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# ON ENERGY–MOMENTUM CONSERVATION

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In this note I show how to use the replicas formalism to study the macroscopic consequences of a random violation of energy–momentum conservation at the microscopic level (i.e. at distances much smaller than $10^{-13}$ cm).

The properties of a quantum field theory defined on a disordered medium have recently been investigated for the first time [1]. If the disordered medium is considered to be not a technical device but a true physical reality two different points of view can be taken: (a) the disorder may be considered as a quantum variable and the usual quantum mechanics (with some modifications) is recovered for the whole system; (b) quantum mechanics does not apply to the variables describing the disorder (e.g. they are classical variables): quantum mechanics is a good description only for the fields, not for the disorder of the medium [2].

The last point of view may be not unreasonable: space–time may have a very complex topology at short distances [3] (i.e. $10^{-19}$ fm) and it is not clear if the evolution of the topology may be described by the familiar quantum mechanics laws. Partially motivated by these wild speculations we address here to a simple well defined question: We consider a conventional field theory in presence of a fixed background field which is disordered at a small scale $l$; translational invariance is obviously broken (the quantum field may exchange momenta of order $1/l$ with the fixed background field). We would like to know the behaviour of the violations of translational invariance and energy–momentum conservation when $l$ goes to zero. The answer to this question is not immediate; indeed it is possible to argue in two opposite ways:

(a) The violations of translational invariance go to zero with $l$ because the theory cannot be dependent on what happens at very short distances.

(b) The violations of translational invariance become very large and translational invariance is completely lost when $l$ goes to zero because the momentum exchanged in an elementary interaction of the quantum field with the background is of order $1/l$.

We now proceed to obtain the correct answer to this question by using the formalism of the replicas [4]; as we shall see the final estimates will be strongly dependent on the type of theory we shall consider.

We start with some definitions in order to formulate the problem in a clear-cut way. We call $G(x, y|B)$ the Green function of the quantum field $\phi$ in presence of the background field $B$:

$$G(x,y|B) \equiv \langle \phi(x)\,\phi(y)\rangle_{\rm B}$$
$$\equiv \int \mathrm{d}\mu_{\rm B}[\phi]\,\phi(x)\,\mathrm{d}(y) \Big/ \int \mathrm{d}\mu_{\rm B}[\phi]\,,$$
$$\mathrm{d}\mu_{\rm B}[\phi] \equiv \mathrm{d}[\phi]\exp[-S(\phi,B)]\,, \tag{1}$$

and we assume that $\langle\phi\rangle_{\rm B} = 0$ in order to simplify the analysis. For convenience I use the notation of euclidean quantum field theory although the usual relation between the minkowski and the euclidean theories is more involved in presence of violations of translational invariance [‡1]; in the worst case eq. (1)

[‡1] The product of unitary matrices is unitary but the product of hermitian matrices is not hermitian in general; moreover the probability distribution for the $B$ field in Minkowsky space may get phases when rotated to euclidean space.



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can be considered as a bookkeeping of the usual diagrammatical expansion.

In the present case $G(x,y|B)$ depends also on $x+y$ and not only on $x-y$ as is usual; as a consequence the Fourier transform

$$\tilde{G}(\omega,\Delta|B) \equiv \int \mathrm{d}^D x\, G(x,x+\Delta|B)\exp(\mathrm{i}\omega\cdot x), \qquad (2)$$

will be non-zero also at $\omega$ different from zero.

The value of

$$\Gamma(\omega,\Delta) \equiv \overline{|\tilde{G}(\omega,\Delta|B)|^2} \equiv \int \mathrm{d}P[B]\, |\tilde{G}(\omega)\Delta|B)|^2\,, \qquad (3)$$

(the bar denotes the average over the configurations of the classical background field $B$) at non-zero $\omega$ will be taken as the signal of violations of translational invariance. If, after averaging over the $B$ fields, translational invariance is recovered, we have that:

$$\Gamma(\omega,\Delta) = \int \mathrm{d}x\,\mathrm{d}y\, \exp[\mathrm{i}(x-y)\cdot\omega]$$
$$\times\, \Gamma^{(4)}(x, x+\Delta, y, y+\Delta)\,,$$
$$\Gamma^{(4)}(x,y,u,v) = \overline{G(x,y|B)\,G(u,v|B)}$$
$$-\, \overline{G(x,y|B)}\;\overline{G(u,v|B)}\,. \qquad (4)$$

Possible ultraviolet divergences due to coinciding points in eq. (4) may be eliminated if we take the precaution of smearing the field $\phi$ with a test function.

The original question becomes now: which is the behaviour of $\Gamma^{(4)}$ when $l$ goes to zero. The simplest and most compact tool for investigating this question is the replica method [4].

We consider a system composed by $n$ replicas of the field $\phi$ ($\phi_i(x)$, $i=1,n$) and by the field $B$ with global probability distribution:

$$\mathrm{d}P[B] \sum_{i=1}^{n} \mathrm{d}\mu_{\mathrm{B}}[\phi_i]\,. \qquad (5)$$

The theory is invariant under the group $\mathrm{P}_n$ of permutations of $n$ elements (the replicas), which we assume to be unbroken ‡2. Simple group theory tells us that ‡3

$$\langle\phi_i(x)\,\phi_j(y)\rangle = \delta_{i,j}G^{(2)}(x-y)\,,$$
$$\langle\phi_i(x)\,\phi_j(y)\phi_l(u)\phi_m(v)\rangle = \delta_{i,j}\delta_{j,l}\delta_{l,m}G_0^{(4)}(x,y,u,v)$$
$$+\,(\delta_{i,j}\delta_{l,m}+\delta_{i,l}\delta_{j,m}+\delta_{i,m}\delta_{l,j})G_1^{(4)}(x,y,u,v)\,. \qquad (6)$$

It is easy to see that if we continue eqs. (5), (6) from integer $n$ to non-integer $n$ (if the replica symmetry is unbroken no difficulties should be present) we have that:

$$G^{(2)}(x-y) = \overline{G(x,y|B)}\,,$$
$$G_1^{(4)}(x,y,u,v) = \Gamma^{(4)}(x,y,u,v)\,. \qquad (7)$$

Our system in the presence of a fixed inhomogeneous field $B$ can be considered as the $n=0$ limit of an $n$-component theory; indeed in this limit the distribution of the $B$ field is not modified by the contributions to the vacuum polarization due to the $\phi$ fields. In this framework violations of the translational invariance manifest themselves as non-vanishing correlations between fields belonging to different replicas.

After these preliminaries the study of the limit $l\to 0$ can be done using the familiar apparatus of quantum field theory. Standard decoiupling theorems tell us that after the integration over the $B$ field we can write a local effective low energy action for the $\phi$ fields of the type (in four dimensions):

$$\prod_{i=1}^{n} \mathrm{d}[\phi_i]\exp(-l^{-2}S_2+S_4+l^2S_6+l^8S_8+\ldots)\,, \qquad (8)$$

where $S_d$ contains local operators of dimensions $d$. In most of the cases the terms proportional to $S_2$ are mass counterterms which can be removed by mass renormalization and do not couple different replicas. The violations of energy–momentum conservation will go to zero like $l^{d_c-4}$, $S_{d_c}$ being the lowest dimensional local operator which couples different replicas and may appear in the effective action. The computation of $d_c$ can be done by inspection. We consider a few representative cases.

For a pure scalar theory with a $\phi^4$ self-interaction we can have a term in the effective action proportional to

$$\left(\sum_{i=1}^{n}\phi_i^2\right)^2\,. \qquad (9)$$

In this case, neglecting logarithms, we expect viola-

‡2 At the present moment it is not clear to me if the original proposal of ref. [2] where the replica symmetry is broken, does not involve too strong violations of the energy momentum conservation.

‡3 We assume that $\langle\phi_i\rangle = 0$ in each replica.

tions of order one of the energy–momentum conservation also when $l$ goes to zero.

If in a pure gauge theory gauge invariance remains exact for any choice of the $B$ field, we can do independent gauge transformations in each of the replicas; the first operator, which couples two replicas and is gauge invariant, has dimension 8 and it is given by:

$$\left(\sum_{i=1}^{n} F_{\mu\nu}^{(i)} F_{\mu\nu}^{(i)}\right)^2 . \tag{10}$$

In this case violations of translational invariance goes to zero like $l^4$.

When we add fermion dimension-six operators like

$$\left(\sum_{i=1}^{n} \bar{\psi}_{(i)}\psi_{(i)}\right)^2, \quad \sum_{i=1}^{n} \bar{\psi}_{(i)}\gamma_\mu\psi_{(i)} \sum_{j=1}^{n} \bar{\psi}_j\gamma_\mu\psi_j , \tag{11}$$

are allowed; however if the coupling of the $B$ fields to the fermions violates neither chiral invariance, nor $C$ invariance (as happens for a background gravitational field) these operators are not allowed and violations of energy–momentum conservation are of $O(l^4)$.

A phenomenonological analysis of the consequences of the presence of violations of energy–momentum conservation is beyond the scope of this paper. We only note that effects of order $l^4$ are quite likely undetectable while the situation could be more interesting as far as the effects $O(l^2)$ are concerned.

It may be suggestive that the effects of non-conservation of energy–momentum could be much more marked in rare events which involve short distance physics. It would be therefore not wise to reject all experimental candidates for proton decay with violations of energy–momentum conservation as neutrino induced reactions.

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*References*

[1] See e.g. R. Friedberg and T.D. Lee, Nucl. Phys. B225 [FS9] (1983) 1, and references therein.
[2] G. Parisi, Randomness as a source of massless particles, in: Unification of the fundamental particle interaction, eds. S. Ferrara, J. Ellis and P. van Nieuwenhuizen (Plenum, New York, 1980).
[3] S.W. Hawking, D.N. Page and C.N. Pope, Nucl. Phys. B170 [FS1] (1980) 283.
[4] See e.g., G.P. Parisi, in: Proc. Les Houches Summer School (1982) (North Holland, Amsterdam) to be published, and references therein.