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E. De Sanctis, P. Di Giacomo, C. Guaraldo, V. Lucherini, E. Polli, A.R.  
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ABSOLUTE PHOTOFISSION CROSS SECTION FOR  $^{238}\text{U}$  BY 120-280 MeV QUASI-MONOCROMATIC PHOTONS

E. De Sanctis, P. Di Giacomo, C. Guaraldo, V. Lucherini, E. Polli and A.R. Reolon  
Istituto Nazionale di Fisica Nucleare, Laboratori Nazionali di Frascati (Italy)

and

V. Bellini, E. Emma, S. Lo Nigro, C. Milone and G.S. Pappalardo  
Istituto di Fisica dell'Università di Catania\* and Istituto Nazionale di Fisica Nucleare, Sezione di Catania (Italy)

ABSTRACT

The LEALE quasi-monochromatic photon beam, produced by positron annihilation, was used to measure the  $^{238}\text{U}$  photofission yields. Measurements were performed at 17 positron energies, from 120 MeV up to 280 MeV, and collecting the annihilation photons at an angle  $\approx 1^\circ$ . Fission fragments were detected with glass plates. The photofission cross-section, deduced by means of an appropriate unfolding method, agrees very well with the results of a recent tagged photon experiment. The calculated nuclear fissility results constant as a function of energy and equal to  $0.87 \pm 0.05$ .

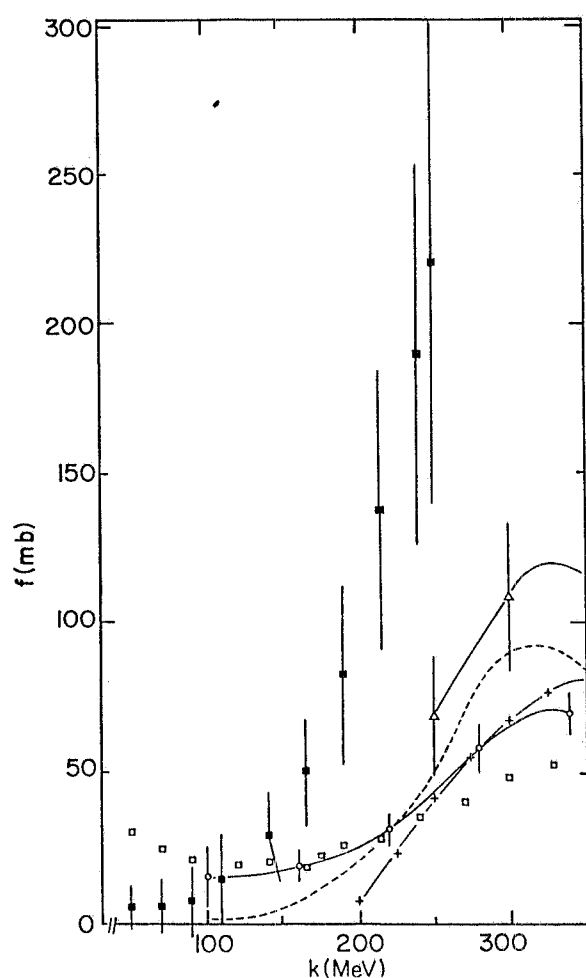
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## 1. - INTRODUCTION

Photofission reactions make use of the simplicity and directness of the electromagnetic interaction as a powerful tool with which to explore the process of nuclear fission. However, owing to the difficulty of performing photofission measurements, especially with monoenergetic photons, very little accurate, detailed, and systematic data have been obtained.

Informations on the photofission cross section of  $^{238}\text{U}$  in the photon energies from 40 up to 300 MeV have been already deduced by various authors<sup>(1-9)</sup> on the basis of measured electron- and bremsstrahlung- induced fission cross sections. All the obtained results show a rapid increase of the photofission cross section at energies greater than 140 MeV. Nevertheless absolute cross section values present a remarkable disagreement, as it can be seen in Fig. 1. This fact may be partially ascribed to



the known difficulties<sup>(10,11)</sup> in determining the cross section from the measured fission yields versus the electron or photon-spectrum end-point energy  $E_0$ . In fact, the exact knowledge of the shape of the photon spectrum from the threshold energy up to  $E_0$  is required to evaluate the cross section with a good degree of reliability. Unfortunately, there is not yet an exact bremsstrahlung theory which simultaneously takes into account all the effects in the interaction between electrons and the bremsstrahlung target.

FIG. 1 - Photofission cross section of  $^{238}\text{U}$  versus the photon energy. Experimental data: ■ Ref. (2), □ Ref. (3), + Ref. (4), Δ Ref. (5), ○ Ref. (15); the solid line curves represent calculations given in the quoted references. The dashed curve represents the cross section assumed in Ref. (6).

An improvement in the quality of measurements was obtained by some of us by using the quasi-monoenergetic photon beam produced by coherent bremsstrahlung of electrons from the Frascati synchrotron striking a diamond single crystal<sup>(12-16)</sup>. This photon beam was characterized by a main peak, lying above a continuous spectrum, whose energy ranged between 200 and 500 MeV.

The aim of the present experiment was to deduce better information on the photofission of  $^{238}\text{U}$  at energies near the photomesonic threshold, where the photofission cross section seems to exhaust the whole photoabsorption cross section. To this end we took advantage of the characteristics of the

LEALE quasi-monochromatic photon beam, produced at Frascati by positron annihilation. The best experimental conditions for performing this kind of measurements were discussed in our previous paper<sup>(17)</sup>.

Tagged photons have been recently used to measure the photofission cross section of Uranium isotopes in the energy range between 40 and 105 MeV<sup>(18)</sup> and in the  $\Delta$ -resonance region<sup>(19)</sup>.

The experimental apparatus and method will be discussed in Sects. 2 and 3, and the measured cross sections, with a discussion of the adopted data analysis procedure, are given in Sect. 4. In Sect. 5 our results are presented and discussed in terms of fissility along with the comparison to recent measurements performed with a tagged photon beam<sup>(19)</sup>.

## 2. - DESCRIPTION OF THE EXPERIMENT

The experiment was performed at Frascati using the LEALE photon beam produced by in-flight annihilation of positrons on a liquid hydrogen target. This experimental facility has already been described in detail<sup>(20)</sup>, so only its main characteristics are recalled here.

This photon beam exhibits a monoenergetic peak at the correct annihilation energy, together with an unavoidable bremsstrahlung continuous tail. Photons can be collected at an angle between 0° and 1.5° respect to the positron flight direction, and so it is possible to improve the annihilation-to-bremsstrahlung photon ratio at a reasonable expense of peak intensity and energy resolution.

The positron intensity is monitored both by a Faraday cup, put in the focal plane of a dumping magnet behind the hydrogen target, and a ferrite toroid mounted just in front of the hydrogen target.

The photon beam spectrum is measured on-line by a pair spectrometer<sup>(21)</sup> connected to a PDP 15/76 computer. Due to the magnet characteristics and detector geometry, it is not possible to measure the spectrum for photon energies lower than  $\approx 50$  MeV. In order to extend the knowledge of the spectrum up to the fission threshold energy, a Monte Carlo code has been used<sup>(22)</sup>. This program, which had as input quantities the measured positron beam characteristics and the geometry of the facility, enabled us to calculate correctly both the emittance and the energy spectrum of the photon beam and gave the absolute value of the photon flux per incident positron. The agreement between the calculated and measured photon spectra has proved to be excellent in widely different experimental conditions, so emphasizing the reliability of Monte Carlo results.

The fission fragments were detected by means of glass sandwiches<sup>(23)</sup> containing a thin target of  $UF_4$  deposited by thermal evaporation onto the surface of one of two glass plates. The contamination of the  $^{235}U$  isotope in the used natural Uranium target resulted to be 0.07%.

In order to obtain absolute values of the cross section, a very thin Uranium layer was used. The thickness and uniformity of the layer were measured in three ways: by an optical interferometer<sup>(24)</sup>, through the backscattering method<sup>(25)</sup> and by counting the natural  $\alpha$ -decay. The target surface -  $5 \times 5$  cm<sup>2</sup> area - resulted 200  $\mu\text{g}/\text{cm}^2$  thick, with an accuracy better than 3%.

Each glass plate used to detect the fission fragments was tightened with the plate containing the Uranium layer. In all measurements the same Uranium sample was irradiated, in order to keep constant the systematic error due to the uncertainty in the knowledge of the target thickness.

The photon beam was finally absorbed into a quantameter, located downstream from the Uranium target, and which monitored the beam intensity in equivalent quanta.

### 3. - EXPERIMENTAL RESULTS

The fission fragment yields of  $^{238}\text{U}$  were measured at 17 positron energies, between 120 and 280 MeV with  $\approx 10$  MeV steps, and by collecting the photons at an angle  $\approx 1^\circ$  by respect to the positron line of flight.

The collimated photon beam had a circular spot ( $\emptyset \approx 4$  cm) on the target position and struck the glass sandwiches at right angle. For each sandwich, only the glass plate which detected the forward emitted fission fragments was processed.

Concurrently glass sandwiches containing a thick Uranium target (0.1 mm thickness,  $5 \times 5$  cm<sup>2</sup> area) were also irradiated, in order to get a large number of fission events in the same exposure time. This supplementary irradiation was performed for deducing information both on the behaviour of fission yields, with good statistics, and on the photon beam profile.

After the irradiation each glass plate was submitted to the etching procedure described in a previous work<sup>(13)</sup>. The fission tracks were observed by an optical microscope. For each plate about  $10^4$  fission events were collected, by entirely scanning the irradiated surface. The extension of this surface was deduced by scanning both glass plates of sandwiches with thick Uranium target. This scanning gave also information on the forward-backward ratio of the detected fragments. This ratio resulted weakly dependent on the photon energy through the whole investigated energy range.

The fission cross sections per equivalent quantum (shortly called "yields") were evaluated from the number of fission tracks counted in each glass plate and by taking into account both the exposure dose and the detection efficiency of glass plates, deduced as in a previous work<sup>(26)</sup>.

To obtain the total cross section, the fission fragment angular distributions were assumed isotropic in the energy range covered by the experiment, as measured in Ref. (3).

The total experimental error resulted to be 4%. It accounts for the statistical error as well as for the uncertainties in target thickness, detector efficiency value and scanning method. This latter was estimated in the same way as in previous experiments<sup>(13,14)</sup>.

We ignored correction due to the contribution of  $^{235}\text{U}$  fission because negligible. In fact in the investigated energy region the two Uranium isotopes have almost equal fission cross sections, as shown in literature<sup>(6,19,27)</sup>.

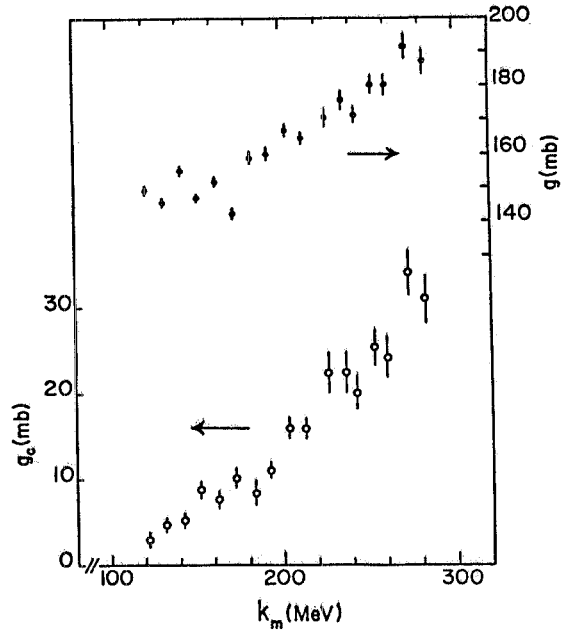
Special care was devoted to background measurements. The contribution of spurious events, due to fission processes of light nuclei in the glass detectors, was evaluated by irradiating glass plates without Uranium sample with photon doses equal to ones used in measurements. In this way the effect of radiation damages in the glass sandwiches was also checked. This turned out to be negligible. Another check was performed by scanning for each sandwich the glass surface not in contact with the Uranium sample.

Background due to neutron induced fission was also measured with a glass sandwich containing a thick Uranium target set 5 cm apart from the beam. Also this contribution was found to be negligible.

In the upper part of Fig. 2 are reported the experimental yields,  $g(k_m)$ , versus the maximum photon energy  $k_m$ . The observed oscillations in the experimental points reflect the different experimental conditions (positron beam emittance and photon collection angle) of each run.

The experimental yields  $g(k_m)$  are connected to the fission cross section  $f(k)$  by the Volterra linear equation

FIG. 2 - Photofission yields per equivalent quantum of  $^{238}\text{U}$  as a function of the maximum photon energy  $k_m$ :  $\bullet$  - experimental yields,  $\circ$  - yields obtained after having subtracted the contribution due to photons with energies  $k \leq 100$  MeV.



$$g(k_m) = \int_{k_T}^{k_m} \mathcal{N}(k, k_m) f(k) dk, \quad (1)$$

where  $\mathcal{N}(k, k_m)dk$  is the number of photons in the energy interval from  $k$  to  $k+dk$  and  $k_T$  is the fission threshold energy.  $k_m$  was assumed to be equal to the incident positron energy. In the case of  $^{238}\text{U}$  a relevant contribution to the fission arises from the photon absorption in the giant resonance region. Thus, in order to deduce the cross section in the energy range from 100 MeV to 280 MeV, Eq. (1) was written in the form:

$$g(k_m) = \int_{k_T}^{100 \text{ MeV}} \mathcal{N}(k, k_m) f(k) dk + \int_{100 \text{ MeV}}^{k_m} \mathcal{N}(k, k_m) f(k) dk = g_0(k_m) + g_c(k_m). \quad (2)$$

In this Equation  $g_c(k_m)$  represents the yield one obtains after subtraction of the  $g_0(k_m)$  contribution due to photons with energies  $\leq 100$  MeV.

The  $g_0(k_m)$  contribution was evaluated by folding the  $f(k)$  data reported in the literature<sup>(27,28)</sup> at energies  $k_T \leq k \leq 100$  MeV and the  $\mathcal{N}(k, k_m)$  values deduced from each measured photon spectrum.

The obtained  $g_c(k_m)$  results are reported in the lower part of Fig. 2. The same absolute errors estimated for the experimental yields were attributed to the  $g_c(k_m)$  values.

#### 4. - ANALYSIS OF THE EXPERIMENTAL RESULTS

The fission cross section  $f(k)$  was calculated by solving the integral equation of the process with an unfolding method similar to the numerical one proposed by Cook<sup>(10)</sup>. We improved the accuracy in the representation of the  $f(k)$  solution, which now is approximated by a natural spline function<sup>(29,30)</sup> instead of a step-wise function.

The fission cross section was evaluated for 10 photon energies at intervals of 20 MeV in the 100 MeV to 280 MeV range. The unfolding method<sup>(10,13)</sup> applied to the experimental yields gives a  $\hat{f}$

vector, which represents an estimate of the photofission cross section averaged in energy by a matrix  $R$ , whose meaning is that of an energy resolution function, as shown by Cook. The cross section values, as well as the shape of the  $R$ -matrix rows, depend also on the value of a smoothing parameter  $\gamma$ , chosen to regularize the  $f(k)$  solution. The parameter  $\gamma$  is generally selected by using the criterion of the  $\chi^2$  values, that can act only if the experimental errors are accurately known.

In this work the application of a Bayesian method, suggested by Turchin et al.<sup>(31)</sup>, based on selecting a priori a probability density for the solution, was preferred. This method is described in our previous paper<sup>(16)</sup>. The  $\gamma$  value, which gets a maximum in the probability density  $P(\gamma/g)$  of obtaining some  $\gamma$  values for a fixed set of experimental  $g$  yields, was chosen. The  $P(\gamma/g)$  probability obtained is shown in Fig. 3 as a function of some  $\gamma$ -parameter values.

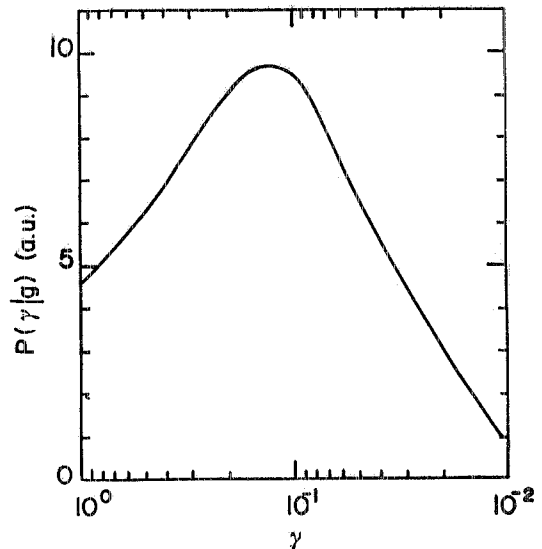
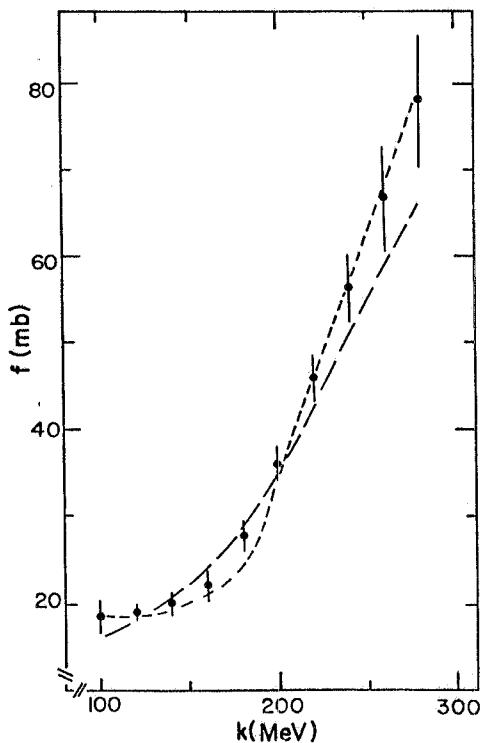


FIG. 3 - Probability density  $P(\gamma/g)$  as a function of the smoothing parameter  $\gamma$ .



The photofission cross section of  $^{238}\text{U}$  obtained for  $\gamma=0.12$  is reported in Fig. 4, by filled points. The errors, which were evaluated by the usual propagation rule, account for both experimental errors and auxiliary conditions imposed on the solution.

The effect of the value of the  $\gamma$ -parameter on the photofission cross section behaviour is shown in Fig. 4, where the result obtained for  $\gamma=0.12$  is compared with the solutions deduced assuming  $\gamma=1.0$  (dashed curve) and  $\gamma=0.03$

FIG. 4 - Photofission cross section estimated by our unfolding method. The experimental points represent the results obtained by  $\gamma=0.12$  smoothing parameter. The dashed curve is obtained by  $\gamma=1.0$ , the short-dashed one by  $\gamma=0.03$  (see text).

(dotted curve), in correspondence of which the probability  $P(\gamma/g)$  assumes the half value respect to the maximum (see Fig. 3). As shown the two curves and the chosen results are well in agreement within the errors.

In Fig. 5 are plotted the rows of the energy resolution R-matrix for some maximum photon energies. The R-matrix rows actually have the suitable form of an energy resolution function, except for some small physically meaningless undershoots, with the maximum at about the correct energy up to  $k \approx 220$  MeV. At higher energies the R-matrix rows are less resolute and this produces large errors in the relevant cross section values. These effects in the resolution functions are generally observed<sup>(10-16)</sup> near the photon maximum energy at which the measurements were stopped.

To allow a comparison, in Fig. 6 the present results are plotted together with those obtained by some of us<sup>(15)</sup> by means of coherent bremsstrahlung photons. These data show the same behaviour of the present results but the absolute values are lower, as a consequence of the average on a large energy range operated by the R-matrix obtained in that experiment, as one can observe looking at Fig. 9 of Ref. (13).

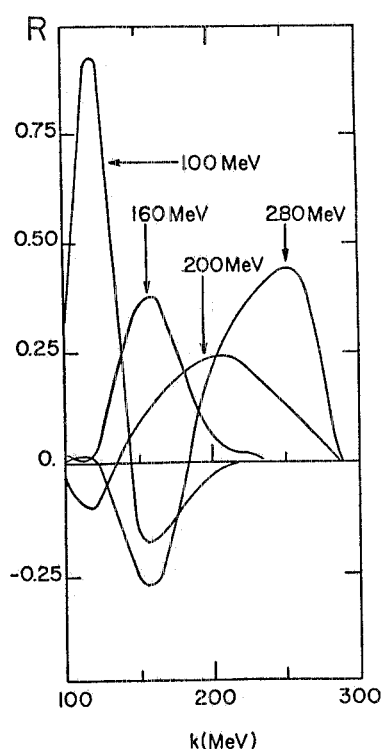


FIG. 5 - Rows of the energy resolution matrix R obtained for  $^{238}\text{U}$  by  $\gamma=0.12$  at different maximum photon energies  $k_m$ .

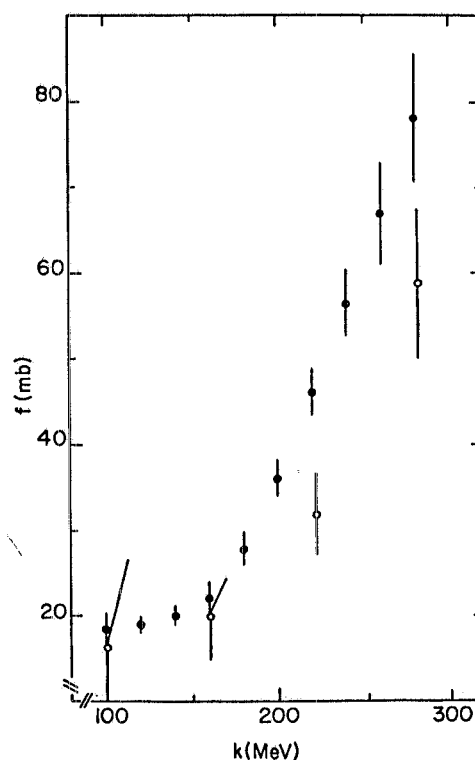


FIG. 6 - Photofission cross section of  $^{238}\text{U}$ . ● - present results obtained by  $\gamma = 0.12$  smoothing parameter, o - data of Ref. (15).

## 5. - DISCUSSION

Tagged photons have been recently used at Bonn to measure the photofission cross section of  $^{235}\text{U}$  and  $^{238}\text{U}$  in the energy range from 120 up to 460 MeV<sup>(19)</sup>. The obtained results of the normalized cross section per nucleon, are compared in Fig. 7 with relevant values deduced from present data. The excellent agreement between experimental points from different experiments emphasizes the reliability of measurements performed with quasi-monochromatic photon beams.



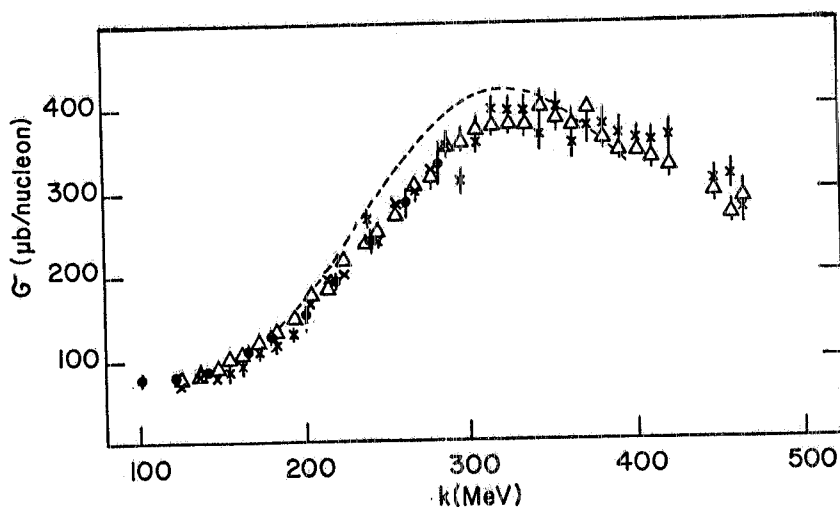


FIG. 7 - Normalized cross section per nucleon versus the photon energy  $k$ . Experimental points:  $\bullet$  - present results on  $^{238}\text{U}(\gamma, f)$ ,  $\Delta$  - data from Ref. (18) for  $^{238}\text{U}(\gamma, f)$ ,  $\times$  - data from Ref. (18) for  $^{235}\text{U}(\gamma, f)$ . The dashed curve represents the renormalized total hadronic photoabsorption cross section for  $^9\text{Be}$ , from Ref. (32).

From the knowledge of the photofission cross section  $f(k)$  it is possible to deduce information on the nuclear fissility, defined as the ratio of the fission cross section to the total photoabsorption cross section:

$$P_f(k) = \frac{f(k)}{\sigma_t(k)} \quad (3)$$

The total photonuclear cross section has been recently measured for  $\text{Be}^{(32)}$  and  $\text{Pb}^{(33)}$  in the  $\Delta$ -resonance region. No significant difference has been observed between the cross section per nucleon derived from the two measurements. This fact has suggested a simple linear dependence of  $\sigma_t(k)$  on  $A$ , for  $9 \leq A \leq 208$ . The extension of this result to the higher  $A$  region can be inferred from Fig. 7; as shown, the normalized Uranium photofission cross section agrees well with the normalized total hadronic cross section for  $^9\text{Be}$  (dashed curve in the Figure). With this assumption the total photoabsorption cross section for the  $^{238}\text{U}$  was evaluated.

Moreover, in order to deduce the fissility from Eq. (3), it must be considered that the obtained photofission cross section values were averaged in energy by the matrix  $R$ . Consequently the nuclear fissility was calculated as follows:

$$R P_f \hat{\sigma}_t = \hat{f} \quad (4)$$

where the product  $P_f \hat{\sigma}_t$  was also averaged in energy by means of the  $R$ -matrix.

The fissility values obtained are given in Fig. 8, as a function of the photon energy. As shown, the fissility is constant in the considered energy region, and equal to  $0.87 \pm 0.05$ . This result disagrees with that given in Ref. (15). Apart from the quoted differences in the absolute photofission cross section values, this disagreement must be ascribed to different values for the total photoabsorption cross section used in the analysis. In fact, in the quoted reference, this cross section was estimated by taking into account photon absorption contributions from the quasi-deuteron model as well as from the

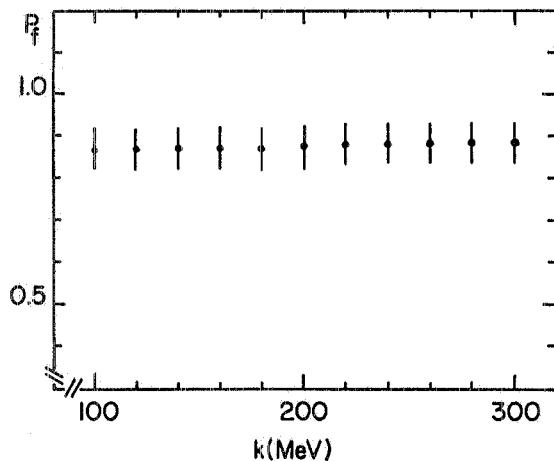


FIG. 8 - Nuclear fissility  $P_f$  for  $^{238}\text{U}$ , versus photon energy  $k$ , deduced from the photofission cross section of Fig. 4.

photomesonic mechanism originated by the production of neutral and charged pions from individual nucleons of the nucleus.

The present fissility value agrees well with those of Refs. (6, 9, 34) and also with the result of a calculation<sup>(35)</sup> made on a basis of a cascade-evaporative model of the nuclear fission induced by photons at energies less than 1 GeV.

#### 6. - CONCLUSIONS

The  $^{238}\text{U}$  photofission yields produced by monochromatic photon were measured in the energy range from 120 up to 280 MeV, using the LEALE photon beam facility at Frascati. The calculated nuclear fissility resulted constant as a function of energy and equal to  $0.87 \pm 0.05$ . Therefore the measurements of photofission cross sections for fissile nuclei can represent an alternative method to estimate the total photoabsorption cross sections in the investigate energy region.

The experiment has also stressed that the use of a quasi-monochromatic photon beam, the simultaneous measurement of energy spectrum, total energy and profile of the photon beam are important improvements for a reliable determination of the absolute value of the photofission cross section.

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