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V. Azcoiti : MORE EVIDENCE FOR UNIVERSALITY IN THE SU(2) LATTICE GAUGE THEORY

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ABSTRACT

The 0^{-+} glueball mass in the fundamental-adjoint SU(2) lattice gauge theory is extracted from the Monte-Carlo data on the correlation functions at time distances t = 1 and t = 2 on a 8^4 lattice. The ratio $m_g/\sqrt{\sigma}$ is constant in the β_F - β_A region explored giving more evidence in favour of the universality hypothesis.

The universality problem in the context of lattice gauge theories comes from the possibility to define an infinite number of lattice actions given the same classical action when the lattice spacing a is removed. If the lattice is a useful tool, this ambiguity can not have any physical meaning and we hope all these lattice formulations will give the same results for physical quantities in the continuum limit.

On a practical level, when we compute some physical quantity on a finite lattice, we should check that the parameters of the theory (the bare coupling g for the pure gauge theory) are in the scaling region where renormalization group behaviour holds. Only in this region, is one able to extract physical quantities .

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In the case of a SU(2) lattice gauge theory, one possible action (besides the standard Wilson action) is the fundamental-adjoint action given by

$$S = \beta_F \sum_{\text{plaq.}} \left[1 - \frac{1}{2} \mathrm{Tr}_F(U_p)\right] + \beta_A \sum_{\text{plaq.}} \left[1 - \frac{1}{3} \mathrm{Tr}_A(U_p)\right] \tag{1}$$

where the summation is over all plaquettes, U_p is the product of the four SU(2) matrices around each plaquette and Tr_F (Tr_A) stands for the trace in the fundamental (adjoint) representation of SU(2) respectively (throughout this paper the normalization is the same as in ref.[8]). It has been shown [1] that the action (1) presents in the $\beta_F - \beta_A$ plane a phase diagram with a line of first order phase transition (in the $\beta_A > 0$ half-plane) terminating at a critical point located at a small value of β_A ,near the β_F range corresponding to the crossover region for the standard Wilson theory [2].

Some time ago, Bhanot and Dashen [3] measured the string tension in the fundamental-adjoint SU(2) lattice gauge theory (1) and from the Monte-Carlo data they got the lattice Λ parameter , finding a discrepancy between the Monte-Carlo results and the theoretical expectations based on a weak coupling calculation (essentially they found that the ratio σ/Λ changes when β_F, β_A are changed). Gonzalez-Arroyo et al. [4] have shown that the string tension data of ref.[3] are well fitted by a strong coupling expansion,casting doubt on the applicability of the weak coupling expansion to analyse the data.

Almost all the string-tension data of ref.[3] were obtained for $\beta_A < 0$. For $\beta_A > 0$, one is near the critical end point where unphysical effects may appear, being responsible for the large fluctuations present in the Monte-Carlo data. On the other hand, the most interesting region ($\beta_A < 0$) being very close to the $g_0^2 = \infty$ line, cast doubt on the use of perturbation theory in such a region. In refs.[5,6,7] an effective formula in the large N limit was obtained by noticing that the $N = \infty$ limit of the theory given by the action (1) is equivalent to a standard Wilson theory with some effective coupling constant β_{eff}. Using this formula, the string-tension data of ref.[3] were reanalysed with a very good agreement.

Another point of view to analyse the universality problem was taken in ref.[8]. To avoid the dangerous use of a perturbative formula, the point of view taken in [4] was to measure the ratio of two physical quantities such as the 0^{++} glueball mass over the string-tension. If we are near enough to the continuum limit, this ratio must be constant (independent of β_F and β_A) and universal (independent of the chosen lattice action). The authors of ref.[8] computed the plaquette-plaquette correlation function in the fundamental-adjoint SU(2) lattice gauge theory and from the Monte-Carlo data, they extracted the 0^{+} glueball "mass" $m_g a$. By comparing the values of $m_g a$ with the string-tension data of ref.[3] a region of β_F was found between 2.0 and 2.6, where the ratio was constant within error bars for almost all the points considered. The time separations of the plaquette-plaquette correlation function used in [8] were $t = 0$ and $t = 1$, the main reason for that being that correlations at $t > 2$ are very small and suffer from severe statistical fluctuations ($m_g a$ is of order one and therefore the correlation function decreases quickly). However, real glueball mass should be extracted in the large t region. Additionally, the physical meaning of the correlation function at $t = 0$ is doubtfull.

In this letter, new results with high statistics for the 0^{+} glueball "mass" in the fundamental-adjoint SU(2) lattice gauge theory are reported. This calculation has been made on an 8^4 lattice and the glueball "mass" has been extracted from correlation functions at time distances $t = 1$ and $t = 2$. Periodic boundary conditions have been imposed and we have used the Icosahedron which in the region under investigation, is indistinguishable from the full SU(2) group.

The glueball mass is evaluated in the following way: one can define the time-sliced plaquette-plaquette correlation function by the following expression

$$G(\tau) = N_s \left[\langle W(\tau)W(0)\rangle - \langle W(0)\rangle^2\right]$$

where

$$W(\tau) = \frac{1}{N_s} \sum_{\substack{\text{all spatiall} \\ \text{plaquettes}}} \frac{1}{2} \mathrm{Tr}_F(U_p)$$

and N_s is the number of spatial like plaquettes. Then, the glueball "mass" is defined by

$$m_g a = - \log\left[G(2)/G(1)\right]$$

All the points in the $\beta_F - \beta_A$ plane we have explored, are in the region where universality (in the sense previously defined) was observed in [8]. In all the measured points the expectation values of the Polyakov loop was about 0.05 which assures we are in the confined phase.

In Table I, the numerical results for the glueball "mass" $m_g a$ are reported. In the numerical simulation we have used the Metropolis algorithm to generate equilibrium configurations for the gauge fields. For all the points we have run about 16000 Monte-Carlo iterations and we did not use the first 2100 iterations to take averages. It can be seen (see table I) that the results for the first two points, which are in the same line of constant physics (one of the lines of constant σa^2 in

TABLE I - Values of the 0^+ glueball as a function of β_F and β_A. In the third column, the values obtained on an 8^4 lattice from correlation functions at time distances t = 1 and t = 2 are reported. The last column stands for the corresponding results in ref.(8).

β_F	β_A	$m_g a^{12}$	$m_g a^{01}$
2.5	- 0.21	1.94 ± 0.18	2.28
2.1	0.2	1.78 ± 0.13	1.90
2.5	- 0.43	2.35 ± 0.08	2.52

ref.[3]), are constant within error bars. On the other hand, if we compare these results with the corresponding results of ref.[8] obtained on a 4^4 lattice from correlation functions at time separations $t = 0$ and $t = 1$ (last column in table I) , a systematic decreasing for all the analysed values of $m_g a$ can be observed.

In Table II we show the values of the $m_g/\sqrt{\sigma}$ ratio. The data in this table tell us that this ratio is constant within error bars in all the three points measured.

TABLE II - $m_g/\sqrt{\sigma}$ ratio as a function of β_F and β_A.

β_F	β_A	$m_g/\sqrt{\sigma}$
2.5	- 0.21	3.23 ± 0.31
2.1	0.2	2.97 ± 0.23
2.5	- 0.43	3.29 ± 0.12

All these results strongly suggest that there is a window in the β_F - β_A plane where universality holds (at least for the mass gap of the pure gauge theory). A similar conclusion in the hadronic sector was found in ref.[9] where the hadronic mass spectrum of a SU(2) quenched gauge theory in the β_F - β_A plane was analysed.

In conclusion, if we collect all the results reported in the literature , I think that at the present moment there is no serious objection to the universality hypothesis.

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