ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

LNF-84/26

F.Palumbo: CONSTANT GAUGE FIELD CONFIGURATIONS AND GALELIAN GAUGE THEORIES

EStratto da:

Phys. Letters 137B, 404 (1984)

Servizio Documentazione
dei Laboratori Nazionali di Frascati
Cas. Postale 13 - Frascati (Roma)

CONSTANT GAUGE FIELD CONFIGURATIONS AND GALILEAN GAUGE THEORIES

F. PALUMBO
INFN, Laboratori Nazionali di Frascati, P.O. Box 13, 00044 Frascati, Italy

Received 5 December 1983

Galilean gauge theories are characterized by constraints which are compatible only with the zero-momentum mode of the gauge fields. These theories are quantized according to the Dirac theory of constrained systems and are explicitly formulated in terms of the canonical variables.

1. Looking for the features of non-abelian gauge theories which could possibly provide the confinement of color, it is desirable to introduce the most drastic approximations which could preserve such features. We have considered the limit of infinite light velocity, which gives rise to galilean gauge theories. These theories are characterized by gauge invariant constraints which are compatible only with the zero-momentum mode of the gauge fields. Quantization of galilean gauge theories is therefore equivalent to the quantization of constant gauge field configurations.

We have checked in previous papers [1] that the low-energy behaviour of relativistic theories for which such behaviour in known is reproduced by the corresponding galilean theories. This has been shown for the Goldstone and Higgs model, the Schwinger mechanism for spontaneous mass generation, the massless Wess–Zumino and Fayet–Iliopoulos models of supersymmetry, and for QED.

The big simplification achieved in the galilean limit is that only the zero-momentum component of massless fields survives. The fact that such a drastic approximation retains the essential infrared features can be easily understood in the framework of spontaneous symmetry breaking, where it corresponds to the tree approximation plus some quantum effect. In the case of QED the explanation requires a detailed analysis showing that the zero-momentum component of the photon field gives rise to a finite effect due to a mechanism analogous to Bose–Einstein condensation in statistical mechanics [2].

The above results give some confidence that also the infrared behaviour of QCD should not qualitatively change in the galilean approximation. Relativistic effects can in any case be accounted for by means of a $1/c$ expansion [3].

In the abelian case we have both taken the $c \to \infty$ limit of relativistic QED [3] and quantized the classical galilean theory of electrodynamics [2], by finding the same infrared behaviour.

In the non-abelian case we will restrict ourselves to the quantization of the classical galilean theory in a new gauge. The formal $c \to \infty$ limit of relativistic QCD has also been obtained [3], but this theory is not known in the gauge that we well adopt here, while there are problems with many other gauges.

In order to illustrate this point we will show at the end of the paper that in the galilean limit the Faddeev–Popov determinant vanishes in the coulombic, Landau and $A_3 = 0$ gauges. This implies that these gauges are not acceptable also in relativistic QCD, unless galilean field configurations have zero-measure.

2. The galilean gauge field lagrangian density in first order formulation is

$$\mathcal{L}_G = E_i^a \partial_t A_i^a - \tfrac{1}{2} E_i^a E_i^a - V^a \mathcal{D}_i^{ab} E_i^a + \Lambda_i^a \phi_i^a \tag{1}$$

In the above equation

$$\mathcal{D}_i^{ab} = \partial_i \delta^{ab} - g f_c^{ab} A_i^c = (\partial_i - g A_i)^{ab}, \tag{2}$$

$$\phi_i^a = \tfrac{1}{2} \epsilon_{ijk} (\partial_j A_k^a - \partial_k A_j^a + g f_{bc}^a A_j^b A_k^c), \tag{3}$$

f_{bc}^a being the structure constants of the color group.

The dynamical variables are A_i^a with canonical mo-

0.370-2693/84/$ 03.00 © Elsevier Science Publishers B.V. (North-Holland Physics Publishing Division)

menta

$$E_i^a = \partial \mathcal{L}_G / \partial \partial_t A_i^a \,, \tag{4}$$

while V^a and Λ_i^a are Lagrange multipliers for the primary constraints

$$\phi_i^a = 0, \quad \phi^a = \mathcal{D}_k^{ab} E_k^b = 0 \,. \tag{5,6}$$

Constraint (6) is common to relativistic QCD.

The galilean matter field lagrangian density is

$$\mathcal{L}_M = \psi^* \mathrm{i} \mathcal{D}_t \psi + \bar{\psi}^* \mathrm{i} \mathcal{D}_t^* \bar{\psi} + \psi^* (\mathcal{D}^2/2m)\psi + \bar{\psi}^* (\mathcal{D}^{*2}/2m)\bar{\psi} - mc^2(\psi^*\psi + \bar{\psi}^*\bar{\psi}) \,, \tag{7}$$

where ψ, $\bar{\psi}$ are matter, antimatter fields respectively,

$$\mathcal{D}_t = \partial_t + \mathrm{i} g t^a V^a \,, \tag{8}$$

and t^a are the generators of the color group in the appropriate representation.

Eq. (8) holds both for spinors and scalars. The canonical variables are ψ and $\bar{\psi}$ with canonical momenta

$$\pi = \partial \mathcal{L}_M / \partial \partial_t \psi = \mathrm{i}\psi^* , \quad \bar{\pi} = \partial \mathcal{L}_M / \partial \partial_t \bar{\psi} = \mathrm{i}\bar{\psi}^* \tag{9}$$

The nonvanishing Poisson brackets in the spinor and scalar case respectively are

$$\{\psi_\alpha^r(x), \pi_\beta^s(y)\} = \{\bar{\psi}_\alpha^r(x), \bar{\pi}_\beta^s(y)\} = \delta_{\alpha\beta}\delta^{rs}\delta^3(x-y) \,,$$

$$[\psi^r(x), \pi^s(y)] = [\bar{\psi}^r(x), \bar{\pi}^s(y)] = \delta^{rs}\delta^3(x-y) \,. \tag{10}$$

When gauge fields are coupled to matter fields the constraint (6) becomes

$$\phi^a = \mathcal{D}_k^b E_k^b + g\rho^a = 0 \,, \tag{11}$$

where

$$\rho^a = \psi^* t^a \psi - \bar{\psi}^* t^a \bar{\psi} \,. \tag{12}$$

We quantize in a cubic box of volume L^3. We require that the fields be periodic or vanish on the surface of such box. We will refer to such conditions as periodic boundary conditions (p.b.c.) or vanishing boundary conditions (v.b.c.).

We can therefore expand fields and constraints in Fourier series

$$A_i^a = \frac{1}{L^{3/2}} \sum_n A_{in}^a \exp[\mathrm{i}(2\pi/L) n \cdot x] \,, \quad A_{in}^a = A_{i,-n}^{a*} \,,$$

$$E_i^a = \frac{1}{L^{3/2}} \sum_n E_{in}^a \exp[-\mathrm{i}(2\pi/L) n \cdot x] \,, \quad E_{in}^a = E_{i,-n}^{a*} \,,$$

$$\phi_i^a = \frac{1}{L^{3/2}} \sum_n \phi_{in}^a \exp[\mathrm{i}(2\pi/L) n \cdot x] \,, \quad \phi_{in}^a = \phi_{i,-n}^{a*} \,, \tag{13}$$

and so on.

We now apply Dirac's theory [4] of canonical quantization of constrained systems.

We find that the only secondary constraints are

$$\chi_i^a = \epsilon_{ijk} \mathcal{D}_j^{ab} E_k^b = 0 \,, \tag{14}$$

and the only first class constraints are ϕ^a. We use an intermediate procedure with respect to the standard ones of using first class constraints on physical states or adding gauge fixing constraints. We use the zero-momentum components ϕ_0^a of ϕ^a as constraints on the physical states and add gauge fixing for ϕ_n^a, $n \neq 0$. One of the reasons to select ϕ_0^a is that it has a direct physical meaning, being proportional to the total color charge Q^a

$$\phi_0^a = g L^{3/2} Q^a \,. \tag{15}$$

Now since ϕ_0^a comes from the variation of V_0^a it is only present when $V_0^a \neq 0$, i.e. for p.b.c., so that with these conditions only singlet color states can exist, while all color states are compatible with v.b.c. This consequence of b.c. has already been discussed in detail [2].

We must now choose the gauge fixing for $n \neq 0$. We see immediately that the constraint

$$A_{3n}^a = 0, \quad n \neq 0 \tag{16}$$

is not acceptable, because of the vanishing of the determinant of the brackets

$$[A_{3m}^a, \chi_n^b] = -\mathrm{i}(2\pi/L) n_3 \delta_{mn} \delta^{ab} \,. \tag{17}$$

A gauge fixing which we will show in a longer paper [5] to be consistent is the following:

$$A_{3n} = 0, \quad n_3 \neq 0 \,,$$
$$A_{2n} = 0, \quad n_3 = 0, \quad n_2 \neq 0 \,,$$
$$A_{1n} = 0, \quad n_3 = n_2 = 0, \quad n_1 \neq 0 \,. \tag{18}$$

It will be shown by lengthy calculations that the solution to eqs. (5), (18) for SU(2) with p.b.c. is

$$A_{in}^a = \delta_{n0} q_i \hat{v}^a, \quad \hat{v}^a = v^a/v \,, \tag{19}$$

where q_i and $\hat{v}^a$ depend only on time. The constraints (11) and (14) (omitting ϕ_0^a) can now be rewritten in

Fourier components

$$\mathcal{D}_k(n)E_{kn} + g\rho_n = 0, \quad n \neq 0,$$

$$\mathcal{D}_i(n)E_{jn} - \mathcal{D}_j(n)E_{in} = 0, \tag{20}$$

where

$$\mathcal{D}_i^{ab}(n) = -\mathrm{i}(2\pi/L)n_i\delta^{ab} - g\epsilon_c^{ab}\hat{\upsilon}^c q_i. \tag{21}$$

The solution to eqs. (20) with p.b.c. is

$$E_{in}^a = \delta_{n0}(p_i\hat{\upsilon}^a + q^{-2}l_\perp^a q_i) - (1-\delta_{n0})\cdot g\mathcal{D}^{-2}(n)\mathcal{D}_i(n)\rho_n, \tag{22}$$

with

$$l_\perp^a = (\pi^a - \hat{\upsilon}^a\hat{\upsilon}\cdot\pi)\upsilon. \tag{23}$$

From Dirac's brackets the new variables are shown to satisfy the canonical commutation relations

$$[q_h, p_k] = \delta_{hk}, \quad [\upsilon^a, \pi^b] = \delta^{ab}. \tag{24}$$

The resulting hamiltonian is

$$H = \tfrac{1}{2}p^2 + \tfrac{1}{2}\omega^2q^2 + \tfrac{1}{2}(\upsilon^2\wedge\pi)^2/q^2 - \tfrac{1}{2}g^2\int \mathrm{d}^3x\,\mathrm{d}^3y\,\rho^a(x)[\mathcal{D}^{-2}x-y]^{ab}\rho^b(y) - \frac{1}{2m}\int \mathrm{d}^3x(\psi^*\Delta\psi + \bar{\psi}^*\Delta\bar{\psi}) + mc^2\int \mathrm{d}^3x(\psi^*\psi + \bar{\psi}^*\bar{\psi}) - 2\omega q_k\hat{\upsilon}^a I_k^a, \tag{25}$$

where

$$\omega^2 = Ng^2/4mL^3, \quad N = \int \mathrm{d}^3x(\psi^*\psi + \bar{\psi}^*\bar{\psi}), \tag{26,27}$$

$$I_k^a = (m/N)^{1/2}\,\mathrm{d}^3x(\mathrm{i}m)^{-1}(\psi^*\tfrac{1}{2}\sigma^a\partial_k\psi - \bar{\psi}^*\tfrac{1}{2}\sigma^a\partial_k\bar{\psi}), \tag{28}$$

$$[\mathcal{D}^{-2}(x)]^{ab} = -(4\pi)^{-1}|x|^{-1}[\exp(gL^{-3/2}\hat{\upsilon}q\cdot x)]^{ab} = -(4\pi)^{-1}|x|^{-1}[\delta^{ab} - (1-\cos gL^{-3/2}q\cdot x) \times(\delta^{ab} - \hat{\upsilon}^a\hat{\upsilon}^b) + \epsilon^{abc}\hat{\upsilon}^c\sin gL^{-3/2}q\cdot x]. \tag{29}$$

Physical states Ψ are constrained according to ϕ_0^a

$$(\upsilon\wedge\pi)^a - \int \mathrm{d}^3x(\psi^*\tfrac{1}{2}\sigma^a\psi - \bar{\psi}^*\tfrac{1}{2}\sigma^a\bar{\psi}) = Q^a\cdot\Psi = 0. \tag{30}$$

The new feature with respect to galilean QED is that the "gluon" state depends on the intrinsic state of matter through the Green function $\mathcal{D}^{-2}(x)$ beyond its coupling with the current.

With v.b.c. $A_{in} = 0$ and E_{in} is given by the second term on the r.h.s. of eq. (22), so that the hamiltonian contains only a purely coulombic interaction.

3. In the course of our quantization procedure we have seen that the determinant of the Faddeev–Popov operator vanishes in the gauge $A_3 = 0$. Let us show that it also vanishes in the Coulomb and Landau gauges which coincide in the Galilean limit.

The constraints ϕ_i^a can be rewritten

$$\mathcal{D}_iA_k = \partial_kA_j. \tag{31}$$

By taking the $\mathcal{D}_k$-derivative of the above equation and using the commutivety of covariant derivates which follows from $\phi_i^a = 0$ we have

$$\mathcal{D}_k\partial_kA_j = \mathcal{D}_j\mathcal{D}_kA_k = \mathcal{D}_j\partial_kA_k = 0, \tag{32}$$

showing that the Faddeev–Popov operator has the eigenstate A_j with vanishing eigenvalue.

References

[1] F. Palumbo, Nucl. Phys. B182 (1981) 261; B197 (1982) 334;
S. Ferrara and F. Palumbo, in: the 2nd Europhysics Study Conf. on Unification of the fundamental particle interactions (Erice, Sicily, October 1982).
[2] F. Palumbo, Phys. Lett. B132 (1983) 165;
F. Palumbo and G. Pancheri, Phys. Lett. 137B (1984) 401.
[3] F. Palumbo, Lett. Nuovo Cimento 30 (1981) 72.
[4] P.A.M. Dirac, Proc. R. Soc. A246 (1958) 326; Lectures in Quantum mechanics, Belfer Graduate School of Science, Yeshiva University (New York, 1964).
[5] E. Hilf and L. Polley, Phys. Lett. B131 (1983) 412.