C.R.Natoli and F.Palumbo : THE MANY-BODY CONTENT OF QUANTUM GAUGE THEORIES AND ITS CONNECTION TO MASS GENERATION MECHANISMS

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1. Introduction

In the last years many fundamental concepts in Quantum Field Theory
have been the result of a fruitful interplay with the Theory of
Condensed Matter. In particular symmetry breaking and mass generation
mechanisms are among those concepts for which the parallel between
the two areas of research has been most fruitful and productive of
deep physical understanding. In this connection the exchange of ideas
between the two fields goes back to the early sixties.

Indeed, soon after Schwinger[1] put forward its conjecture that local
gauge invariance does not necessarily imply masslessness for the gauge
boson, Anderson[2] remarked that "the familiar plasmon theory of the
free-electron gas exemplifies Schwinger's theory in a very straight-
forward manner. In the plasma, transverse electromagnetic waves do
not propagate below the plasma frequency, which is usually thought of
as the frequency of long-wavelength longitudinal oscillations of the
electron gas. At and above this frequency, three modes exist, in close
analogy (except for problems of galilean invariance implied by the
inequivalent dispersion of longitudinal and transverse modes) with
the massive vector boson mentioned by Schwinger.

The plasma frequency is equivalent to the mass, while the finite
density of electrons leading to divergent "vacuum" current fluctuations
resembles the strong renormalized coupling of Schwinger's theory. In

spite of the absence of low-frequency photons, gauge invariance and particle conservation are clearly satisfied in the plasma".

Anderson was able to draw a direct parallel between the dielectric constant treatment of plasmon theory and Schwinger argument, by showing that there is a close relationship between the two arguments, dispite the fact that the usual plasmon theory does not treat the electromagnetic field quantum-mechanically or discuss vacuum fluctuations. Hence, the quantum nature of the gauge field is irrelevant in the analogy.

Referring the interested reader to the cited works for more details, what we would like to point out in this paper is that the connection between the two arguments is more than a mere analogy.
Indeed such a connection can be framed in a more general contest by explicitly constructing a procedure that brings out the non-relativistic content of a Q.F.T. so that one can make use of the more solid knowledge that in general we have about many-body systems and their properties, in order to get insight into the structure of the theory itself. The way to achieve this is to perform the galilean limit of the relativistic theory by sending in this latter the speed of light c to infinity. This limiting process exposes the low energy behaviour of the relativistic theory.

It is in general not unique, but, by requiring that the Poincaré invariance contract into Galilei invariance while preserving all the other symmetries of the relativistic theory (gauge or chiral invariance, charge symmetry, etc.) a definite limiting procedure is set up which leads to meaningful results.
One of us[4] has indicated how this can be achieved. We take the limit $c \longrightarrow \infty$ within the path-integral formulation of Q.F.T. by performing a simple functional transformation on the fields. Such transformations are c-dependent and give rise to terms in the lagrangian which diverge with c, terms independent of c and terms of order 1/c. The limiting galilean lagrangian can contain c only in the mass term which is the centre of the Galilei algebra. Therefore the functional transformations must be performed in such a way that the terms divergent with c either cancel out or give rise to mass terms and Galilei-invariant constraints while the c-independent terms must be Galilei-invariant. Both

lagrangian and constraints must be invariant with respect to all the symmetries of the original relativistic lagrangian except, of course, space-time ones.

If the limit is performed in this way, in the few cases studied the known features of the phase structure as a function of the coupling constants present in the relativistic theory are preserved in the limit. This has been shown for the massless Wess-Zumino model, the Fayet-Iliopoulos model[5] and the infrared behaviour of QED with periodic boundary conditions[6]. It will also be shown here for the model of a complex scalar field interacting with an abelian gauge field (Goldstone and Higgs model).

Hence, by studying the many-body content of the original relativistic theory one can not only gain some insight about the structure of its ground state ("vacuum") and the low lying excitations, but also decide about which order parameter to take into consideration when studying the phase structure of the vacuum.

In particular we shall show that the non relativistic limiting procedure naturally suggests the occurence of a degenerate type of vacuum of the Nambu[7] type (i.e. particle-antiparticle condensate with superconducting properties, as far as the excitation spectrum is concerned) both in the case of fermion and boson systems interacting with an electromagnetic field. For fermions the transition to a condensate occurs for a large enough value of the electric charge, thus substantiating Schwinger's conjecture. For bosons, provided a self repulsive interaction is present, the transition to a condensate occurs for a definite relation between the coupling constants of the two interactions. In both cases the boson field acquires a mass.

Hence the limiting procedure strongly suggests that Schwinger's mechanism for mass generation is connected with the structure of the "vacuum" ground state, not with the type of particles involved.

Finally we would like to point out that the connection so established between relativistic and nonrelativistic theories can be exploited in the reverse direction. As will turn out, the limiting nonrelativistic theory describes a many-body interacting system with some constraints.

Solving for these latters will provide a quantum system without constraints for which one has to find the ground state and the quasi

particle excitations. The freedom inherent in the solution of the constraints can be exploited to optimize the choice of the collective variables that most easily describe the quasi particle excitations of the system. This alternative point of view might also be promising even for the study of the excitation spectrum of known many-body systems under new physical conditions.

2. The many-body content of the limiting galilean theory

The relativistic model we intend to investigate in the light of its connection to the many-body physics is described by a complex scalar field interacting with an abelian gauge field with lagrangian density

$$\mathscr{L}(x) = -\frac{1}{4}\mathscr{F}_{\mu\nu}\mathscr{F}^{\mu\nu} + \frac{1}{2}(\partial_\mu\Phi_1\partial^\mu\Phi_1 + \partial_\mu\Phi_2\partial^\mu\Phi_2) -$$

$$- e\mathscr{A}_\mu(\Phi_1\partial^\mu\Phi_2 - \Phi_2\partial^\mu\Phi_1) - \frac{1}{2}e^2\mathscr{A}_\mu^2(\Phi_1^2 + \Phi_2^2) - \qquad (1)$$

$$- \sigma\mu c^2\frac{1}{2}(\Phi_1^2 + \Phi_2^2) - \frac{\lambda}{4}(\Phi_1^2 + \Phi_2^2)^2$$

where $\mathscr{A}_\mu$ is the gauge field, $\mathscr{F}_{\mu\nu} = \partial_\mu\mathscr{A}_\nu - \partial_\nu\mathscr{A}_\mu$, $\sigma = \pm 1$, and we have introduced the real components of the complex scalar field $\Phi = \frac{1}{\sqrt{2}}(\Phi_1 + i\Phi_2)$. $\mathscr{L}$ is invariant under local gauge transformations

$$\mathscr{A}_\mu \longrightarrow \mathscr{A}_\mu + \partial_\mu\chi , \qquad (2)$$

$$\Phi_1 \longrightarrow \Phi_1\cos\chi + \Phi_2\sin\chi , \qquad \Phi_2 \longrightarrow -\Phi_1\sin\chi + \Phi_2\cos\chi .$$

The known properties of this model as a function of the coupling constants e^2 and λ of the theory and the parameter σ are summarized in the following, where the expression e^2 small or large is to be understood with reference to a certain positive function of the self-repulsion constant to be specified later on in the nonrelativistic limit:

a) $e^2 = 0$, $\sigma = +1$

We have the $|\Phi|^4$ theory with global gauge invariance. The vacuum is a normal vacuum with a gap in the excitation spectrum for $\mu \neq 0$.

b) $e^2 = 0$, $\sigma = -1$

We have the Goldstone $|\Phi|^4$ theory with the "wrong" sign of the mass. Particle condensation occurs in the ground state driven by the mass term acting as a chemical potential. The low energy spectrum extends

down to zero due to the presence of a massless scalar, the goldstone boson.

c) e^2 "small", $\sigma = 1$

The model describes scalar electrodynamics. We have transversality and masslessness of the gauge field. The longitudinal field does not describe a physical degree of freedom. The "vacuum" ground state has all the symmetries of the lagrangian.

d) e^2 "large" and $\sigma = 1$

We are in the Schwinger regime. The vacuum is described by a dynamical condensation of particle-antiparticle pairs with superconducting properties in the sense that the structure of the low lying excited states is similar to that of a superconducting system.

As for the Higgs case, the gauge field acquires a mass with its longitudinal component describing a physical degree of freedom.

e) $e^2 \neq 0$, $\sigma = -1$

We have the Higgs model. The gauge field becomes massive, its longitudinal component describing a physical degree of freedom. The vacuum ground state is described in terms of a particle condensate which is not spontaneous but somehow "forced" by the mass term which acts as chemical potential to impose a constrained occupation of the vacuum.

What we want to show is that these properties are preserved in the nonrelativistic limit of the model, which is given in terms of a many-body hamiltonian describing its low-energy behaviour.

The nonrelativistic lagrangian, obtained through the limiting procedure briefly described in the introduction and applied to Eq.(1), is given by

$$\mathscr{L} = \varphi_1^* i \mathscr{D}_t \varphi_1 + \varphi_2^* i \mathscr{D}_t^* \varphi_2 + \frac{1}{2\mu} (\varphi_1^* \mathscr{D}^2 \varphi_1 + \varphi_2^* \mathscr{D}^{*2} \varphi_2) -$$

$$ - \sigma \mu^2 c^2 (\varphi_1^* \varphi_1 + \varphi_2^* \varphi_2) - \frac{\lambda}{8\mu} \left[3(\varphi_1^* \varphi_1 + \varphi_2^* \varphi_2)^2 - \right. \tag{3}$$

$$ \left. - (\varphi_1^* \varphi_1 - \varphi_2^* \varphi_2)^2 \right] + \frac{1}{2} F_{oj}^2 $$

supplemented by the constraints $F_{ij} = 0$. As usual, latin indices indicate the spatial components of vector or tensor quantities and

$$F_{ij} = \partial_i A_j - \partial_j A_i \ , \qquad\qquad F_{oj} = \partial_t A_j - \partial_j V \ , \tag{4}$$

$$\mathscr{D}_t = \partial_t - ieV \ , \qquad \mathscr{D}_k = \partial_k - ieA_k , \qquad \mathscr{D}^2 = \mathscr{D}_k \mathscr{D}_k$$

where the gauge field described by A_j and V is related to the one appearing in Eq.(1) by

$$\mathscr{A}_o = -V , \qquad \mathscr{A}_1 = cA_j .$$

The matter fields φ_1 and φ_2 are complex scalar fields describing particles and antiparticles respectively, in the nonrelativistic limit. Since under Galilei transformation

$$A_j \rightarrow A_j , \qquad V \rightarrow V + v_k A_k$$

so that

$$F_{ij} \rightarrow F_{ij} , \qquad F_{oj} = F_{oj} + v_k F_{kj}$$

we easily see that the lagrangian (3) is Galilei invariant if supplemented by the contraints $F_{ij} = 0$. Moreover this lagrangian preserves the local gauge invariance and particle-antiparticle symmetry of the original lagrangian Eq. (1). It has one more symmetry, though, compared to this latter, since it conserves separately the number of particles and the number of antiparticles, as appropriate to a nonrelativistic situation.

From the lagrangian (3), using Dirac's theory of quantization of constrained systems we obtain the hamiltonian

$$H = \frac{1}{2} p^2 - \frac{1}{2} \int \mathscr{E}(x) \Delta^{-1} \mathscr{E}(x) d^3x + \int d^3x \left[\frac{e^2}{2\mu} (\partial_k \mathscr{A} + \frac{1}{\Omega^{1/2}} q_k)^2 \right. \cdot$$

$$\cdot (\varphi_1^* \varphi_1 + \varphi_2^* \varphi_2) + e(\partial_k \mathscr{A} + \frac{1}{\Omega^{1/2}} q_k) J_k - \frac{1}{2\mu} \varphi_1^* \Delta \varphi_1 - \qquad (5)$$

$$- \frac{1}{2\mu} \varphi_2^* \Delta \varphi_2 + \sigma\mu c^2 (\varphi_1^* \varphi_1 + \varphi_2^* \varphi_2) + \frac{\lambda}{8\mu} \left[3(\varphi_1^* \varphi_1 + \varphi_2^* \varphi_2)^2 \right. -$$

$$\left. \left. - (\varphi_1^* \varphi_1 - \varphi_2^* \varphi_2^2)^2 \right] \right]$$

supplemented by the Gauss constraint

$$\mathscr{E}(x) - e(\varphi_1^* \varphi_1 - \varphi_2^* \varphi_2) = 0 \qquad (6)$$

and the commutation relations

$$[q_i, \mathscr{E}] = [q_i, \mathscr{A}] = [p_i, \mathscr{E}] = [p_i, \mathscr{A}] = 0 ,$$

$$[q_i, p_j] = [-i \delta_{ij}] , \qquad [\mathscr{A}(x), \mathscr{E}(y)] = \delta^3(x-y) \qquad (7)$$

together with the relations

$$A_j(x,t) = \partial_j \mathcal{A}(x,t) + \frac{1}{\Omega^{1/2}} q_j(t) \, ,$$

$$F_{oj}(x,t) = - \partial_j \Delta^{-1} \mathcal{E}(x,t) + \frac{1}{\Omega^{1/2}} p_j(t) \, .$$

$$(8)$$

In Eq. (8) we have singled out the zero momentum components of the Fourier transforms of A_j and F_{ij} assuming for $\mathcal{A}$ and $\mathcal{E}$ periodic boundary conditions in the quantization volume Ω. Also

$$J_k(x) = \frac{i}{2\mu} \left[\varphi_1^* \partial_k \varphi_1 - \partial_k \varphi_1^* \varphi_1 - \varphi_2^* \partial_k \varphi_2 + \partial_k \varphi_2^* \varphi_2 \right]$$

is the usual current density vector for the matter field.

The properties of the many-body system described by the hamiltonian (5), together with the constraints (6) and the commutation relations (7) should then relate to the low energy behaviour of the original relativistic lagrangian in Eq. (1).

We reexamine the various cases:

a) $\underline{e^2 = 0, \quad \sigma = +1}$

The hamiltonian (5) reduces to the galilean version of the $|\phi|^4$ model:

$$H_M = \int d^3x \left[- \frac{1}{2\mu} \varphi_1^* \Delta \varphi_1 - \frac{1}{2\mu} \varphi_2^* \Delta \varphi_2 + \mu c^2 (\varphi_1^* \varphi_1 + \varphi_2^* \varphi_2) + \right.$$

$$\left. + \frac{\lambda}{8\mu} \left[3(\varphi_1^* \varphi_1 + \varphi_2^* \varphi_2)^2 - (\varphi_1^* \varphi_1 - \varphi_2^* \varphi_2)^2 \right] \right]$$

$$(9)$$

which describes a gas particles and antiparticles with self-repulsion in each of the Hilbert subspaces with a constant number of particles and antiparticles separately. The ground state is the bare vacuum and the excitation spectrum has a gap for $\mu \neq 0$.

b) $\underline{e^2 = 0, \quad \sigma = -1}$

The corresponding hamiltonian describes a system of two types of bosons with positive chemical potential. As a consequence a condensate of bosons of zero momentum is formed in the ground state with average density $\langle \varphi_1^* \varphi_1 + \varphi_2^* \varphi_2 \rangle = \mu^2 c^2 / \lambda$.

Its properties are well described by a Bogolubov model for a super-

fluid boson gas with self-repulsion, whose excitations are linear in the momentum, in complete analogy with the relativistic case.

c,d) $e^2 \neq 0, \quad \sigma = 1$

To discuss the model, it is expedient in this case to use the Coulomb gauge $\nabla_k A_k = 0$, which, through use of Eq. (8), becomes $\Delta \mathscr{A} = 0$. For periodic boundary conditions, this entails $\mathscr{A} = 0$. Eliminating $\mathscr{E}$ through the Gauss constraint (6) we obtain the hamiltonian

$$H = \frac{1}{2} p^2 + \frac{1}{2} \omega^2 q^2 - \omega q_k I_k - \frac{e^2}{2} \int d^3x (\varphi_1^* \varphi_1 + \varphi_2^* \varphi_2) \cdot$$

$$\cdot \Delta^{-1} (\varphi_1^* \varphi_1 + \varphi_2^* \varphi_2) + \int d^3x \left[- \frac{1}{2\mu} \varphi_1^* \Delta \varphi_1 - \frac{1}{2\mu} \varphi_2^* \Delta \varphi_2 + \right. \tag{10}$$

$$\left. + \mu c^2 (\varphi_1^* \varphi_1 + \varphi_2^* \varphi_2) + \frac{\lambda}{8\mu} \left[3(\varphi_1^* \varphi_1 + \varphi_2^* \varphi_2)^2 - (\varphi_1^* \varphi_1 - \varphi_2^* \varphi_2)^2 \right] \right]$$

where

$$I_k = \int d^3x J_k(x) / \int d^3x (\varphi_1^* \varphi_1 + \varphi_2^* \varphi_2)$$

and $\tag{11}$

$$\omega^2 = \frac{e^2}{\mu \Omega} \int d^3x (\varphi_1^* \varphi_1 + \varphi_2^* \varphi_2) = \frac{e^2 N}{\mu \Omega} \qquad (N = N_1 + N_2)$$

This hamiltonian describes a system of charged bosons interacting via Coulomb forces plus self-repulsion. The photon field is represented only by its zero momentum components in the galilean limit. It has a mass only if the quantity $\lim_{N \to \infty} N / \Omega(N)$ exists and is different from zero.

To investigate this point we need to investigate the nature of the ground state of the matter field.

It is an established result (Dyson 1967)[8] that for a neutral system $(\frac{N}{2} = N_1 = N_2)$ of charged bosons interacting via Coulomb forces only, the energy per particle E_N/N is not bounded from below, i.e. $E_N < - AN^{7/5} \hbar^2 / \mu a_o^2$ where A is a constant and a_o is the "Bohr" radius $\hbar^2 / \mu c^2$. At the same time the volume occupied by the system collapses to zero as N goes to infinity as $\Omega(N) = a_o^3 N^{-3/5}$. For such a system, which is the nonrelativistic limit of the lagrangian in (1) with $\lambda = 0$, the plasma frequency in Eq. (11) is not defined, since

it depends on the number of particles N and goes to infinity with N. Hence it is likely that in the corresponding relativistic theory, problems arise due to the non existence of the galilean limit. The connection of this fact with the nonrenormalizability of the relativistic theory described by Eq. (1) with $\lambda = 0$ would be worth investigating.

When $\lambda \neq 0$ in the hamiltonian (10), it is possible to show that the energy per particle is bounded from below and that $\Omega(N) \sim N$, so that the quantity in Eq. (11) is well defined. More precisely, introducing the two quantities, having dimension of a length

$$a_o = \hbar^2/\mu e^2, \qquad \alpha_o = \hbar^2\lambda/\mu, \qquad a_o/\alpha_o = 1/\lambda e^2 \qquad (12)$$

one can rigorously prove[9] that

$$- A'(\frac{a_o}{\alpha_o})^{1/2} \frac{\hbar^2}{\mu a_o^2} < \frac{E_N}{N} < - A(\frac{a_o}{\alpha_o})^{1/3} \frac{\hbar^2}{\mu a_o^2} \qquad (13)$$

where A and A' are constants independent of N and

$$\Omega(N) = a_o^3 (\frac{\alpha_o}{a_o})^{4/3} N . \qquad (14)$$

To establish the lower bound in Eq. (13), correlations between particles were neglected, whereas the upper bound was obtained with the help of a pair correlated variational wavefunction of the Dyson's type[8]. One can then reasonably assume that

$$E_N = - \gamma (\frac{a_o}{\alpha_o})^{1/3} N \hbar^2/\mu a_o^2 \qquad (15)$$

where γ is a constant of order 1.

If $\gamma(a_o/\alpha_o)^{1/3}\hbar^2 N/\mu a_o^2 < \mu c^2 N$, we see from hamiltonian (10) that in each Hilbert subspace of N particles, the ground state is given by the bare vacuum $|B\rangle$ for which $\langle B| \varphi_1^*\varphi_1 + \varphi_2^*\varphi_2 |B\rangle = 0$. In such a case $\omega^2 = 0$. This situation then corresponds to the "small" e^2 case discussed above for the relativistic lagrangian.

If instead $\gamma(a_o/\alpha_o)^{1/3}\hbar^2 N/\mu a_o^2 > \mu c^2 N$, then it becomes advantageous for the system to create particle-antiparticle pairs so that in each subspace the ground state $|C\rangle$ is a condensate of these pairs with superconducting properties, meaning that there is a finite gap for single particle

excitations. For this state $\langle c| \varphi_1^* \varphi_1 + \varphi_2^* \varphi_2 |c\rangle \neq 0$. As a consequence, because of (11) and (15) we have

$$\omega^2 = \frac{e^2}{\mu} \frac{1}{a_o^3} (\frac{a_o}{\alpha_o})^{4/3} \tag{16}$$

so that the photon field has a finite mass. This situation corresponds to the "large" e^2 case discussed above for the lagrangian (1) and substantiates Schwinger's conjecture for the relativistic theory in the boson case.

In the nonrelativistic limit we can also give a more precise meaning to the expression e^2 "large" or "small" according to whether

$$\gamma(\frac{a_o}{\alpha_o})^{1/3} \frac{\hbar^2}{\mu a_o^2} \gtrless \mu c^2 \qquad \text{or} \qquad \frac{a_o}{\alpha_o} \gtrless \gamma^{-3}(\frac{\hbar c}{e^2})^6 \tag{17}$$

or, remembering (12), whether

$$e^2 \gtrless k\lambda^{1/5} . \tag{18}$$

With the quantity $\varrho = \langle c| \varphi_1^* \varphi_1 + \varphi_2^* \varphi_2 |c\rangle$ playing the role of an order parameter we have the phase diagram in Fig. 1.
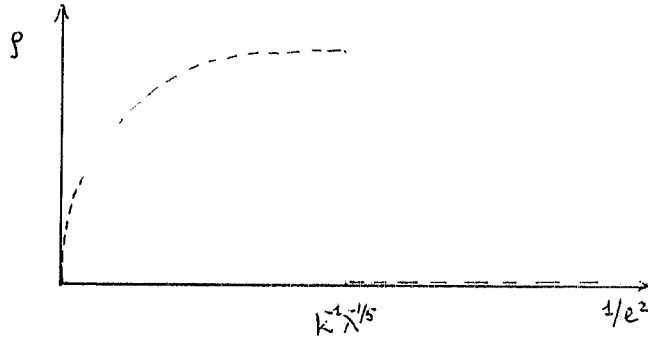

Fig.1 - Phase diagram for a system of interacting bosons with self-repulsion.

The value of ϱ in the condensed phase is fixed by the chemical potential μc^2 which imposes equality in (17), so that from Eq.(14)

$$\varrho = \frac{N}{\Omega} = a_o^{-3} \gamma^{-4} (\frac{\hbar c}{e^2})^8 \tag{19}$$

and

$$\omega^2 = \frac{e^2}{\mu} \, a_o^{-3} \gamma^{-4} \, (\frac{\hbar c}{e^2})^8 \; . \tag{20}$$

e) $\underline{e^2 \neq 0, \quad \sigma = -1}$

Again the existence of a positive chemical potential forces a condensation of bosons into the ground state with such a density that the positive energy arising from the repulsion term overcomes the attraction of bosons with opposite charge. Again the ground state energy and volume are proportional to N so that the plasma frequency is well defined. There is however a difference between the structure of the dynamical (Schwinger) condensate in the case $\sigma = 1$ and the forced (Higgs) condensate when $\sigma = -1$. This latter is more similar to a plasma with no or little particle-antiparticle correlation, whereas in Schwinger's condensate the particle-antiparticle correlation is complete, giving the state superconducting properties.

As is clear from examination of the various limiting cases, the similarity with the relativistic behaviour is striking. In the above discussion we have limited ourselves to boson systems. However, had we started from the relativistic model describing a spinor field interacting with an electromagnetic field, we would have found the same limiting hamiltonian for fermions as we found for bosons, without of course the repulsive part. Such a hamiltonian would describe a system of charged fermions interacting via Coulomb forces. It is known[10] that for such a system, due to the Pauli exclusion principle providing the internal pressure to prevent collapse, the energy per particle is bounded from below $\frac{E_N}{N} = \gamma' \frac{\hbar^2}{\mu a_o^2}$, γ' being of order one), exactly as it happens for the boson system, where the self-repulsion plays the same stabilizing role as the Pauli principle for fermions. Hence we could repeat the same argument as above, with condensation appearing when $\gamma' \frac{\hbar^2}{\mu a_o^2} = \mu c^2$, i.e. when $\gamma' = (\frac{\hbar c}{e^2})^2$. In both cases, the longitudinal oscillations in the condensate describe the longitudinal degree of freedom for the massive boson. If the analogy with many-body physics could be pursued further, one would expect a damping of these excitations at some critical value of the momentum.

The implication of this would be that the longitudinal boson field is not a fundamental field, a fact which can be related to the recent finding that even in the relativistic model, the longitudinal gauge field is the gradient of a compact variable, i.e.

$$\mathcal{A}_{Lk}(x) = \partial_k \mathcal{A}(x) \ , \qquad -\frac{\pi}{e} \leq \mathcal{A}(x) \leq \frac{\pi}{e}$$

rather than the gradient of a noncompact field, as it would be if it were fundamental[11].

Finally we want to add a brief comment about the possibility, mentioned in the introduction, of defining collective variables in the many-body theory by comparison with their relativistic parent. The hamiltonian (5) together with the constraint (6) and the commutation relations (7) provides an example of such a procedure. Indeed in order to describe longitudinal density fluctuations for this system we can use the constraint (6) either to eliminate the fields $\mathcal{A}$ and $\mathcal{E}$ in favour of the matter fields φ_1, φ_2 or to eliminate the matter field φ_1 in favour of φ_2, $\mathcal{A}$ and $\mathcal{E}$.

In this latter case $\mathcal{A}$ and $\mathcal{E}$ describe, as is possible to show by a direct calculation, the longitudinal density fluctuations of the system.

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