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N.Lo Iudice and F.Palumbo: M2 COLLECTIVE EXCITATIONS IN LIGHT
DEFORMED NUCLEI AND THEIR RELATIONSHIP TO THE ONE PION
EXCHANGE POTENTIAL

M2 collective excitations in light deformed nuclei and their relationship to the one pion exchange potential

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(Received 12 August 1983)

The combined effect of the one pion exchange potential, the spin isospin dependent short range interaction, and the deformation on the M2 states in light deformed nuclei is studied in the framework of a semiclassical model which describes these magnetic states as longitudinal (along the symmetry axis) and transverse spin isospin oscillations. It is argued that these states might present quite distinctive properties depending on the shape of the nuclei and on the strength of the short range repulsive interaction, due to the anisotropic character of the one pion exchange potential, which gives most attraction to the longitudinal mode in oblate nuclei only.

I. INTRODUCTION

Magnetic excitations in nuclei have been the object of several investigations in recent years, mainly in connection with the study of precursor phenomena.[1] The softening of some of these excitation modes, usually called spin-isospin ($\sigma\tau$) or pionlike modes, and the enhancement of the corresponding magnetic transition probabilities would in fact indicate proximity of the nucleus to a static $\sigma\tau$ phase[2] or pion condensation.[3]

In all those analyses the possible role of nuclear deformation has been ignored. It was pointed out, however, in Ref. 4, to be referred to as I, that light oblate nuclei could be quite natural candidates for exhibiting a precursor behavior.

This can be seen from the analogy with the $\sigma\tau$ static phase in nuclear matter. This phase is characterized by a laminated structure due to one-dimensional crystallization along the direction of spin quantization (z axis), which is normal to the planes alternatingly occupied by $\sigma_3\tau_3=1$ and $\sigma_3\tau_3=-1$ nucleons. This model was invented in order to get a nonvanishing direct Hartree contribution from the $\sigma\tau$-dependent component of the nucleon-nucleon potential

$$\langle V_{\sigma\tau}\rangle_d = \frac{1}{2}\sum_1\sum_2\int d\vec{r}_1\int d\vec{r}_2\rho_{\sigma_3(1)\tau_3(1)}(\vec{r}_1)\rho_{\sigma_3(2)\tau_3(2)}(\vec{r}_2)\langle\sigma_3(1)\tau_3(1)\sigma_3(2)\tau_3(2)\,|\,V_{\sigma\tau}\,|\,\sigma_3(1)\tau_3(1)\sigma_3(2)\tau_3(2)\rangle , \quad (1.1)$$

where the sums are over $\sigma_3(1)\tau_3(1)$ and $\sigma_3(2)\tau_3(2)$, respectively, and $\rho_{\sigma_3\tau_3}(\vec{r})$ is the $\sigma\tau$ density while the $\sigma\tau$ matrix element of $V_{\sigma\tau}$ is given by

$$\langle\sigma_3(1)\tau_3(1)\sigma_3(2)\tau_3(2)\,|\,V_{\sigma\tau}\,|\,\sigma_3(1)\tau_3(1)\sigma_3(2)\tau_3(2)\rangle = [V_c + V_T(2z^2 - \vec{r}_T^2)]\sigma_3(1)\tau_3(1)\sigma_3(2)\tau_3(2) , \quad (1.2)$$

where $\vec{r}_T$ is the component of the nucleon-nucleon distance in the x-y plane and V_c and V_T are the central and tensor components of $V_{\sigma\tau}$. Equation (1.1) can be rewritten

$$\langle V_{\sigma\tau}\rangle_d = \frac{1}{2}\int d\vec{r}_1\int d\vec{r}_2\langle S_{33}(\vec{r}_1)\rangle\langle S_{33}(\vec{r}_2)\rangle$$
$$\times[V_c + V_T(2z^2 - \vec{r}_T^2)] , \quad (1.3)$$

where

$$\langle S_{33}(\vec{r})\rangle = \sum_{\sigma_3\tau_3}\rho_{\sigma_3\tau_3}(\vec{r})\sigma_3\tau_3 . \quad (1.4)$$

For $\langle V_{\sigma\tau}\rangle$ not to vanish we need $\langle S_{33}\rangle\neq 0$, i.e., $\sigma\tau$ correlations. But if we want, in addition, to get an attrac-

tive contribution from the tensor potential we must require that

$$\langle 2z^2 - \vec{r}_T^2\rangle < 0 , \quad (1.5)$$

a condition which is realized by the laminated structure as illustrated in Fig. 1(a).

This condition can also be fulfilled in an oblate nucleus by displacing $\sigma_3\tau_3=1$ with respect to $\sigma_3\tau_3=-1$ nucleons along the symmetry (longitudinal) axis, as shown in Fig. 1(b). The short range $\sigma\tau$-dependent interaction, however, prevents this configuration from being realized statistically.

We, therefore, suggested in I that the one pion exchange potential (OPEP) could prevail over the short range forces

If the spin quantization is chosen to coincide with the symmetry axis (longitudinal spin quantization), expanding up to the second power in d (harmonic approximation), the following collective Hamiltonian will result:

$$H = \frac{P^2}{2M} + \tfrac{1}{2}(\bar{C}_Z + C_Z)d_Z^2 + \tfrac{1}{2}(\bar{C}_T + C_T)\vec{d}_T^2 , \quad (2.3)$$

where d_Z and $\vec{d}_T$ are to be considered now as quantum variables, as prescribed by the unified theory of collective motion,[9] M is the reduced mass of the two oscillating fluids, $\bar{C}_Z$ and $\bar{C}_T$ are the $\sigma\tau$-independent restoring force constants, and C_Z and C_T are the $\sigma\tau$-dependent ones. These latter quantities are determined by the equation

$$\langle \psi_{\vec{d}} \mid V_{\sigma\tau} \mid \psi_{\vec{d}} \rangle \simeq \tfrac{1}{2}C_Z d_Z^2 + \tfrac{1}{2}C_T \vec{d}_T^2 . \quad (2.4)$$

The potential used has the form

$$V_{\sigma\tau} = V_\pi + V_\rho + V_g , \quad (2.5)$$

where V_π is the OPEP given in momentum space by

$$V_\pi = - \left[\frac{f_\pi}{m_\pi} \right]^2 \Gamma_\pi^2(q^2) \frac{(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q})}{q^2 + m_\pi^2} \vec{\tau}_1 \cdot \vec{\tau}_2 , \quad (2.6)$$

where V_ρ is the ρ-exchange potential

$$V_\rho = - \left[\frac{f_\rho}{m_\rho} \right]^2 \Gamma_\rho^2(q^2) \frac{(\vec{\sigma}_1 \times \vec{q}) \cdot (\vec{\sigma}_2 \times \vec{q})}{q^2 + m_\rho^2} \vec{\tau}_1 \cdot \vec{\tau}_2 , \quad (2.7)$$

and V_g is the Landau-Migdal contact potential

$$V_g = \left[\frac{f_\pi}{m_\pi} \right]^2 \Gamma_\pi^2(q^2) g_0'(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\tau}_1 \cdot \vec{\tau}_2) . \quad (2.8)$$

m_π and m_ρ are, respectively, the pion and ρ masses, f_π and f_ρ are π-nucleon and ρ-nucleon coupling constants, Γ_π and Γ_ρ are the π-nucleon and ρ-nucleon form factors given by

$$\Gamma_{\pi(\rho)} = \frac{\Lambda_{\pi(\rho)}^2 - m_{\pi(\rho)}^2}{\Lambda_{\pi(\rho)}^2 + q^2} , \quad (2.9)$$

and g_0' is the Landau-Migdal free parameter. As we mentioned in the Introduction, the use of the contact interaction (2.8) to account for the short range contribution has been justified at least in nuclear matter by a fully microscopic calculation.[6] Although these nuclear matter results do not necessarily hold unchanged in finite deformed nuclei, especially if anisotropic properties are intended to be detected,[10] we maintain the same parametrization for simplicity.

On the other hand an immediate test of the plausibility of such a parametrization is provided by the effect of V_ρ. Were the contribution of this latter potential short ranged itself but anisotropic, such as to change drastically the

pattern of the results due to V_π plus V_g, then the Landau-Migdal force would certainly appear to be an inadequate approximation to the short range interaction in deformed nuclei.

If $V_{\sigma\tau}$ is included in Eq. (2.4), the following expressions of C_Z and C_T are obtained:

$$C_i = - \frac{1}{\pi^2} \int_{-\infty}^{+\infty} dq_z \int_0^\infty dq_T q_T f_i(q_z, q_T)$$

$$\times [Q_\pi + Q_g + Q_\rho] G^2(q_z, q_T) , \quad (2.10)$$

where $i = z, T$, and

$$f_z(q_z, q_T) = q_z^2 , \quad f_T(q_z, q_T) = \tfrac{1}{2}q_T^2 ,$$

$$Q_\pi + Q_g = \left[\frac{f_\pi}{m_\pi} \right]^2 \left[\frac{q_z^2}{q^2 + m_\pi^2} - g_0' \right] \Gamma_\pi^2(q^2) ,$$

$$Q_\rho = \left[\frac{f_\rho}{m_\rho} \right]^2 \frac{q_T^2}{q^2 + m_\rho^2} \Gamma_\rho^2(q^2) , \quad (2.11)$$

$$G(q_z, q_T) = \sum_{n_z, nm} G_{n_z}(q_z) G_{nm}(q_T) ,$$

and $G_{n_z}(q_z)$, $G_{nm}(q_T)$ are the Fourier transforms of $\mid \varphi_{n_z} \mid^2$, $\mid \varphi_{nm} \mid^2$,

$$G_{n_z}(q_z) = (\varphi_{n_z}, e^{iq_z z} \varphi_{n_z}) ,$$

$$G_{nm}(q_T) = (\varphi_{nm}, e^{i\vec{q}_T \cdot \vec{r}_T} \varphi_{nm}) . \quad (2.12)$$

The first excited states are characterized by the quantum numbers $K^\pi = 0^-$ for $\nu_T = 0$ and $\nu_Z = 1$ (longitudinal oscillation), and by $K^\pi = 1^-$ for $\nu_T = 1$ and $\nu_Z = 0$ (transverse oscillation).

Their excitation energies are, respectively,

$$\omega_{Z(T)} = \bar{\omega}_{Z(T)} \left[1 + \frac{C_{Z(T)}}{\bar{C}_{Z(T)}} \right]^{1/2} , \quad (2.13)$$

where $\bar{\omega}_z$ and $\bar{\omega}_T$ are the unperturbed $\sigma\tau$-independent longitudinal and transverse frequencies related to $\bar{C}_Z$ and $\bar{C}_T$ by

$$\bar{C}_Z = M\bar{\omega}_z^2, \quad \bar{C}_T = M\bar{\omega}_T^2 . \quad (2.14)$$

These states are excited by the $M2$ operator

$$\mathcal{M}(M2,\mu) = \frac{A}{8} \left[\frac{5}{4\pi} \right]^{1/2} \left[\frac{(2+\mu)!(2-\mu)!}{(1+\mu)!(1-\mu)!} \right]^{1/2}$$

$$\times d_\mu(g_n - g_p)\frac{e\hbar}{2mc} \quad (2.15)$$

with a transition probability given by

$$B(M2, I = K = 0 \rightarrow I, K) = \frac{2}{1 + \delta_{K0}} \mid \langle \Phi_K \mid \mathcal{M}(\lambda = I = 2, \mu = K) \mid \phi_0 \rangle \mid^2$$

$$= \frac{5}{32\pi} \frac{\hbar^2}{m} A \left[\frac{1}{\hbar\omega_z} \delta_{K0} + \frac{3}{2} \frac{1}{\hbar\omega_T} \delta_{K1} \right] (g_n - g_p)^2 \left[\frac{e\hbar}{2mc} \right]^2 \text{fm}^2 ; \quad (2.16)$$

$K = 0 \rightarrow K = 2$ transitions are absent.

The previous results hold for longitudinal spin quantization as stated before. This is a natural choice, which also has the merit of maximizing the attractive contribution of the OPEP in oblate nuclei. In prolate nuclei, however, the contribution of the OPEP gets larger for transverse spin quantization, as suggested by Eq. (1.3) and the findings in I. Such a choice does not fit easily in the present semiclassical model, since it breaks axial symmetry. Let, in fact, the spin quantization axis be the x axis. The contribution of $V_{\sigma\tau}$ will make $C_x \neq C_y \neq C_z$. For $V_{\sigma\tau} = V_\pi + V_g$ we have,

$$C_i = -\frac{1}{\pi^2}\left[\frac{f_\pi}{m_\pi}\right]^2 \int_{-\infty}^{+\infty} dq_z \int_0^\infty dq_T q_T f_i(q_z,q_T)\left[\alpha_i \frac{q_T^2}{q^2+m_\pi^2}-g_0'\right]\Gamma_\pi'^2(q^2)G^2(q_z,q_T), \tag{2.17}$$

where $i = x,y,z$, and

$$f_x = f_y = \tfrac{1}{2}q_T^2, \quad f_z = q_z^2,$$

$$\alpha_x = \tfrac{3}{4}, \quad \alpha_y = \tfrac{1}{4}, \quad \alpha_z = \tfrac{1}{2}. \tag{2.18}$$

The states with good K quantum numbers are in general linear combinations of the eigenstates of the harmonic Hamiltonian. This can be seen by inspecting the expression of the transition probability. The $M2$ operator is in fact given by

$$\mathcal{M}(2,K) = \frac{1}{8}\left[\frac{15}{2\pi}\right]^{1/2} A \sum_{\nu\mu} \langle\,|\nu|\mu|2K\rangle\hat{x}\cdot\hat{e}_\mu d_\nu(g_n - g_p)\frac{e\hbar}{2mc} \tag{2.19}$$

from which the following expression for the transition probability is obtained:

$$B(M2,0\rightarrow K) = \frac{5}{128\pi}\frac{\hbar^2}{m}A\left[\frac{1}{\hbar\omega_x}\delta_{K0}+\frac{3}{\hbar\omega_z}\delta_{K1}+\frac{3}{2}\left[\frac{1}{\hbar\omega_x}+\frac{1}{\hbar\omega_y}\right]\delta_{K2}\right](g_n-g_p)^2\left[\frac{e\hbar}{2mc}\right]^2 \text{fm}^2. \tag{2.20}$$

It is to be observed also that the $K^\pi = 2^-$ states are excited in this case. Just these latter states do not correspond to a single oscillation along one of the axes, which makes the model not completely consistent in this case. For this reason we will consider both transverse and longitudinal spin quantization for prolate nuclei.

III. CALCULATIONS AND RESULTS

The values of the parameters entering into the calculations are the currently used ones: $m_\pi = 0.70$ fm^{-1}, $m_\rho = 3.90$ fm^{-1}, $\Lambda_\pi = 1000$ MeV, $\Lambda_\rho = 2000$ MeV, $f_\pi^2/4\pi = 0.08$, and $f_\rho^2/4\pi = 5$.

The single particle level sequences used are, in cylindrical coordinates,

$$(n_z,n,m) = (0,0,0);(0,1,\pm1);(1,0,0);(0,2,\pm2);$$

$$(0,2,0);(1,1,\pm1);\text{ etc.,}$$

appropriate for oblate nuclei, and,

$$(n_z,n,m) = (0,0,0);(1,0,0);(0,1,\pm1);(2,0,0);$$

$$(1,1,\pm1);(0,2,\pm2);\text{ etc.,}$$

as required for prolate nuclei.

The $\sigma\tau$-independent restoring force constants $\bar{C}_z$ and $\bar{C}_T$ are determined by Eq. (2.14), where the reduced mass is taken to be $M = (A/4)m$; and $\bar{\omega}_z$ and $\bar{\omega}_T$ are chosen to be the sp HO frequencies, following the unified theory of nuclear vibrations.[9] These frequencies satisfy the usual volume conserving condition

$$\bar{\omega}_z\bar{\omega}_T^2 = \bar{\omega}^3, \tag{3.1}$$

where $\bar{\omega}$ is the sp HO frequency at zero deformation.

We used for this quantity the usual estimate

$$\bar{\omega} = 41A^{-1/3} \text{ MeV} \tag{3.2}$$

which should be appropriate for the nuclei of the (s,d) region, less so for ^{12}C. Given the semiclassical nature of the calculation we stick to Eq. (3.2) also for ^{12}C.

The deformation parameter $\bar{\omega}_z/\bar{\omega}_T$ is fixed so as to reproduce the observed deformation of the nucleus under consideration through the relation
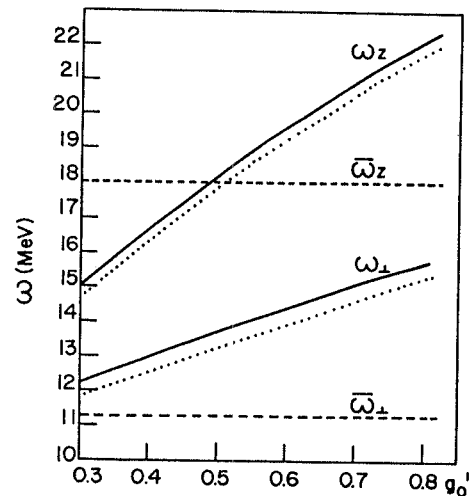

FIG. 2. Frequencies of the longitudinal and transverse $\sigma\tau$ oscillations versus g_0' in ^{28}Si. The full lines are obtained using $V_{\sigma\tau} = V_\pi + V_g$, the dotted lines when V_ρ is also included.
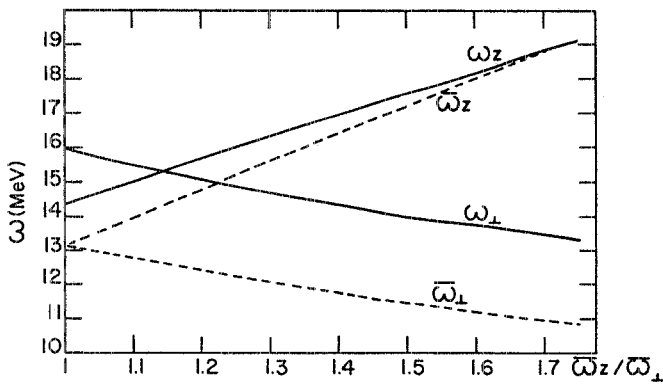
FIG. 3. Frequencies of the longitudinal and transverse $\sigma\tau$ oscillations versus deformation on ^{28}Si.

$$\delta = \frac{3}{4} \frac{\left\langle \sum_i {}^A (2z_i^2 - \vec{r}_{iT}^2) \right\rangle}{\left\langle \sum_i {}^A r_i^2 \right\rangle} , \tag{3.3}$$

where the mean values are evaluated using a Slater determinant of sp wf's. The values of $\bar{\omega}_z / \bar{\omega}_T$ so determined might not result in the most realistic ones. We will see,
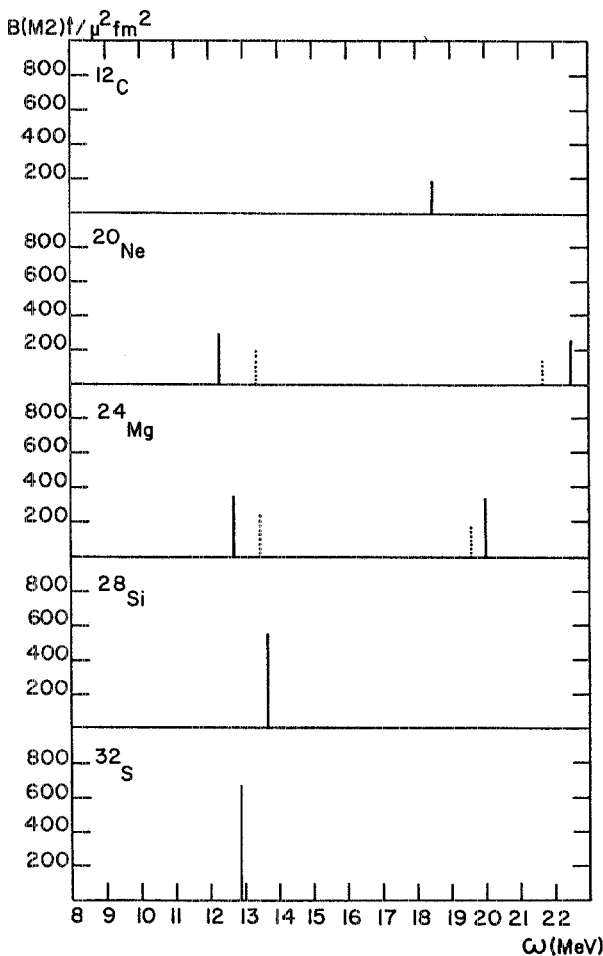

FIG. 4. Excitation energies and $M2$ transition probabilities estimated for $g_0' = 0.5$ ($g_0' = 0.6$ for ^{12}C). The $B(M2)$'s corresponding to a transverse spin quantization axis are also shown for prolate nuclei (dotted lines).
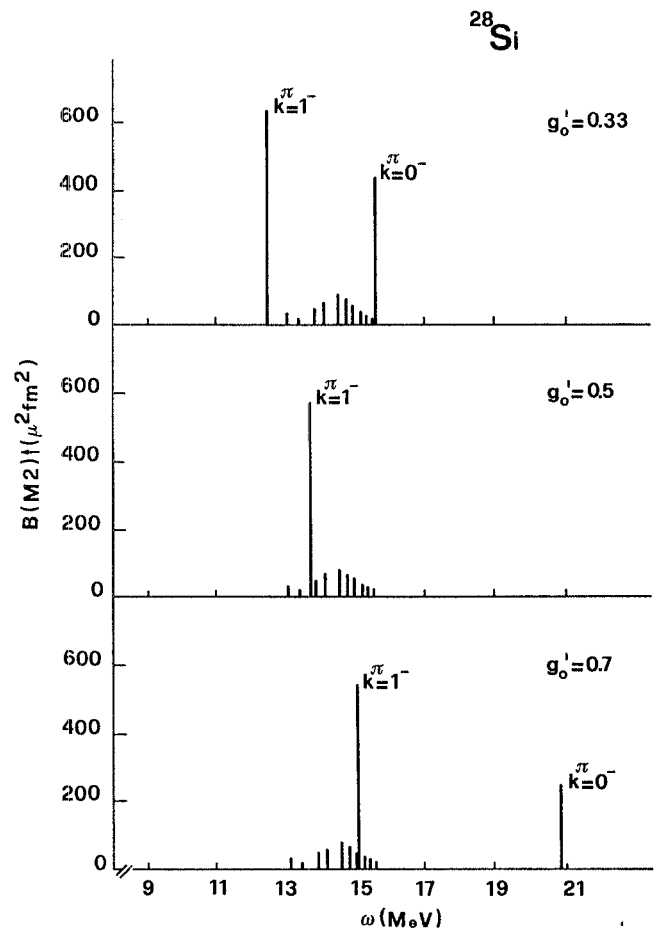

FIG. 5. Experimental and theoretical values of excitation energies and $M2$ strengths for different values of g_0' in ^{28}Si. The theoretical lines are assigned the K^π quantum numbers of the states which get excited.

however, that our conclusions do not depend critically on the actual value of the deformation parameter.

We observe first of all that the effect of V_ρ is practically negligible for all values of g_0' (and all reasonable deformations) (Fig. 2). This result is therefore in support of the plausibility of the Landau-Migdal parametrization also in deformed nuclei, at least for low energy phenomena.

What instead should be noted is the extreme sensitivity of the levels to the value of g_0', especially for the states corresponding to longitudinal oscillations. In this latter case it is worth it to observe that for $g_0' \approx 0.5$ the mutual cancellation of the contributions due to V_π ($+V_\rho$) and V_g is complete.

The collectivity of such a state would therefore disappear completely for such a value of g_0'. The transverse mode instead remains collective for practically all values of g_0'. These results remain valid for a large range of the values assumed by the deformation parameter $\bar{\omega}_z / \bar{\omega}_T$, as shown in Fig. 3. (From here on we will show only the results due to $V_\pi + V_g$, given the negligible effect of V_ρ.) This does not imply that the deformation is not important. As one can check from the figure, in fact, transverse and longitudinal levels get dramatically split as $\bar{\omega}_z / \bar{\omega}_T$ increases. Again, the important effect of the OPEP is to be noted. This, resulting strong (and increasingly with the

deformation) attraction in the z direction only, opposes the splitting.

All comments made so far are based on the ^{28}Si results, but remain true for all oblate nuclei considered. Prolate nuclei, instead, exhibit two collective $M2$ states whatever is the spin quantization axis. In case of longitudinal polarization, in fact, the OPEP does not give enough attraction to cancel one of the modes for any value of g_0'. If the quantization axis is the x axis, the contributions of V_π and V_g to C_x mutually cancel for $g_0' \cong 0.5$, so that only the oscillation modes along the y and z axis remain collective.

In Fig. 4 the levels of the collective states with their corresponding $M2$ strengths are shown for both type of nuclei. (The results relative to the noncollective states are not reported since the semiclassical description is not adequate for these states.) It is to be observed that the strengths are never too large, as a result of the competing effect of V_π, V_g, and deformation.

The most interesting prediction of the model is, in our opinion, that for reasonably but not exceedingly high values of g_0' ($g_0' \simeq 0.5$—0.6) oblate nuclei should exhibit only one collective $M2$ level. Such a prediction does not seem in blatant contradiction with the known levels and strengths. As shown in Fig. 5, the known experimental $M2$ levels of ^{28}Si are in fact clearly centered around an energy value $E_c = 14.5$ MeV with an integrated strength

$$\sum_K B(M2) \simeq 400 \mu^2 \text{ fm}^2 ,$$

in reasonable agreement with our results.

In ^{12}C (Fig. 6) the $M2$ strength is practically all concentrated in one single level at 19.3 MeV, but is far too large with respect to the strength estimated here. It seems, however, that the experimental values in ^{12}C are not the result of a direct measure but follow from a theoretical analysis[11] of the data[12] carried out using the random-phase approximation (RPA) with spherical sp wf's.

Our schematic model cannot account for the detailed properties of the nuclei considered here. Only a RPA calculation in a deformed basis, which includes the effect of the Δ isobar, the spin orbit terms, etc., would allow a
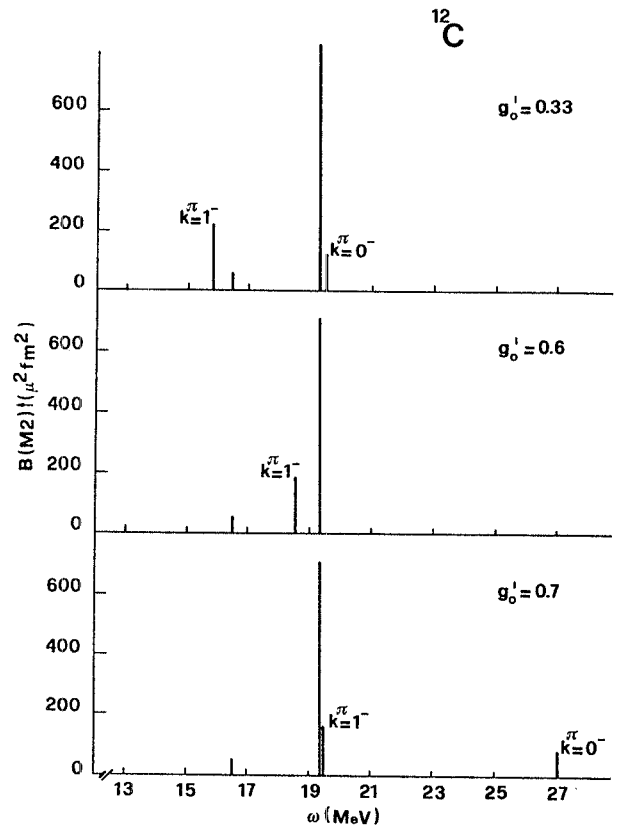

FIG. 6. The same as in Fig. 5 for ^{12}C.

complete description. Such a microscopic calculation for instance would account for the fragmentation of the $M2$ states observed in ^{28}Si and would give a more realistic estimate of the Landau-Migdal parameter g_0' for deformed nuclei. The schematic analysis carried out in the present paper has, however, strongly suggested that light deformed nuclei might present quite interesting magnetic properties which are worth investigating more carefully by means of a fully microscopic calculation.

We are grateful to A. Richter for pointing out Refs. 11 and 12 to us.

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