{\varepsilon^2}+\frac{2}{\varepsilon}\ln\frac{Q^2}{\mu^2}-\ln^2\frac{Q^2}{\mu^2}-\frac{3}{\varepsilon}+3\ln\frac{Q^2}{\mu^2}+\pi^2-8\right). \tag{37}$$

(iii) Simplification of the formula by use of the identity

$$\int_{x_1^0}^1 \frac{dz}{z}\frac{f(z)-f(1)}{1-z} = \int_{x_1^0}^1 \frac{dz}{z} f(z)\frac{1}{(1-z)_+} + f(1)\ln\frac{x_1^0}{1-x_1^0}. \tag{38}$$

The "+" distribution in the above formula is defined on the range 0 to 1 such that

$$\int_0^1 dz\,\frac{f(z)}{(1-z)_+} = \int_0^1 dz\,\frac{f(z)-f(1)}{1-z}. \tag{39}$$

Upon completion of these manipulations we obtain

$$\begin{aligned}
\frac{d\sigma}{dq_T^2\,dy} = N\frac{\alpha_s}{2\pi}C_F\Bigg\{ & H(x_1^0, x_2^0)\left[\frac{2}{(q_T^2)_+}\ln((1-x_1^+)(1-x_2^+)S) - 2\left(\frac{\ln q_T^2}{q_T^2}\right)_+ \right.\\
& \left. + \left(3\ln\frac{Q^2}{A_T^2}-\ln^2\frac{Q^2}{A_T^2}\right)\delta(q_T^2)\right]\\
& + \frac{1}{(q_T^2)_+}\left[\int_{\sqrt{\tau_+}e^{y}}^1 \frac{dx_1}{x_1-x_1^+} + \int_{\sqrt{\tau_+}e^{-y}}^1 \frac{dx_2}{x_2-x_2^+}\right]\\
& \times\left[H(x_1,x_2)\left(1+\left(\frac{\tau}{x_1x_2}\right)^2\right) - 2H(x_1^0, x_2^0)\right]\\
& + \delta(q_T^2)\left(\ln\left(\frac{A_T}{\mu}\right)^2 - \frac{1}{\hat{\varepsilon}}\right)\left[\int_{x_1^0}^1 \frac{dz}{z}P_{qq}(z)H(x_1^0/z, x_2^0)\right.\\
& \left. + \int_{x_2^0}^1 \frac{dz}{z}P_{qq}(z)H(x_1^0, x_2^0/z)\right]\\
& + \delta(q_T^2)\left[\int_{x_1^0}^1 \frac{dz}{z}c_q(z)H(x_1^0/z, x_2^0) + \int_{x_2^0}^1 \frac{dz}{z}c_q(z)H(x_1^0, x_2^0/z)\right]\\
& - \left[\int_{\sqrt{\tau_+}e^{y}}^1 \frac{dx_1}{x_1-x_1^+} + \int_{\sqrt{\tau_+}e^{-y}}^1 \frac{dx_2}{x_2-x_2^+}\right]\frac{2H(x_1,x_2)}{x_1x_2S}\Bigg\},
\end{aligned} \tag{40}$$

where

$$P_{qq}(z)=\frac{1+z^2}{(1-z)_+}+\frac{3}{2}\delta(1-z),$$

$$c_q(z)=(1-z)+\delta(1-z)\left(\tfrac{1}{2}\pi^2-4\right),$$

$$\frac{1}{\hat{\varepsilon}}=\frac{1}{\varepsilon}+\ln(4\pi)-\gamma_E. \tag{41}$$

We can finally write the result of the order α_s $q\bar{q}$ annihilation term:

$$\frac{d\sigma}{dq_T^2\,dy}=X_q\left(q_T^2,Q^2,y\right)+Y_q\left(q_T^2,Q^2,y\right), \tag{42}$$

where

$$X_q\left(q_T^2,Q^2,y\right)=X_q^{(0)}+X_q^{(1)},$$

$$X_q^{(0)}=NH\left(x_1^0,x_2^0\right)\delta\left(q_T^2\right),$$

$$X_q^{(1)}=N\frac{\alpha_s}{2\pi}C_F\left\{H\left(x_1^0,x_2^0\right)\left[2\left(\frac{\ln\left(Q^2/q_T^2\right)}{q_T^2}\right)_+-\frac{3}{\left(q_T^2\right)_+}\right.\right.$$
$$\left.+\left(-3\ln\frac{A_T^2}{Q^2}-\ln^2\frac{A_T^2}{Q^2}\right)\delta\left(q_T^2\right)\right]$$
$$+\left[\frac{1}{\left(q_T^2\right)_+}+\delta\left(q_T^2\right)\left(\ln\frac{A_T^2}{\mu^2}-\frac{1}{\hat{\varepsilon}}\right)\right]$$
$$\times\left[\int_{x_1^0}^1\frac{dz}{z}H\left(x_1^0/z,x_2^0\right)P_{qq}(z)+\int_{x_2^0}^1\frac{dz}{z}H\left(x_1^0,x_2^0/z\right)P_{qq}(z)\right]$$
$$+\delta\left(q_T^2\right)\left[\int_{x_1^0}^1\frac{dz}{z}c_q(z)H\left(x_1^0/z,x_2^0\right)\right.$$
$$\left.\left.+\int_{x_2^0}^1\frac{dz}{z}c_q(z)H\left(x_1^0,x_2^0/z\right)\right]\right\}. \tag{43}$$

The + distributions in the above equation are defined with respect to the upper limit of the q_T^2 integration A_T^2. A different choice B^2 for the upper limit would modify the distributions such that

$$\frac{1}{(q_T^2)_+} \to \frac{1}{(q_T^2)_{B_+}} + \ln\frac{B^2}{A_T^2}\delta(q_T^2),$$

$$\left(\frac{\ln(Q^2/q_T^2)}{q_T^2}\right)_+ \to \left(\frac{\ln(Q^2/q_T^2)}{q_T^2}\right)_{B_+} + \tfrac{1}{2}\ln\frac{B^2}{A_T^2}\ln\left(\frac{Q^4}{A_T^2 B^2}\right)\delta(q_T^2). \tag{44}$$

The second term in eq. (42), Y_q contains only terms which are finite in the limit $q_T^2 \to 0$. The explicit form of Y_q will be given in the next section. This completes step III.

The fourth step is to perform the Fourier transform of X_q to impact parameter space. The expression for X_q becomes

$$\begin{aligned} X_q(b^2, Q^2, y) = N\Bigg\{ & H(x_1^0, x_2^0)\left[1 + F(Q^2, y) + S(b^2, Q^2, y)\right] \\ & + \frac{\alpha_s}{2\pi}C_F\left(\ln\frac{P^2}{\mu} - \frac{1}{\hat{\varepsilon}}\right)\left[\int_{x_1^0}^1 \frac{dz}{z} H(x_1^0/z, x_2^0) P_{qq}(z)\right. \\ & \left. + \int_{x_2^0}^1 \frac{dz}{z} H(x_1^0, x_2^0/z) P_{qq}(z)\right] \\ & + \frac{\alpha_s}{2\pi}C_F\left[\int_{x_1^0}^1 \frac{dz}{z} c_q(z) H(x_1^0/z, x_2^0)\right. \\ & \left. + \int_{x_2^0}^1 \frac{dz}{z} c_q(z) H(x_1^0, x_2^0/z)\right]\Bigg\}, \end{aligned} \tag{45}$$

where

$$F(Q^2, y) = \frac{\alpha_s}{2\pi}C_F\left[-3\ln\frac{A_T^2}{Q^2} - \ln^2\frac{A_T^2}{Q^2}\right], \tag{46}$$

$$\ln P^2 = \ln A_T^2 + \int_0^{A_T^2} \frac{d^2 q_T}{\pi}\left(e^{i b\cdot q_T} - 1\right)\frac{1}{q_T^2}, \tag{47}$$

and $S(b)$ is given by eq. (15). The collinear singularities are factored and the finite terms are fixed in terms of the deep inelastic structure function F_2 at the scale Q^2:

$$q(x,Q^2) = q_0(x) + \int_x^1 \frac{dz}{z}\frac{\alpha_s}{2\pi} C_F \left[\left(\ln\frac{Q^2}{\mu^2} - \frac{1}{\hat{\varepsilon}}\right) P_{qq}(z) + c_q^{(2)}(z)\right] q_0\left(\frac{x}{z}\right)$$
$$+ \int_x^1 \frac{dz}{z}\frac{\alpha_s}{2\pi} T_R \left[\left(\ln\frac{Q^2}{\mu^2} - \frac{1}{\hat{\varepsilon}}\right) P_{qg}(z) + c_g^{(2)}(z)\right] g_0\left(\frac{x}{z}\right), \tag{48}$$

where $T_R = \frac{1}{2}$ and [5]

$$c_q^{(2)}(z) = \left\{(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ - \frac{3}{2}\frac{1}{(1-z)_+} - \frac{1+z^2}{1-z}\ln z + 3\right.$$
$$\left. +2z - \left(\tfrac{9}{2} + \tfrac{1}{3}\pi^2\right)\delta(1-z)\right\},$$
$$c_g^{(2)}(z) = \left\{\left[z^2 + (1-z)^2\right]\ln\frac{1-z}{z} + 6z(1-z)\right\}. \tag{49}$$

After resummation of the form factor S we obtain in b space:

$$X_q(b^2,Q^2,y) = N\left\{\exp S(b^2,Q^2,y)\left[H(x_1^0,x_2^0,P^2)(1+F(Q^2,y))\right]\right.$$
$$\left. + \frac{\alpha_s}{2\pi} C_F \left[\int_{x_1^0}^1 \frac{dz}{z} f_q(z) H(x_1^0/z, x_2^0) + \int_{x_2^0}^1 \frac{dz}{z} f_q(z) H(x_1^0, x_2^0/z)\right]\right\}, \tag{50}$$

where

$$f_q(z) = c_q(z) - c_q^{(2)}(z). \tag{51}$$

The gluon term in eq. (48) will be included in the Compton contribution which we now consider.

The result from the quark–gluon scattering (fig. 2) is given by

$$\frac{s}{\pi}\frac{d\sigma^{qg}}{dt\,du} = N'\frac{\alpha_s}{2\pi} T_R \frac{1-\varepsilon}{\Gamma(1-\varepsilon)}\left(\frac{4\pi\mu^2}{q_T^2}\right)^{\varepsilon} M_2(s,t,u)\,\delta(s+t+u-Q^2), \tag{52}$$

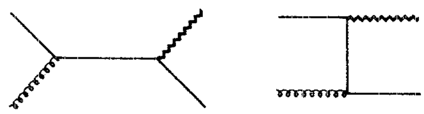

Fig. 2. The production of a vector boson from an initial state gluon.

where the matrix element squared is

$$M_2(s,t,u)=\left\{\frac{1}{q_T^2}\left[-(1-\varepsilon)\frac{u}{s}-\frac{2u^2Q^2}{s^3}\right]-(1-\varepsilon)\frac{t}{s^2}\right\}. \tag{53}$$

Proceeding through the first three steps we easily obtain

$$\begin{aligned}\frac{d\sigma}{dq_T^2\,dy}=N\frac{\alpha_s}{2\pi}T_R\Bigg\{\Bigg[\Bigg[&\int dx_1\,dx_2\,\delta(x_1x_2-x_1x_2^+-x_2x_1^++x_1^0x_2^0)K_1(x_1,x_2)\\
&\times\left[\frac{1}{q_T^2}\left(\frac{x_2x_1^+-x_1^0x_2^0}{x_1x_2}-\frac{2x_1^0x_2^0(x_2x_1^+-x_1^0x_2^0)^2}{(x_1x_2)^3}\right)\right.\\
&\left.+\frac{x_1x_2^+-x_1^0x_2^0}{(x_1x_2)^2S}\right]\\
&-\int_{x_2^0}^1 dx_2\,K_1(x_1^0,x_2)\frac{1}{q_T^2}\left(\frac{1}{x_2}-\frac{2(x_2-x_2^0)x_2^0}{x_2^3}\right)\Bigg]+[(1\leftrightarrow 2)]\\
&+\left[\frac{1}{(q_T^2)_+}+\ln\frac{A_T^2}{\mu^2}-\frac{1}{\hat{\varepsilon}}\right]\left[\int_{x_2^0}^1\frac{dz}{z}P_{qg}(z)K_1(x_1^0,x_2^0/z)\right.\\
&\left.+\int_{x_1^0}^1\frac{dz}{z}P_{qg}(z)K_2(x_1^0/z,x_2^0)\right]\\
&+\delta(q_T^2)\left[\int_{x_2^0}^1\frac{dz}{z}K_1(x_1^0,x_2^0/z)c_g(z)+\int_{x_1^0}^1\frac{dz}{z}c_g(z)K_2(x_1^0/z,x_2^0)\right]\Bigg]\Bigg\},\end{aligned} \tag{54}$$

where

$$\begin{aligned}P_{qg}(z)&=\left[z^2+(1-z)^2\right],\qquad c_g(z)=1,\\
K_1(x_1,x_2)&\sim\left[q(x_1)+\bar{q}(x_1)\right]g(x_2),\\
K_2(x_1,x_2)&\sim\left[q(x_2)+\bar{q}(x_2)\right]g(x_1).\end{aligned} \tag{55}$$

Eq. (54) is written for proton (anti-)proton scattering. With obvious modifications to take into account the differing distributions of quarks in pions and protons, it would also be valid for πP scattering.

3. Analytic results

The final analytic results suitable for use in numerical calculations are given here. The results form a self-contained set which can be used almost without reference to previous sections. The principal analytic result of our paper is the expression for the cross section for the production of a vector boson:

$$\frac{d\sigma}{dq_T^2\, dy} = N\left\{\int \frac{d^2\boldsymbol{b}}{4\pi} e^{-iq_T \cdot b} \exp S(b^2, Q^2, y) R(b^2, Q^2, y) + Y(q_T^2, Q^2, y)\right\}, \tag{56}$$

where

$$\begin{aligned} R(b^2, Q^2, y) = {} & H(x_1^0, x_2^0, P^2)\left[1 + \frac{\alpha_s}{2\pi}\frac{4}{3}\left(-3\ln\frac{A_T^2}{Q^2} - \ln^2\frac{A_T^2}{Q^2}\right)\right] \\ & + \frac{\alpha_s}{2\pi}\frac{4}{3}\left[\int_{x_1^0}^1 \frac{dz}{z} f_q(z) H(x_1^0/z, x_2^0, P^2)\right. \\ & \left. + \int_{x_2^0}^1 \frac{dz}{z} f_q(z) H(x_1^0, x_2^0/z, P^2)\right] \\ & + \frac{\alpha_s}{2\pi}\frac{1}{2}\left[\int_{x_1^0}^1 \frac{dz}{z} f_g(z) K_2(x_1^0/z, x_2^0, P^2)\right. \\ & \left. + \int_{x_2^0}^1 \frac{dz}{z} f_g(z) K_1(x_1^0, x_2^0/z, P^2)\right]. \end{aligned} \tag{57}$$

In this formula the scale at which the parton densities in H, K_1 and K_2 are probed is given by

$$\ln P^2 = \left[\ln A_T^2 + \int_0^{A_T^2} \frac{d^2 q_T}{\pi} (e^{-ib\cdot q_T} - 1) \frac{1}{q_T^2}\right] \sim \ln \frac{b_0^2}{b^2}, \tag{58}$$

where $b_0^2 = 4\,e^{-2\gamma_E} \sim 1.261$ [19] and the correction terms are defined as

$$f_q(z) = \left\{ \frac{3}{2} \frac{1}{(1-z)_+} - (1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ \right.$$
$$\left. + \frac{1+z^2}{1-z} \ln z - 2 - 3z + \left(\tfrac{1}{2} + \tfrac{5}{6}\pi^2\right)\delta(1-z) \right\},$$
$$f_g(z) = \left\{ 1 - 6z(1-z) - \left[z^2 + (1-z)^2\right] \ln \frac{1-z}{z} \right\}. \tag{59}$$

The quark distributions used in the above formula are defined beyond the leading order in terms of the deep inelastic structure function F_2 at the scale Q^2 and are given in eqs. (66). The form factor S is obtained by Fourier transforming the expression in eq. (13):

$$S(b^2, Q^2, y) = \int_0^{A_T^2} dq^2 \left[J_0(bq) - 1\right] \frac{\alpha_s(q^2)}{2\pi} \frac{4}{3} \left[2 \frac{\ln(Q^2/q^2)}{q^2} - \frac{3}{q^2}\right]. \tag{60}$$

Note that there is no ambiguity in the value of the upper limit of integration A_T^2. Given the total cross section constraint a change of A_T^2 should be compensated by a corresponding change of R in eq. (57). For example, if one prefers Q^2 as an upper limit in the exponential, the replacement

$$\exp \int_0^{A_T^2} \simeq \left(1 + \int_{Q^2}^{A_T^2}\right) \exp \int_0^{Q^2} \tag{61}$$

can be made. This is allowed because $\alpha_s(q_T^2)$ is small for $Q^2 < q_T^2 < A_T^2$. The resulting additional contribution to R cancels in this case the logarithms appearing in eq. (57) in the large-b limit. Actually, the replacement in eq. (61) is demanded when $A_T^2 \gg Q^2$, as for example is the case of W production at tevatron energies, in order to keep under numerical control the large $\log(A_T^2/Q^2)$ terms.

The residual finite term Y can be divided into the parts due to the annihilation and Compton scattering graphs:

$$Y(q_T^2, Q^2, y) = \frac{\alpha_s}{2\pi} \frac{4}{3} Y_q(q_T^2, Q^2, y) + \frac{\alpha_s}{2\pi} \frac{1}{2} Y_g(q_T^2, Q^2, y), \tag{62}$$

where

$$Y_q(q_T^2, Q^2, y) = -\frac{2}{S}\left[\int_{\sqrt{\tau_+}\,e^{y}}^{1}\frac{dx_1}{(x_1-x_1^+)}\,\frac{H(x_1,x_2^*)}{x_1x_2^*}+\int_{\sqrt{\tau_+}\,e^{-y}}^{1}\frac{dx_2}{(x_2-x_2^+)}\,\frac{H(x_1^*,x_2)}{x_1^*x_2}\right]$$

$$+\frac{1}{q_T^2}\left\{\int_{\sqrt{\tau_+}\,e^{y}}^{1}\frac{dx_1}{(x_1-x_1^+)}\left[H(x_1,x_2^*)\left(1+\left(\frac{\tau}{x_1x_2^*}\right)^2\right)-2H(x_1^0,x_2^0)\right]\right.$$

$$-\int_{x_1^0}^{1}\frac{dx_1}{(x_1-x_1^0)}\left[H(x_1,x_2^0)\left(1+\left(\frac{x_1^0}{x_1}\right)^2\right)-2H(x_1^0,x_2^0)\right]$$

$$+\int_{\sqrt{\tau_+}\,e^{-y}}^{1}\frac{dx_2}{(x_2-x_2^+)}\left[H(x_1^*,x_2)\left(1+\left(\frac{\tau}{x_1^*x_2}\right)^2\right)-2H(x_1^0,x_2^0)\right]$$

$$-\int_{x_1^0}^{1}\frac{dx_2}{(x_2-x_2^0)}\left[H(x_1^0,x_2)\left(1+\left(\frac{x_2^0}{x_2}\right)^2\right)-2H(x_1^2,x_2^0)\right]$$

$$\left.+2H(x_1^0,x_2^0)\ln\frac{(1-x_1^+)(1-x_2^+)}{(1-x_1^0)(1-x_2^0)}\right\}, \qquad (63)$$

$$Y_g(q_T^2, Q^2, y) = \left\{\frac{1}{q_T^2}\int_{\sqrt{\tau_+}\,e^{y}}^{1}\frac{dx_1}{(x_1-x_1^+)}K_1(x_1,x_2^*)\left[\frac{x_2^*x_1^+-\tau}{x_1x_2^*}-\frac{2\tau(x_2^*x_1^+-\tau)^2}{(x_1x_2^*)^3}\right]\right.$$

$$+\frac{1}{q_T^2}\int_{\sqrt{\tau_+}\,e^{-y}}^{1}\frac{dx_2}{(x_2-x_2^+)}K_1(x_1^*,x_2)\left[\frac{x_2x_1^+-\tau}{x_1^*x_2}-\frac{2\tau(x_2x_1^+-\tau)^2}{(x_1^*x_2)^3}\right]$$

$$-\frac{1}{q_T^2}\int_{x_2^0}^{1}\frac{dx_2}{x_2}K_1(x_1^0,x_2)\left[1-2\frac{x_2^0}{x_2}\left(1-\frac{x_2^0}{x_2}\right)\right]$$

$$+\frac{1}{S}\left[\int_{\sqrt{\tau_+}\,e^{y}}^{1}\frac{dx_1}{(x_1-x_1^+)}K_1(x_1,x_2^*)\,\frac{x_1x_2^+-\tau}{(x_1x_2^*)^2}\right.$$

$$\left.\left.+\int_{\sqrt{\tau_+}\,e^{-y}}^{1}\frac{dx_2}{(x_2-x_2^+)}K_1(x_1^*,x_2)\,\frac{x_1^*x_2^+-\tau}{(x_1^*x_2)^2}\right]+(1\leftrightarrow 2)\right\}, \qquad (64)$$

where $x_{1,2}$, $x^+_{1,2}$, $x^*_{1,2}$, $\sqrt{\tau_+}$ are defined in table 1 and the exchange $(1 \leftrightarrow 2)$ als includes $K_1(x_1, x_2^*) \leftrightarrow K_2(x_1^*, x_2)$, $K_1 x_1^*, x_2) \leftrightarrow K_2(x_1, x_2^*)$, $K_1(x_1^0, x_2) \leftrightarrow K_2(x_1, x_2^0)$ and $y \leftrightarrow -y$.

The overall normalization factors in eq. (56) for the production of the three type of vector bosons are given by

$$N(\gamma^*) = \frac{4\pi^2\alpha}{3S}, \qquad N(\mathrm{Z}) = \frac{\pi^2\alpha_{\mathrm{w}}}{12S\cos^2\theta_{\mathrm{w}}}, \qquad N(\mathrm{W}) = \frac{\pi^2\alpha_{\mathrm{w}}}{3S}, \tag{65}$$

and $\alpha_{\mathrm{w}} = \alpha/\sin^2\theta_{\mathrm{w}}$. The differential cross section $\mathrm{d}\sigma/\mathrm{d}Q^2\mathrm{d}q_{\mathrm{T}}^2\mathrm{d}y$ usually quote for the production of a lepton pair of invariant mass Q is obtained by multiplyin $N(\gamma^*)$ by a further factor of $(\alpha/3\pi Q^2)$. We also define the products of the parto distribution functions:

$$H^{\gamma}(x_1, x_2, Q^2) = \sum_f e_f^2 \{ q_f(x_1, Q^2)\bar{q}_f(x_2, Q^2) + (1 \leftrightarrow 2) \},$$

$$H^{\mathrm{Z}}(x_1, x_2, Q^2) = \sum_f n_f^z \{ q_f(x_1, Q^2)\bar{q}_f(x_2, Q^2) + (1 \leftrightarrow 2) \},$$

$$\begin{aligned} H^{\mathrm{W}^+}(x_1, x_2, Q^2) = \{ & [u(x_1, Q^2)\bar{d}(x_2, Q^2) + c(x_1, Q^2)\bar{s}(x_2, Q^2)]\cos^2\theta_{\mathrm{c}} \\ & + [u(x_1, Q^2)\bar{s}(x_2, Q^2) + c(x_1, Q^2)\bar{d}(x_2, Q^2)]\sin^2\theta_{\mathrm{c}} \}, \end{aligned} \tag{66}$$

and a similar expression for H^{W^-}. A quark of flavour f has a charge e_f, and

$$n_f^z = \left[\left(1 - 4|e_f|\sin^2\theta_{\mathrm{w}}\right)^2 + 1 \right]. \tag{67}$$

The analogous results involving the gluon distribution functions are

$$K_1^{\gamma}(x_1, x_2, Q^2) = \sum_f e_f^2 [q_f(x_1, Q^2) + \bar{q}_f(x_1, Q^2)] g(x_2, Q^2),$$

$$K_1^{\mathrm{Z}}(x_1, x_2, Q^2) = \sum_f n_f^z [q_f(x_1, Q^2) + \bar{q}_f(x_1, Q^2)] g(x_2, Q^2),$$

$$K_1^{\mathrm{W}^+}(x_1, x_2, Q^2) = [u(x_1, Q^2) + c(x_1, Q^2) + \bar{d}(x_1, Q^2) + \bar{s}(x_1, Q^2)] g(x_2, Q^2) \tag{68}$$

while $K_2^i(x_1, x_2, Q^2)$ are obtained by interchanging quark and gluon densit

$(K_2 - q(x_2)g(x_1))$. In eqs. (62)–(64), no scale is written for α_s and the product of the parton densities in $Y_{q,g}$ which is undetermined to this order in α_s. We have taken both these scales to be Q^2. A different choice leads to numerical variations of the results which have to be taken into account in the estimate of the theoretical error.

Our result for the form factor is in agreement with the general formalism of Collins and Soper [12, 13] in the leading DLA. After some manipulations their result for the form factor can be written as

$$S_{CS}(b^2, Q^2) = -\frac{C_F}{\pi}\int_{c_1^2/b^2}^{c_2^2 Q^2}\frac{dk^2}{k^2}\left[\ln\frac{c_2^2 Q^2}{k^2}A(k^2) + B(k^2)\right], \tag{69}$$

where in the $\overline{\mathrm{MS}}$ scheme:

$$A(k^2) = \left[\alpha_s(k^2) + D\alpha_s^2(k^2) - \ln\frac{c_1^2}{b_0^2}\frac{d}{d\ln k^2}\alpha_s(k^2)\right] + O(\alpha_s^3),$$

$$B(k^2) = \alpha_s(k^2)\left[-\tfrac{3}{2} + \ln\frac{c_1^2}{b_0^2} - \ln c_2^2\right] + O(\alpha_s^2), \tag{70}$$

and c_1 and c_2 are arbitrary constants. A change in c_1 or c_2 in the form factor is exactly compensated by corresponding variations elsewhere in their complete formula.

Our result corresponds to the natural choice of the parameters:

$$c_1 = b_0 = 2\,e^{-\gamma_E},$$

$$c_2 = \frac{A_T}{Q} = \frac{\left[\left(1 - x_1^{0^2}\right)\left(1 - x_2^{0^2}\right)\right]^{1/2}}{\left(x_1^0 + x_2^0\right)}. \tag{71}$$

With these choices our expression in terms of the Bessel functions J_0 agrees with eq. (69) at the DLA. The term $-\frac{3}{2}\alpha(q_T)$, subleading in the limit $q_T^2 \to 0$, is directly present in our formula. D is the correction term given by [17, 18]

$$D = \frac{1}{2\pi}\left[\tfrac{67}{6} - \tfrac{1}{2}\pi^2 - \tfrac{10}{18}n_f\right]. \tag{72}$$

In sect. 4 we shall study the quantitative importance of this non-leading correction.

Since data exist over a large range of y we report here also the expression for the cross section integrated over y. This expression has a form similar to eq. (56):

$$\frac{d\sigma}{dq_T^2} = N\left\{\int\frac{d^2\boldsymbol{b}}{4\pi}e^{-i\boldsymbol{q}_T\cdot\boldsymbol{b}}\exp S'(b^2, Q^2)R'(b^2, Q^2) + Y'(q_T^2, Q^2)\right\}. \tag{73}$$

The upper limit of the transverse momentum is now given by $A_T'^2 = \frac{1}{4}S(1-\tau)^2$ and consequently the form factor S becomes

$$S'(b^2,Q^2) = \int_0^{A_T'^2} \mathrm{d}k^2 \left(J_0(bk)-1\right)\frac{\alpha_s(k^2)}{2\pi}\frac{4}{3}\left(2\frac{\ln(Q^2/k^2)}{k^2} - \frac{3}{k^2}\right). \quad (74)$$

The function R′ is given by

$$R'(b^2,Q^2) = \int \mathrm{d}x_1\,\mathrm{d}x_2\,\delta(x_1x_2-\tau)H\left(x_1,x_2,P'^2\right)$$

$$\times\left[1+\frac{\alpha_s}{2\pi}\frac{4}{3}\times\left(3\ln\frac{Q^2}{A_T'^2} - \ln^2\frac{Q^2}{A_T'^2}\right)\right] + \frac{\alpha_s}{2\pi}\frac{4}{3}\int\frac{\mathrm{d}x_1\,\mathrm{d}x_2}{x_1x_2}H\left(x_1,x_2,P'^2\right)$$

$$\times 2f_q(\tau_{12})\theta(x_1x_2-\tau) + \frac{\alpha_s}{2\pi}\frac{1}{2}\int\frac{\mathrm{d}x_1\,\mathrm{d}x_2}{x_1x_2}$$

$$\times\left[K_1\left(x_1,x_2,P'^2\right)+K_2\left(x_1,x_2,P'^2\right)\right]f_g(\tau_{12})\theta(x_1x_2-\tau), \quad (75)$$

where we define

$$\tau_{12} = \frac{Q^2}{x_1x_2S}, \qquad \rho_{12} = \frac{q_T^2}{x_1x_2S}, \qquad \rho = \frac{q_T^2}{S}, \quad (76)$$

and f_q and f_g are given in eq. (59). The corresponding finite pieces are (see eqs. (63) and (64))

$$Y_q'(q_T^2,Q^2) = \frac{4}{q_T^2}\int_\tau^1\frac{\mathrm{d}x_1}{x_1}H\left(x_1,\frac{\tau}{x_1}\right)\ln\left[\frac{\sqrt{(x_1-\tau_+)(x_1-\tau_-)}+x_1-\tau-2\rho}{2(x_1-\tau)}\sqrt{\frac{\tau}{\tau+\rho}}\right.$$

$$-\frac{4}{S}\int\frac{\mathrm{d}x_1\,\mathrm{d}x_2}{(x_1x_2)^2}\theta(x_1x_2-\tau_+)\frac{H(x_1,x_2)}{\sqrt{(1-\tau_{12})^2-4\rho_{12}}}$$

$$+\frac{2}{q_T^2}\left\{\int\frac{\mathrm{d}x_1\,\mathrm{d}x_2}{x_1x_2}\theta(x_1x_2-\tau_+)\frac{\left(1+\tau_{12}^2\right)H(x_1,x_2)-2H(x_1,\tau/x_1)}{\sqrt{(1-\tau_{12})^2-4\rho_{12}}}\right.$$

$$\left.-\int\frac{\mathrm{d}x_1\,\mathrm{d}x_2}{x_1x_2}\theta(x_1x_2-\tau)\frac{\left(1+\tau_{12}^2\right)H(x_1,x_2)-2H(x_1,\tau/x_1)}{1-\tau_{12}}\right\}$$

(77)

The expression for the gluonic piece is most easily expressed in terms of the quantities

$$\phi^{+}=\tfrac{1}{2}\left[1+\tau_{12}+\sqrt{(1-\tau_{12})^{2}-4\rho_{12}}\right],\qquad \phi^{-}=\frac{1}{\phi^{+}},\qquad \left(\phi^{+}\underset{q_{\mathrm{T}}^{2}\to 0}{\to}1\right). \tag{78}$$

In terms of these variables, we find

$$\begin{aligned}
Y_{\mathrm{g}}'(q_{\mathrm{T}}^{2},Q^{2})=\Bigg\{&\int\frac{\mathrm{d}x_{1}\,\mathrm{d}x_{2}}{x_{1}x_{2}}\frac{\theta(x_{1}x_{2}-\tau_{+})}{\sqrt{(1-\tau_{12})^{2}-4\rho_{12}}}\frac{1}{q_{\mathrm{T}}^{2}}\\
&\times\left[(\tau_{12}+\rho_{12})\phi^{-}-\tau_{12}-2\tau_{12}\left(\phi^{-}(\tau_{12}+\rho_{12})-\tau_{12}\right)^{2}\right]\\
&\times K_{1}(x_{1},x_{2})+\frac{1}{S}\int\frac{\mathrm{d}x_{1}\,\mathrm{d}x_{2}}{(x_{1}x_{2})^{2}}\frac{\theta(x_{1}x_{2}-\tau_{+})}{\sqrt{(1-\tau_{12})^{2}-4\rho_{12}}}\\
&\times\left[\phi^{+}-2\tau_{12}+(\tau_{12}+\rho_{12})\phi^{-}\right]K_{1}(x_{1},x_{2})\\
&+\frac{1}{q_{\mathrm{T}}^{2}}\Bigg[\int\frac{\mathrm{d}x_{1}\,\mathrm{d}x_{2}}{x_{1}x_{2}}\theta(x_{1}x_{2}-\tau_{+})\frac{(\phi^{+}-\tau_{12})}{\sqrt{(1-\tau_{12})^{2}-4\rho_{12}}}\\
&\qquad\times\left[1-2\tau_{12}(\phi^{+}-\tau_{12})\right]K_{1}(x_{1},x_{2})\\
&\qquad-\int\frac{\mathrm{d}x_{1}\,\mathrm{d}x_{2}}{x_{1}x_{2}}\theta(x_{1}x_{2}-\tau)\left[1-2\tau_{12}(1-\tau_{12})\right]K_{1}(x_{1},x_{2})\Bigg]\Bigg\}\\
&+\{(1\leftrightarrow 2)\}.
\end{aligned} \tag{79}$$

The results for the integrals over q_T of eqs. (56) and (73) are already present in the literature [5, 20]. We perform the integral over q_T of eq. (56) and obtain

$$\frac{d\sigma}{dy} = N\Bigg\{ H(x_1^0, x_2^0, Q^2)\left[1 + \frac{\alpha_s}{2\pi}\frac{4}{3}\left(3\ln\frac{Q^2}{A_T^2} - \ln^2\frac{Q^2}{A_T^2} + I\right)\right]$$

$$+ \frac{\alpha_s}{2\pi}\frac{4}{3}\left[\int_{x_1^0}^1 \frac{dz}{z} f_q(z) H(x_1^0/z, x_2^0) + \int_{x_2^0}^1 \frac{dz}{z} f_q(z) H(x_1^0, x_2^0/z)\right]$$

$$+ \frac{\alpha_s}{2\pi}\frac{1}{2}\left[\int_{x_2^0}^1 \frac{dz}{z} f_g(z) K_1(x_1^0, x_2^0/z) + \int_{x_1^0}^1 \frac{dz}{z} f_g(z) K_2(x_1^0/z, x_2^0)\right]$$

$$+ W(Q^2, y), \qquad (80)$$

where

$$I = 2\int_0^{A_T^2/S} \frac{dx}{x} \ln\left[\frac{1-\sqrt{\tau + x}\,e^{y}}{1 - x_1^0}\,\frac{1 - \sqrt{\tau+x}\,e^{-y}}{1 - x_2^0}\right]$$

$$= \mathrm{Li}_2\left(\frac{2x_1^0}{1 + x_1^0}\right) + \mathrm{Li}_2\left(\frac{2x_2^0}{1+x_2^0}\right) - \tfrac{1}{3}\pi^2 + 2\ln\frac{x_1^0(1 + x_2^0)}{x_1^0 + x_2^0}\ln\frac{x_2^0(1+x_1^0)}{x_1^0 + x_2^0}$$

$$+ \ln\frac{1 + x_1^0}{1 - x_1^0}\ln\frac{1 + x_1^0}{2x_1^0} + \ln\frac{1 + x_2^0}{1 - x_2^0}\ln\frac{1+x_2^0}{2x_2^0}, \qquad (81)$$

where as usual

$$\mathrm{Li}_2(x) = -\int_0^x \frac{dz}{z}\ln(1 - z); \qquad (82)$$

$$W(Q^2, y) = \frac{\alpha_s}{2\pi}\frac{4}{3} W_q(Q^2, y) + \frac{\alpha_s}{2\pi}\frac{1}{2} W_g(Q^2, y), \qquad (83)$$

$$W_q(Q^2, y) = \int_{x_1^0}^1 \frac{dx_1}{x_1 - x_1^0}\int_{x_2^0}^1 \frac{dx_2}{x_2 - x_2^0}$$

$$\times\left[h(x_1, x_2) - h(x_1, x_2^0) - h(x_1^0, x_2) + h(x_1^0, x_2^0)\right]$$

$$- \int_{x_1^0}^1 \frac{dx_1}{x_1 - x_1^0}\ln\frac{(x_1 + x_1^0)A_T^2}{2x_1^0(x_1 - x_1^0)(1 - x_2^0)S}\left[h(x_1, x_2^0) - h(x_1^0, x_2^0)\right]$$

$$- \int_{x_2^0}^1 \frac{dx_2}{x_2 - x_2^0}\ln\frac{(x_2 + x_2^0)A_T^2}{2x_2^0(x_2 - x_2^0)(1 - x_1^0)S}\left[h(x_1^0, x_2) - h(x_1^0, x_2^0)\right]$$

$$-2\int_{x_1^0}^1 \frac{dx_1}{x_1}\int_{x_2^0}^1\frac{dx_2}{x_2} H(x_1, x_2)\frac{2\tau(\tau + x_1x_2)}{(x_1x_2^0 + x_1^0x_2)^2}; \qquad (84)$$

$$h(x_1, x_2) = \frac{2(\tau + x_1 x_2)}{(x_1 + x_1^0)(x_2 + x_2^0)} \left[H(x_1, x_2)\left(1 + \left(\frac{\tau}{x_1 x_2}\right)^2\right) - 2H(x_1^0, x_2^0)\right], \tag{85}$$

$$\begin{aligned} W_g(Q^2, y) = &\int_{x_2^0}^1 \frac{dz}{z}\left[z^2 + (1-z)^2\right] \ln\left[\frac{2(1 - x_1^0)(1-z)}{(1+z)x_1^0}\right] K_1\left(x_1^0, \frac{x_2^0}{z}\right) \\ &+ \int_{x_1^0}^1 \frac{dx_1}{(x_1 - x_1^0)} \int_{x_2^0}^1 dx_2 \left[K_1(x_1, x_2) - K_1(x_1^0, x_2)\right] \\ &+ \int_{x_1^0}^1 dx_1 \int_{x_2^0}^1 dx_2 \frac{2\tau(\tau + x_1 x_2)}{(x_1 x_2^0 + x_2 x_1^0)^2} \frac{x_2^0}{x_2}\left(1 + \frac{x_1^{0^2}}{x_1^2} + 2\frac{x_2^0}{x_2}\frac{x_1^0}{x_1}\right) \\ &\times K_1(x_1, x_2) + \{(1 \leftrightarrow 2)\}. \end{aligned} \tag{86}$$

After some further manipulation, the correspondence of eq. (80) with the results of refs. [5, 20] can be shown.

Lastly, we record the expression for the total cross section calculated in order α_s:

$$\begin{aligned} \sigma = N \int \frac{dx_1}{x_1}\frac{dx_2}{x_2} H(x_1, x_2, Q^2)\left[\delta(1 - \tau_{12}) + \frac{\alpha_s}{2\pi}\frac{4}{3}\theta(x_1 x_2 - \tau) 2 f_q^T(\tau_{12})\right] \\ + \frac{\alpha_s}{2\pi}\frac{1}{2}\int \frac{dx_1}{x_1}\frac{dx_2}{x_2}\theta(x_1 x_2 - \tau)\left[K_1(x_1, x_2, Q^2) + K_2(x_1, x_2, Q^2)\right] f_g^T(\tau_{12}), \end{aligned} \tag{87}$$

where in this case the functions f^T are given by [5, 6]

$$f_q^T(z) = \left\{\frac{3}{2}\frac{1}{(1-z)_+} + (1 + z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ - 3 - 2z + \left(\tfrac{1}{2} + \tfrac{2}{3}\pi^2\right)\delta(1-z)\right\},$$

$$f_g^T(z) = \left\{\tfrac{1}{2}\cdot 9z^2 - 5z + \tfrac{3}{2} + \left(z^2 + (1-z)^2\right)\ln(1-z)\right\}. \tag{88}$$

4. Numerical results

Before presenting the numerical results, we discuss our treatment of the strong coupling constant and other input parameters. In performing the Fourier transform to impact parameter space, we must integrate the running coupling constant over the low-momentum region. Although this region is of little importance for the final result, for numerical reasons, however, it is convenient to introduce either a "freezing" of the coupling constant or a smearing by an intrinsic transverse momentum.

We have chosen the former procedure and fixed the running coupling constant as

$$\alpha_s(Q^2) = \left[\beta_0^{(3)} \ln \frac{Q^2 + a\Lambda^2}{\Lambda^2}\right]^{-1}, \qquad 0 \leqslant Q^2 \leqslant 4m_c^2,$$

$$\alpha_s(Q^2) = \left[\frac{1}{\alpha_s(4m_c^2)} + \beta_0^{(4)} \ln \frac{Q^2}{4m_c^2}\right]^{-1}, \qquad 4m_c^2 \leqslant Q^2 \leqslant 4m_b^2,$$

$$\alpha_s(Q^2) = \left[\frac{1}{\alpha_s(4m_b^2)} + \beta_0^{(5)} \ln \frac{Q^2}{4m_b^2}\right]^{-1}, \qquad 4m_b^2 \leqslant Q^2, \tag{89}$$

where

$$\beta_0^{(n_f)} = \frac{33 - 2n_f}{12\pi}. \tag{90}$$

This formula takes into account the change in the slope of the running coupling constant as the charm and beauty thresholds are passed, and ensures that α_s is a continuous function of Q^2. In the low-momentum region the form of the coupling constant is controlled by the parameter a ($a > 1$), which can be varied to check the sensitivity to the freezing: we checked that the results are insensitive to value of a for $q_T \gtrsim 1$ GeV^2 and $a\Lambda^2 < 1$ GeV^2.

We did not attempt to describe the smearing from the intrinsic q_T of partons inside the nucleon or that arising from initial state interactions between active and spectator quarks [21]. It is by now settled that the total production cross sections are not affected by initial state interactions [22]. However, a smearing effect can still possibly be present in the q_T distribution. The justification for neglecting these effects is that the average smearing momentum is less than $\langle q_T \rangle$ in the energy domain of interest here. As a consequence, the slight flattening of the q_T distribution from the smearing is well inside the present uncertainty on the parton component, as has been explicitly checked numerically for smearing momenta below 1 GeV.

All cross sections quoted are for the production of real $(W^+ + W^-)$ or Z^0. Before comparison with data they should be multiplied by the branching ratios into the observed decay channel. The values chosen for the boson masses are

$$M_W = 83.0 \text{ GeV}, \qquad M_Z = 93.8 \text{ GeV}, \qquad \sin^2\theta_w = 0.217. \tag{91}$$

The sensitivity of the numerical results to the choice of quark and gluon densities in the proton (antiproton) was tested using different sets of parametrizations. We have taken the two sets of parton densities given by Duke and Owens (DO) [23] and the set proposed by Glück, Hoffmann and Reya (GHR) [24]. In fig. 3 we display the normalized q_T distribution at $y=0$ for the charged W's for the two choices of the parton densities considered by DO. The two sets of densities are both compatible with existing data on deep inelastic scattering. The first set (DO1) has a smaller Λ ($\Lambda = 0.2$ GeV) and a narrower gluon distribution at the evolution starting point $Q_0^2 = 4$ GeV^2. The second set (DO2) has $\Lambda = 0.4$ GeV and a broader gluon distribution.

The dependence on Λ and the choice of parton densities was further studied in fig. 4 using the GHR distributions. These authors used $\Lambda = 0.4$ GeV in their analysis of the data. By comparison of fig. 3 (DO2) and fig. 4 (solid line), it turns out that the results are more sensitive to Λ rather than to the parametrization of densities. To check further the sensitivity to Λ, we also plot in fig. 4 the results for the q_T distribution of the W's obtained in the GHR case by varying Λ between 0.2 GeV and 0.6 GeV. Notice, however, that strictly speaking, the GHR distributions are only guaranteed to work for $\Lambda = 0.4$. The conclusion is that the overall uncertainty due to the choice of Λ and the form of the parton distributions is at most 25%.

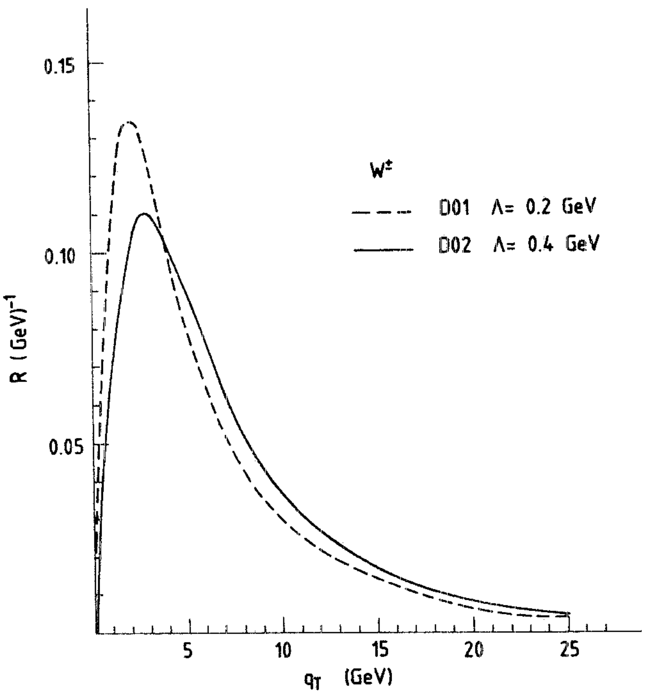

Fig. 3. The ratio $R = (d\sigma/dq_T\,dy)/(d\sigma/dy)$ at rapidity $y=0$ as a function of q_T. The dashed line uses the Duke-Owens parametrization (set 1, $\Lambda = 0.2$ GeV) whilst the solid line shows (set 2, $\Lambda = 0.4$ GeV).

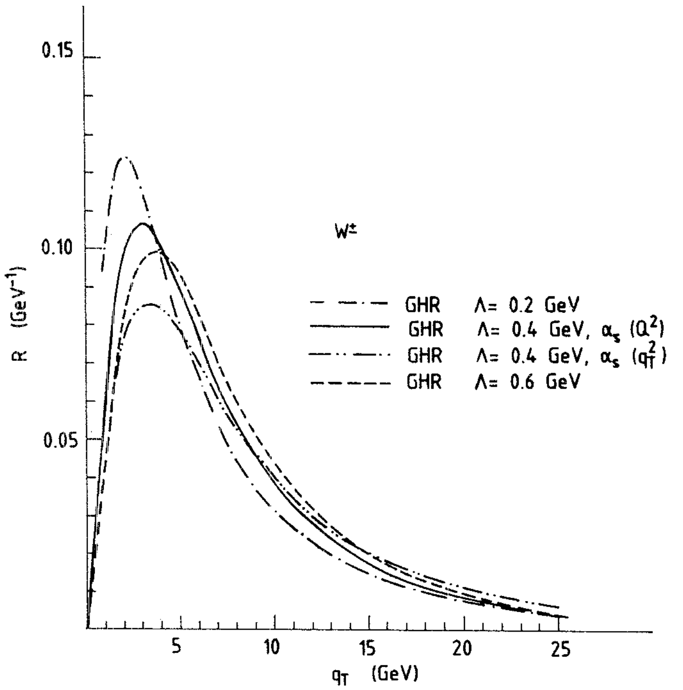

Fig. 4. The ratio $R = (\mathrm{d}\sigma/\mathrm{d}q_T\,\mathrm{d}y)/(\mathrm{d}\sigma/\mathrm{d}y)$ at rapidity $y = 0$ using the densities of GHR. The two curves with $\Lambda = 0.4$ GeV differ by the choice of scale in terms of order α_s which is taken either as Q^2 or q_T^2 (other values of Λ correspond to the choice Q^2).

We also studied the effect of including the Kodaira-Trentadue [17] nonleading term [cf. eqs. (69), (70) and (72)]. We found that it flattens the q_T distribution by an amount that can roughly be reproduced by changing of Λ by a factor of about 1.5. This numerical check makes us confident that the effect of the neglected second-order terms, which we cannot include completely because they are not all known, is comparable with the uncertainty coming from the values of Λ and of the parton densities. All the above statements on the theoretical uncertainty hold good for the case of the Z^0 also. In the following we shall take the GHR parametrization with $\Lambda = 0.4$ GeV as a reference distribution. However, all curves are subject to the theoretical uncertainty as stated above.

In addition we recall that, for the terms of order α_s, there is a further ambiguity connected with the choice of the scale (of order Q^2) for the running coupling and the parton densities which cannot be removed without a complete knowledge of the $O(\alpha_s^2)$ terms. We have mostly taken this scale to be Q^2. An alternative would be to identify this scale with q_T^2 in the q_T distribution. The corresponding variation of the results can be seen from fig. 4 and must be kept in mind in estimating the theoretical error. It is interesting to note that the probability of producing a W with $q_T > 25$ GeV is between 3% and 6%.

In figs. 5 and 6 we plot the q_T distributions at $y = 0$ for the charged and neutral weak bosons respectively. For the case of the charged bosons, we also include the data of the UA1 and UA2 groups [25,26] suitably normalized. Note that the normalization of our curves is determined analytically from $d\sigma/dy$ (eq. (80)). In this way we avoid a numerical normalization error due to the long tail of the q_T distribution. In fig. 7, we plot the y distribution for W production at fixed q_T in order to show that the results at $y = 0$ are actually representative of a quite large range of rapidity values. Fig. 8 displays the x_F distribution of W's plotted together with the suitably normalized histogram of 43 UA1 events.

In table 2 we report the values of the total production cross sections for $W^+ + W^-$ and Z^0 computed from eq. (87) for energies between $\sqrt{S} = 0.54$ and 2 TeV. Decay branching ratios are not included. Also reported is the ratio of the two cross sections which is less affected by theoretical uncertainties. The values shown refer to the case

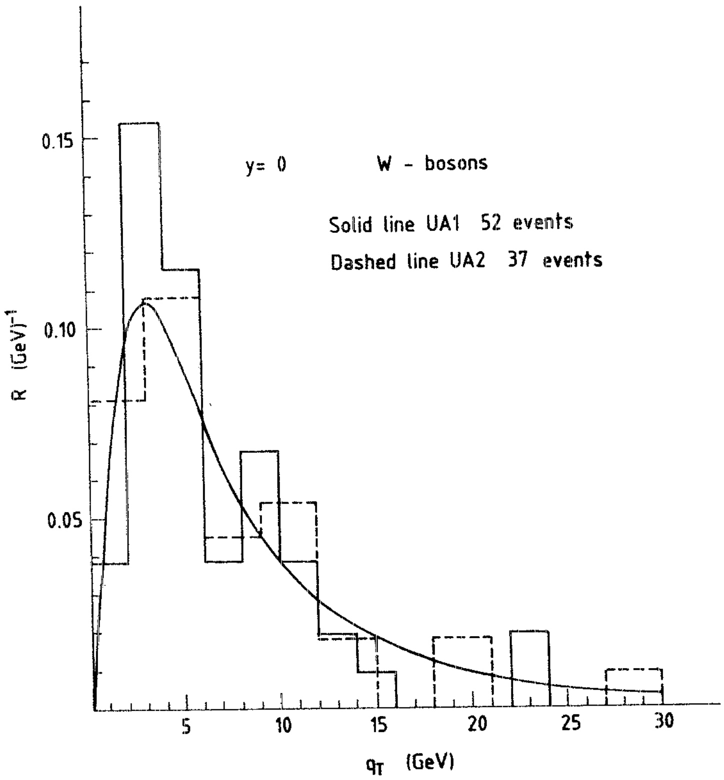

Fig. 5. The data for $W^\pm$ boson production suitably normalized and plotted against q_T. Also shown is our prediction for GHR, $\Lambda = 0.4$ GeV, $y = 0$, $\alpha_s(Q^2)$.

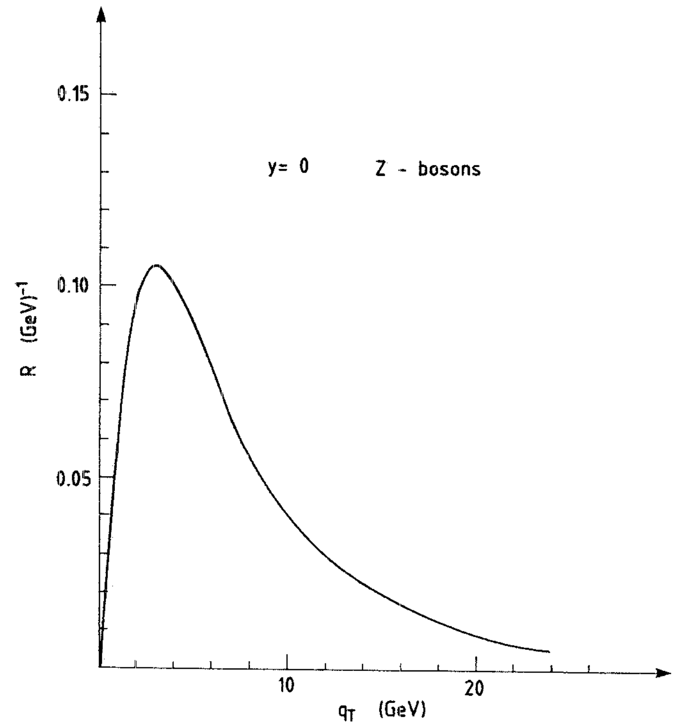

Fig. 6. R at rapidity $y=0$ for Z^0 boson production plotted versus q_T (GHR, $\Lambda = 0.4$ GeV, $\alpha_s(Q^2)$).

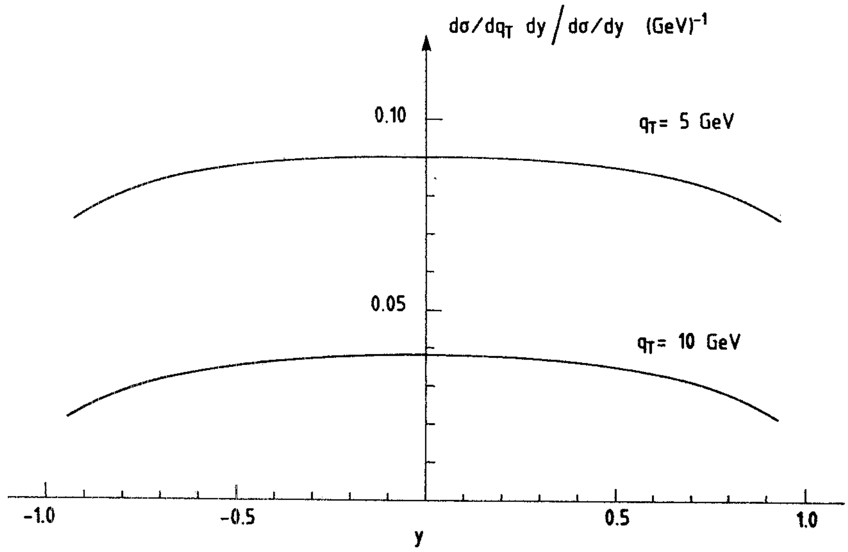

Fig. 7. The ratio $R=(d\sigma/dq_T\,dy)/d\sigma/dy$ versus y at $q_T=5.1$ GeV; the maximum value of y is $y_{max}=1.9$ (GHR, $\Lambda=0.4$ GeV, $\alpha_s(Q^2)$).

where the scale Q^2 is chosen for the terms of order α_s. If so, then the $O(\alpha_s)$ terms included in these results represent about 30% of the total. However, it could be argued that since the bulk of the cross sections arises at relatively small values of q_T^2, the relevant scale could be smaller, in the range $\langle q_T^2 \rangle - Q^2$. This would imply larger values for the total cross sections. We therefore include this theoretical uncertainty in the errors which we quote in the following.

At $\sqrt{S} = 540$ GeV, we find that the total cross sections are

$$\sigma^{W^+ + W^-} = 4.2 \pm ^{1.3}_{0.6} \text{ nb}, \quad \sigma^{Z^0} = 1.3 \pm ^{0.4}_{0.2} \text{ nb}. \tag{92}$$

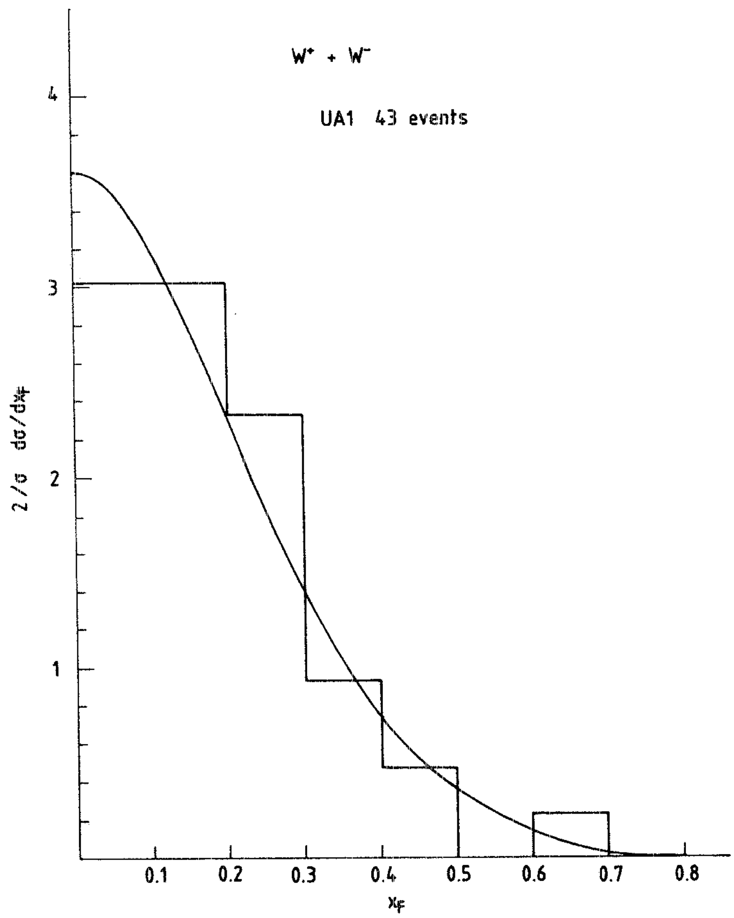

Fig. 8. The prediction for $W^{\pm}$ production versus x_F. The plot shows $(2/\sigma_{TOT})(d\sigma/dx_F)$ versus x_F. The data are from the UA1 Collaboration (GHR, $\Lambda = 0.4$ GeV, $\alpha_s(Q^2)$).

TABLE 2
Values (in nb) of the total cross sections for $W^{\pm}$ and Z^0 production

$\sqrt{S}$ (GeV)	$W^+ + W^-$ GHR	$W^+ + W^-$ DO1	$W^+ + W^-$ DO2	Z^0 GHR	Z^0 DO1	Z^0 DO2	$\frac{\sigma(W^+ + W^-)}{\sigma(Z^0)}$ GHR	$\frac{\sigma(W^+ + W^-)}{\sigma(Z^0)}$ DO1	$\frac{\sigma(W^+ + W^-)}{\sigma(Z^0)}$ DO2
540	4.2	4.3	4.1	1.3	1.3	1.2	3.1	3.4	3.5
700	6.2	6.3	6.1	2.0	1.9	1.8	3.1	3.3	3.4
1000	9.5	9.5	9.6	3.1	3.0	2.9	3.1	3.2	3.3
1300	12.5	12.5	12.9	4.0	3.9	3.9	3.1	3.2	3.3
1600	15.5	15.6	16.5	5.0	4.8	5.0	3.1	3.2	3.3

The ratio of the two cross sections is less affected by theoretical errors:

$$\frac{\sigma^{W^+ + W^-}}{\sigma^{Z^0}} = 3.3 \pm 0.2, \tag{93}$$

and is important in order to obtain Γ_W/Γ_Z from experiment [31]. Multiplying by the branching ratio into electrons:

$$B(W \to e\nu) = 0.089, \qquad B(Z^0 \to e^+e^-) = 0.032, \tag{94}$$

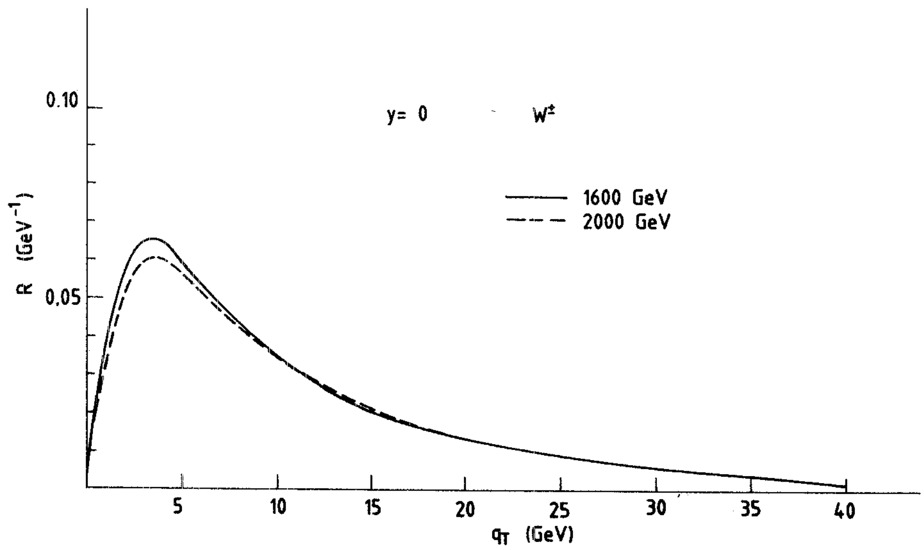

Fig. 9. Prediction for the W distribution in transverse momentum at the tevatron energies: $\sqrt{S} = 1600$ GeV, solid line; $\sqrt{S} = 2000$ GeV, dashed line. Similar curves can be drawn for the Z^0 distribution (GHR, $\Lambda = 0.4$ GeV, $\alpha_s(Q^2)$).

which are the values obtained for the top quark mass $m_t = 40$ GeV and $\alpha_s/\pi = 0.04$, we find that the product of the cross section and decay branching ratio is

$$(\sigma B)^{W^\pm \to e^\pm} = 370 \pm {}^{110}_{60} \text{ pb}, \quad (\sigma B)^{Z^0 \to e^+e^-} = 42 \pm {}^{12}_{6} \text{ pb}. \tag{95}$$

The results in eq. (94) are in perfect agreement with those of ref. [27], where the total cross sections are also computed from eq. (87) with α_s at the scale Q^2. On the other hand, we do not reproduce the results of ref. [28], especially at higher energies. A somewhat larger result has been found in ref. [29], where, however, the total cross section is reconstructed from the lepton p_T distribution by a numerical integration. Finally, a value for the Z^0 production cross section of about 55 pb can be derived from the work of ref. [30]. The corresponding experimental results are [25, 26]

$$\text{UA1:} \quad (\sigma B)^{W^\pm} = 530 \pm 80 \pm 90 \text{ pb}, \qquad (\sigma B)^{Z^0} = 71 \pm 24 \pm 13 \text{ pb},$$

$$\text{UA2:} \quad (\sigma B)^{W^\pm} = 530 \pm 100 \pm 100 \text{ pb}, \quad (\sigma B)^{Z^0} = 110 \pm 40 \pm 20 \text{ pb}.$$

Finally, in fig. 9 we give the distribution in the transverse momentum of the W's at the tevatron energies $\sqrt{S} = 1.6, 2$ TeV.

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