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INCLUSIVE INELASTIC CROSS SECTION FOR α–α SCATTERING AT HIGH ENERGIES

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The inclusive inelastic scattering of two nuclei is analysed in the microscopic model with multiple scattering of constituent nucleons. A good agreement with experiment for the α–α system is obtained if the translational invariance and the correct structure of the projectile profile are included.

Recently experimental data have been published [1,2] for inclusive inelastic scattering of fast α particles on ^{4}He target, extending over a large range of energy and momentum transfer. They challenge for an interpretation in terms of inelastic scattering sum rules, whose attractive feature is that nuclear informations involved pertain only to the ground state. Moreover the simplicity of the α–α system may help to reveal essential effects that determine the inclusive scattering of two nuclei.

The total inelastic cross section, at a given momentum transfer q, is obtained by subtracting from the sum of all purely nuclear scattering the elastic contribution,

$$d\sigma_{\rm inel}/d\Omega = \sum_{n\neq 0} |\langle n|G(q)|0\rangle|^2, \tag{1}$$

$$d\sigma_{\rm inel}/d\Omega = \langle 0|G^+(q)\,G(q)|0\rangle - |\langle 0|G(q)|0\rangle|^2, \tag{2}$$

where $G(q)$ is an operator describing the nuclear transitions induced by the incident hadron. We will use the Glauber form [3] of the transition operator that accounts for multiple scattering of constituent nucleons in fast nuclear collisions:

$$G(q) = (\mathrm{i}p/2\pi)\int d^2b\, \exp(\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{b}) \times \left(1-\prod_{j=1}^{A}[1-\gamma_j(\boldsymbol{b}-\boldsymbol{s}_{jA}+\boldsymbol{S}_A)]\right) \tag{3}$$

$$= (\mathrm{i}p/2\pi)\exp(-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{S}_A)\int d^2b\, \exp(\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{b}) \times \left(1-\prod_{j=1}^{A}[1-\gamma_j(\boldsymbol{b}-\boldsymbol{s}_{jA})]\right), \tag{4}$$

where p is the incident CM momentum and γ_j are the profiles of the target nucleons which depend on their coordinates s_{jA} in the plane of impact parameters. The presence of the respective coordinate of the centre-of-mass of the target $\boldsymbol{S}_A$ is required by the translational invariance [4]. The factorization and the form of the CM correction in eq. (4) means that only the elastic term in the rum rule (2) is affected by this correction.

The profile of each target nucleon as seen by the incident ion may be expressed through the elementary profiles $\gamma_{\rm NN}$ of the nucleon–nucleon interaction as follows:

$$\gamma(b) = \langle 0_B|\left(1-\prod_{K=1}^{B}[1-\gamma_{\rm NN}(\boldsymbol{b}-\boldsymbol{s}_{kB}+\boldsymbol{S}_B)]\right)|0_B\rangle, \tag{5}$$

where $|0_B\rangle$ denotes the ground state of the projectile nucleus and $s_{kB} - S_B$ are the intrinsic coordinates of its nucleons projected onto the impact parameter plane.

Another way of obtaining the profile $\gamma(b)$ is to exploit the experimental properties of the elastic scattering of a nucleon on the projectile nucleus. In either case, microscopic or phenomenological, the projectile ion is treated as quasi-rigid during the collision and a small probability of its virtual excitations [5] is neglected.

We assume that the ground state of the target can be described by means of the independent particle model:

$$|\langle r_1 \ldots r_A|0\rangle|^2 = \prod_{j=1}^{A} \rho_A(r_j), \tag{6}$$

where $\rho_A(r)$ is the single particle density. Then for the inelastic cross section one obtains:

$$\begin{aligned} \mathrm{d}\sigma_{\mathrm{inel}}/\mathrm{d}\Omega = &(p/2\pi)^2 \int \mathrm{d}^2 b_1\, \mathrm{d}^2 b_2 \exp[\mathrm{i}\boldsymbol{q}\cdot(\boldsymbol{b}_1 - \boldsymbol{b}_2)] \\ &\times \{[1 - S(b_1) - S^*(b_2) + U(\boldsymbol{b}_1, \boldsymbol{b}_2)]^A \\ &- \theta_A^2(q)[1 - S(b_1) - S^*(b_2) + S(b_1) S^*(b_2)]^A\}, \end{aligned} \tag{7}$$

where

$$S(b) = \int \mathrm{d}^3 r\, \rho_A(r)\, \gamma(\boldsymbol{b} - \boldsymbol{s}), \tag{8}$$

$$U(\boldsymbol{b}_1, \boldsymbol{b}_2) = \int \mathrm{d}^3 r\, \rho_A(r)\, \gamma(\boldsymbol{b}_1 - \boldsymbol{s})\, \gamma^*(\boldsymbol{b}_2 - \boldsymbol{s}), \tag{9}$$

and θ_A^2 is the CM correction due to the elastic term in (2) which is being subtracted from the total cross section. This correction can indeed be factorized in the case of harmonic oscillator density. Though commonly nelgected [6–8] it turns out to be quite important for light targets.

In our calculations we have used the gaussian density for both the projectile and target nuclei,

$$\rho(r) = \pi^{-3/2} R^{-3} \exp(-r^2/R^2). \tag{10}$$

Then $\theta_A(q) = \exp(q^2 R_A^2/4A)$, R_A being the gaussian radius of the target.

The elementary nucleon profiles were assumed spin and isospin independent as

$$\gamma_{\mathrm{NN}}(b) = [\sigma(1 - \mathrm{i}\,\alpha)/4\pi a] \exp(-b^2/2a), \tag{11}$$

which corresponds to the gaussian dependence of the nucleon–nucleon elastic scattering amplitude at small momentum transfer; the parameters σ (total cross section), α (Re/Im ratio) and a (slope) are energy dependent. Then the microscopic nucleon profile as seen by the projectile nucleus B is

$$\begin{aligned} \gamma(b) = &\sum_{k=1}^{B} \binom{B}{k} (-1)^{k+1} [\sigma(1 - \mathrm{i}\alpha)/2\pi(R_B^2 + 2a)]^k \\ &\times \{(R_B^2 + 2a)/[(B-k) R_B^2/B + 2a]\} \\ &\times \exp\{-kb^2/[(B-k) R_B^2/B + 2a]\}. \end{aligned} \tag{12}$$

In the phenomenological approach $\gamma(b)$ is usually approximated as a single gaussian, i.e. it has the form of eq. (11) with the parameters adequate for the nucleon–projectile elastic scattering at low q [7,8].

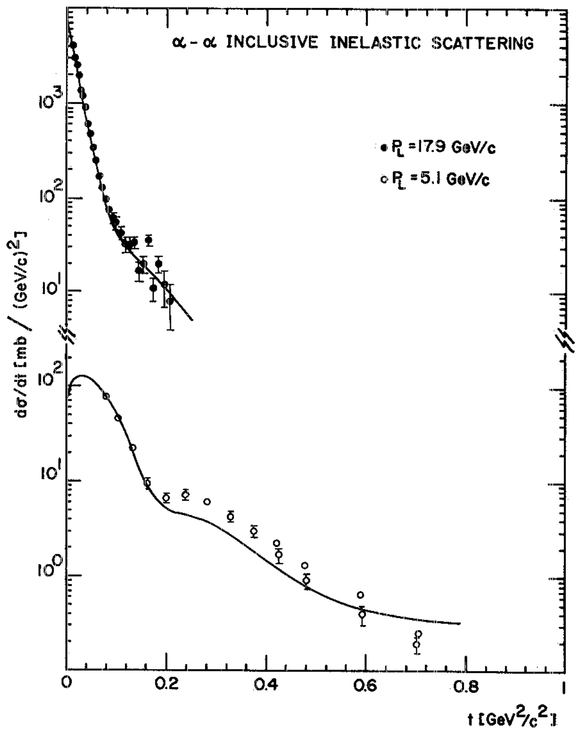

Fig. 1. The inclusive inelastic cross section for α–α scattering. The experimental data at p_L = 17.9 GeV/c are from ref. [1], those at 5.1 GeV/c from ref. [2]. The curves represent our microscopic calculations using the gaussian radius R_α = 1.37 fm. The parameters of nucleon–nucleon scattering are: σ = 41.5 mb, α = −0.35, a = 7.5 GeV^{-2} at 17.9 GeV/c and σ = 40 mb, α = 0.2, a = 2.2 GeV^{-2} at 5.1 GeV/c.

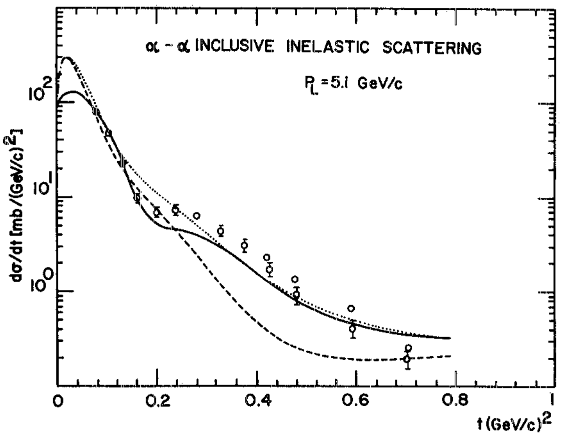

Fig. 2. The inclusive inelastic cross section for $\alpha-\alpha$ scattering at 5.1 GeV/c. The solid and dotted curves represent our microscopic calculations with and without ($\theta_A = 1$) the CM correction, respectively. The same parameters as in fig. 1. The dashed curve has been obtained in the semiphenomenological approach using the gaussian projectile profile (11) with the parameters σ = 135.8 mb, $\alpha = 0$, a = 27.7 GeV^{-2} [7] deduced from the elastic p–α scattering at low q.

Our microscopic calculations of the inelastic sum rule for the $\alpha-\alpha$ system are confronted with the experimental data [1,2] on fig.1. An overall agreement with experiment shows that the inelastic collision of two nuclei at high energies is dominated by multiple scattering of constituent nucleons. It should be pointed out that our calculation does not contain any free parameter.

In fig. 2 are shown the effects of the CM motion and of the structure of the projectile–nucleon profile. The center-of-mass correction makes evident the saddle form of the inclusive cross section around t = 0.25 (GeV/c)^2. Indeed this correction determines the proper height of the second maximum, at t = 0.16 (GeV/c)^2, in the subtracted elastic cross section. The multi-term structure of the projectile profile (12), appearing naturally in the microscopic approach, results to be important in determining the magnitude and the slope of the inclusive cross section below the shoulder. The slope here is indeed much smaller than at low momentum transfers, in agreement with experiment and at variance with the semiphenomenological analysis of ref. [7] which predicts a unique slope. It should be stressed that the microscopic model reproduces very well the angular distributions for $\alpha-\alpha$ elastic scattering.

It would be interesting to follow the way the multiple collision terms in the projectile–nucleon amplitude [eq. (12)] affect the nucleus–nucleus scattering. However for inclusive inelastic scattering the concept of multiple collision looses its transparency since the basic formula (7) appears as a complicated polynomial of the projectile–nucleon profile.

In the case of $\alpha-\alpha$ scattering we have found that all four terms in eq. (12) are important. In fact both the double and triple collision terms reduce the cross section (for all t and for $t > 0.2$ GeV$^2/c^2$, respectively) obtained with the single scattering term alone, while the quadruple scattering again increases the inelastic yield (its increment being most important for $0.05 < t < 0.2$ GeV$^2/c^2$).

As a shortcoming of our calculations we find an insufficient magnitude of the inelastic cross section in the saddle region. This could reflect a need for including the nucleon–nucleon correlations in nuclear wave functions. One should also consider the possibility of virtual excitations [5] of the projectile nucleus during the collision. The estimation of these effects is challenging; however, it requires a considerable computation effort.

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