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LEPTON K_T SPECTRUM FROM W PRODUCTION AT COLLIDER ENERGIES

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ABSTRACT

The effect of subleading corrections to multiple soft gluon emission is shown to be very relevant for the determination of the k_T-spectrum of the decay charged lepton from W produced in $\bar{p}p$ collision . We study the k_T distributions at $\sqrt{s}$=540 GeV and $\sqrt{s}$=2000 GeV and comment on various aspects of these predictions.

1. - INTRODUCTION

The discovery of the W[1] and Z_o[2] weak bosons has given strong support to the electroweak SU(2)xU(1) Standard Model[3]. In addition to the direct observation of these heavy vector bosons, a detailed study of the hadronic production cross sections and of various distributions, provides also an opportunity to probe the strong sector of the process. Conversely, as emphasized by many authors[4], a precise evaluation of QCD effects is quite important for obtaining the accuracy desired to test the electroweak part of the process. This refers in particular to the W-mass which is determined from the transverse momentum spectrum of the decay lepton.

The transerve momentum distributions of both the produced weak boson and of the decay lepton have been recently the object of much investigation. As for the p_T distributions in Drell-Yan process /5/, it has been widely recognized the essential role played by soft gluon effects resummed to all orders, and the relevance of subleading corrections, for understanding the properties of the weak boson p_T spectrum /6/. The question of the relative interplay between the soft anf the hard (to order α_s) components has been also discussed in great detail /7/. The aim of this paper is to present an extensive study of the transverse momentum distribution of the decay lepton in high energy $\bar{p}p$ collisions and to compare it with previous calculations and with the CERN collider data.

In the next section we briefly review the results for the p_T spectrum of the W in terms of the soft gluon resummation formulae including a discussion of the subleading corrections. In Sect. 3 we then combine the W cross section with the essential kinematics relative to the W decay, to obtain the k_T spectrum of the decay lepton. Our numerical results are presented and discussed in Sect. 4 and we give some concluding remarks in Sect. 5.

2. - THE p_T SPECTRUM OF THE W

We first start with the basic formula /5-7/ for the soft p_T distribution of weak bosons produced in a hadronic collision at the c.m. squared energy s

$$\frac{d\sigma}{dQ^2\, dy_W\, dp_T^2} = \frac{2\pi\alpha^2}{9} \frac{K}{(Q^2-M^2)^2+M^2\Gamma^2} \sum_i g_{wi}^2 \times$$
$$\cdot\int bdb\, J_o(bp_T) \exp\left[S(b,q_{T\max})\right] \tag{1}$$
$$\cdot\left\{x_1 q_i^{(1)}(x_1, \frac{1}{b^2})\, x_2\, q_i^{(2)}(x_2, \frac{1}{b^2}) + (1 \leftrightarrow 2)\right\} .$$

M and Γ are the mass and the width of the boson of rapidity y_W, $x_1 = \sqrt{\tau}\, e^{y_W}$ $x_2 = \sqrt{\tau}\, e^{-y_W}$ with $\tau = Q^2/s$ and g^2_{wi} are the appropriate coupling to the quarks and leptons. For $q\bar{q} \to Z_o \to \ell\bar{\ell}$ one has

$$g^2_{wi} = \frac{(A_i^2 + B_i^2)(a^2 + b^2)}{\sin^4 \theta_w \cos^4 \theta_w} \qquad (2)$$

with

$$A_i = \frac{1}{4} - \frac{2}{3}\sin^2\theta_w \qquad B_i = \frac{1}{4} \qquad \text{for up quarks}$$

$$A_i = -\frac{1}{4} + \frac{1}{3}\sin^2\theta_w \qquad B_i = -\frac{1}{4} \qquad \text{for down quarks}$$

$$a = -\frac{1}{4} + \sin^2\theta_w, \qquad b = \frac{1}{4}$$

and we will use $\sin^2\theta_w = 0.226$.

On the other hand for $q\bar{q} \to W \to \ell\nu_\ell$ we have $a=b=\dfrac{\cos\theta_w}{2\sqrt{2}}$ and

$$A_i = B_i = \frac{\cos\theta_w}{2\sqrt{2}}\cos\theta_c \qquad \text{for } \bar{u}d \text{ and } \bar{c}s \text{ coupling}$$

$$A_i = B_i = \frac{\cos\theta_w}{2\sqrt{2}}\sin\theta_c \qquad \text{for } \bar{u}s \text{ and } \bar{c}d \text{ coupling} \qquad (3)$$

where θ_c is the Cabibbo angle.

In Eq. (1), $K \equiv K(Q^2, y_w)$ is the effective K factor obtained through the recent complete analysis of soft and hard terms of Ref. (7). One has $K = 1 + O(\frac{\alpha_s(Q^2)}{2\pi} C_F \pi^2)$, with $C_F = \frac{4}{3}$ and $\alpha_s(Q^2)$ is the QCD running coupling constant given by

$$\alpha_s = \frac{12\pi}{(33-2N_f)\ln(\frac{Q^2}{\Lambda^2})},$$

N_f being the number of flavours. Furthermore in Eq. (1) the parton density $q_i^{(1,2)}$ are evoluted to the scale $1/b^2$ and finally, the Sudakov quark form factor, in impact parameter space, reads,

$$S(b, q_{Tmax}) = \frac{C_F}{\pi} \int_0^{q^2_{Tmax}} \frac{dq^2_T}{q^2_T} \alpha_s (q^2_T) \left[\ln \left(\frac{Q^2}{q^2_T} \right) - \frac{3}{2} \right] (J_o(bq_T)-1) , \qquad (4)$$

where q_{Tmax} is the maximum transverse momentum allowed in the multiple gluon emission.

In Eq. (4) we have included leading (double-logarithmic) and non-leading (single logarithmic) contributions. The latter have been shown to be quite relevant in the phenomenological description of the W p_T spectrum/6/. However, in Eq. (4) we have not included terms of the order α_s^2 which are negligible for the purpose of this paper.

We would like now to make a few comments on Eq. (4) which are relevant for the numerical evaluation of the full distribution (1) as well as for the comparison with previous works on the same subject. We first observe that through a redefinition of the K factor one can take $q_{Tmax} = Q$ and then rewrite Eq. (4) for $1/b^2 \ll Q^2$, i.e. in the soft region, as/7/

$$S(b, q_{Tmax}= Q) \simeq - \frac{C_F}{\pi} \int_{C_1^2/b^2}^{Q^2} \frac{dq^2_T}{q^2_T} \alpha_s(q_T) \left[\ln\left(\frac{Q^2}{q^2_T} \right) - \frac{3}{2} \right] \qquad (5)$$

where $C_1 = 2 e^{-\gamma_E}$, with $\gamma_E = 0.5772$.

Furthermore the following equality holds for $\alpha_s(q_T)$ fixed:.

$$S(b, q_{Tmax}= Q) = S (b, q_{Tmax} = Q/e^{3/2}) \qquad (6)$$

This is due to the particular form of the integrand of Eq. (5). When $\alpha_s(q_T)$ is running, it only induced differences of order $\lesssim$ 5% in the weak boson spectrum given in Eq. (1) at the CERN $\bar{p}p$ collider energy. Of course such approximate equality does not hold when only leading (double-logarithmic) terms are considered in Eq. (4). Actually, the leading formula

$$S_{lead}(b, q_{Tmax}) = \frac{C_F}{\pi}\int_0^{q^2_{Tmax}} \frac{d\,q_T^2}{q_T^2} \ln\left(\frac{Q^2}{q_T^2}\right)\alpha_s(q_T)\,(J_o(b\,q_T) - 1), \qquad (4')$$

with the choice $q_{Tmax}=Q$, has been proved to be completely inadequate to describe the low energy Drell-Yan phenomenology/8/ as well as, the weak boson p_T spectrum/6/. On the other hand it has also been suggested/8/ that the leading formula, once kinematically improved by the use of the relation

$$q_{Tmax} = \frac{\sqrt{\langle s \rangle}}{2}\left(1 - \frac{Q^2}{\langle s \rangle}\right) \qquad (7)$$

$\langle s \rangle$ being the average of the effective parton subenergy, led to a quite good phenomenology. Now it is amusing to note that, always for large b, one can approximate

$$S\,(b, q_{Tmax} = Q/e^{3/2}) = \frac{C_F}{\pi}\,\alpha_s\left(\frac{1}{2}\ln^2 Q^2 b^2 - \frac{3}{2}\ln Q^2 b^2\right) \simeq$$

$$\simeq S_{lead}\,(b, q_{Tmax} = Q/e^{3/2}) = \frac{C_F}{\pi}\,\alpha_s\left(\frac{1}{2}\ln^2 Q^2 b^2 - 9/2\right) \qquad (8)$$

for $\ln Q^2 \langle b^2 \rangle \sim \ln \frac{Q^2}{\langle p_T^2 \rangle} \sim 3$. Indeed that was the situation has been encountered in our previous phenomenological analysis/6,8/ where we also found $q_{Tmax} \simeq \frac{Q}{4}$.

We have gone through this numerics in some detail, in orders to clarify to the reader the mutual consistency of the various analyses of Drell-Yan processes based on different treatments of subleading terms. In the present work we shall take $q_{Tmax} = Q/e^{3/2}$ in Eq. (4) which makes the numerical evaluation much easier.

So far we have not discussed the finite hard terms from the $q\bar{q}$ annihilation diagrams which have to be added to the soft contributions. However, due to their tiny relevance in the peak region of the W p_T spectrum/7/, where the decay lepton k_T distribution gets its major contribution, we will disregard those terms in our numerical analysis, the error induced being much smaller than the uncertainties coming from our poor knowledge of Λ, as discussed in Sect. 4.

After these considerations about the weak bosons p_T distribution, we now turn to the calculation of the k_T spectrum of the decay lepton.

3. - THE DECAY LEPTON SPECTRUM

The Lorentz-invariant cross section of one single lepton of four momentum k, produced in a W decay is [9]

$$E_k \frac{d\sigma}{d^3k} = \int \frac{d^3p}{E_p} \left(E_p \frac{d\sigma^W}{d^3p}\right) \frac{1}{2\pi} \delta\left(p.k - \frac{Mw^2}{2}\right) \tag{9}$$

where $E_p \frac{d\sigma^W}{d^3p}$ is the invariant differential cross section for the production of W and the δ function stands for the two-boby decay, neglecting the lepton mass. We first have to calculate $E_p \frac{d\sigma^W}{d^3p}$ which is clearly related to Eq. (1).

In Eq. (1) we have integrated over the direction of the decay lepton, but we must use here the parton differential cross section which is proportional to

$$(A_i^2 + B_i^2)(a^2 + b^2)(1+\cos^2\hat{\theta}) + 8\, A_i\, B_i\, ab \cos\hat{\theta} \tag{10}$$

where $\hat{\theta}$ specifies the lepton direction with respect to the incident $\bar{p}$ in the W rest frame. By using Eq. (3), this $\hat{\theta}$ dependence reads

$$\frac{\cos^4\theta_W}{16} (1 \pm \cos\hat{\theta})^2 \tag{11}$$

according to whether the (+2/3) quarks come from the proton (+sign) or the antiproton (-sign). We need to express Eq. (11) in terms of the angle θ and transverse k_T of the lepton in the $\bar{p}p$ c.m. system. This can be easily derived and one has

$$1 + \cos\hat{\theta} = \frac{2k_T}{x_1\sqrt{s}} \; \frac{1+\cos\theta}{\sin\theta}$$
$$1 - \cos\hat{\theta} = \frac{2k_T}{x_2\sqrt{s}} \; \frac{1-\cos\theta}{\sin\theta} \quad , \tag{12}$$

where x_1 refers to the $\bar{p}$ and x_2 to the p.

Furthermore one obtains $E_p \frac{d\sigma^W}{d^3p}$ by integration Eq. (1) over Q^2, using the narrow width approximation which gives a factor

$$\frac{\pi}{M_W \Gamma_W} = 12 \sin^2\theta_W \;\; \frac{\pi B_\ell}{\alpha M_W^2} \tag{13}$$

where B_ℓ is the leptonic branching ratio, whose expression is $B_\ell = (N_\ell + \frac{3}{2} N_f)^{-1}$ in a model with N_ℓ lepton doublets. Therefore, assuming that there are no charmed quarks inside the proton one finally gets for the integrand of Eq. (9)

$$\begin{aligned} E_p \frac{d\sigma^W}{d^3p} &= \frac{1}{\pi}\int dQ^2 \; \frac{d\sigma}{dQ^2\, dy\, dp_T^2} \\ &= \frac{\alpha\pi B_\ell K}{6M_W^2 \sin^2\theta_W}\int bdb\; J_o(bp_T)\exp\left[S(b, q_{T\max})\right] \\ &\cdot \Big\{ (u_2\, d_1 \cos^2\theta_c + u_2\, s_1 \sin^2\theta_c)\tfrac{3}{4}(1+\cos\hat{\theta})^2 \\ &+ (\bar{u}_1\, \bar{d}_2 \cos^2\theta_c + \bar{u}_1\, s_2 \sin^2\theta_c)\tfrac{3}{4}(1-\cos\hat{\theta})^2 \Big\} \; , \end{aligned} \tag{14}$$

where q_i stand for $x_i q(x_i, 1/b^2)$, (u, d) include valence and sea contributions, while $(\bar{u},\bar{d},\bar{s})$ only refer to the sea. Eq. (14) can be easily generalized to include heavier flavours.

We have now to perform in Eq. (9) the phase space integration. The δ function will allow us to reduce it to a two-dimensional integral. If $p'_{/\!/}$ and p'_T denote the components of $\vec{p}$ parallel and perpendicular to the three momentum $\vec{k}$ of the lepton, we have from the constraint of the δ function

$$p'_{/\!/} = E_p - \frac{M_w^2}{2k}$$

(15)

which gives

$$p'^2_T = \frac{M_w^2}{k} \left[E_p - \left(k + \frac{M_w^2}{4k} \right) \right]$$

where $k = |\vec{k}|$. If θ' is the angle between $\vec{p}$ and $\vec{k}$ and ϕ is the angle between the plane $(\vec{p}, \vec{k})$ and $(\vec{0}_z, \vec{k})$, where $\vec{0}_z$ is the collision axis, the integration over θ' is immediately done by the δ function and we are left with

$$E_k \frac{d\sigma}{d^3k} = \frac{1}{2\pi k} \int_{E_o}^{\frac{\sqrt{s}}{2}} dE_p \int_o^{2\pi} d\phi \left[E_p \frac{d\sigma^w}{d^3p} (y_w, p_T^2) \right] \tag{16}$$

where $E_o = k + \frac{M_w^2}{4k}$. In $E_p \frac{d\sigma^w}{d^3p}(y_w, p_T^2)$, given by Eq. (14), we must take

$$p_T^2 = (p'_T \cos\theta \cos\phi + p'_{/\!/} \sin\theta)^2 + (p'_T \sin\phi)^2 \tag{17}$$

and make use of Eq. (15).

This completes the description of the evaluation of the final lepton cross section. We will present the numerical results in the next section.

4. - NUMERICAL RESULTS AND DISCUSSION

In the previous sections we have derived all the formulae for the calculation of the lepton spectrum. Before going on to the discussion of the results, we have to specify which of the parton densities we have used. This choice has important consequences for the theoretical uncertainties associated to the final results, particularly for the contribution of the sea at very high energies. We have done the numerical evaluation with two different non-scaling parametrizations of the structure function. The first is due to Glück-Hoffmann-Reya (GHR)/10/ and the other is due to Baier-Engels-Petersson (BEP)/11/ and we shall compare the two corresponding sets of predictions.

In Fig. 1 we show the lepton invariant cross section versus $k_T = k \sin\theta$ for $\theta = 90°$ (y=0) at $\bar{p}p$ collider energy $\sqrt{s} = 540$ GeV, using the GHR parametrization. The leading approximation (solid curve) corresponding to Eq. (4') gives rise to a much broader k_T distribution than the one resulting from the inclusion of subleading effects (dashed curve) corresponding to Eq. (14), which is reminescent of what we observed/6/ for the p_T distribution of the W. The Jacobian peak obtained from the naive Drell-Yan is partially restored. We have used $\Lambda = 0.20$ GeV. A variation of Λ from 0.1 to 0.3 GeV would induce a change in the theoretical predictions of order of 10%. The addition of the charm component in the sea in Eq. (14), with c=s, does not change appreciably our results. In Fig. 2 by using the BEP parametrization we find similar results, although the cross section is slightly lower than in the previous case. The comparison between Figs. 1-2

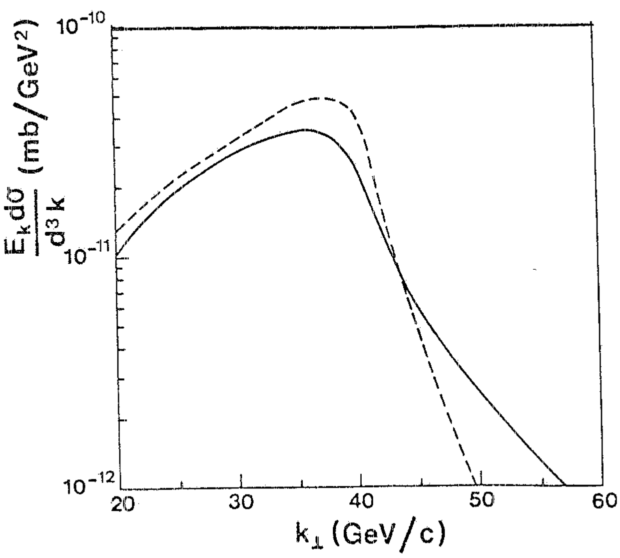

FIG. 1 - The k_T spectrum of the charged lepton from $W^+ \to \ell^+ \nu_\ell$ at $\sqrt{s} = 540$ GeV and $\theta = 90°$. The solid curve corresponds to the leading approximation, whereas the dashed curve corresponds to the subleading contributions. We use Ref. /10/ for the parton densities.

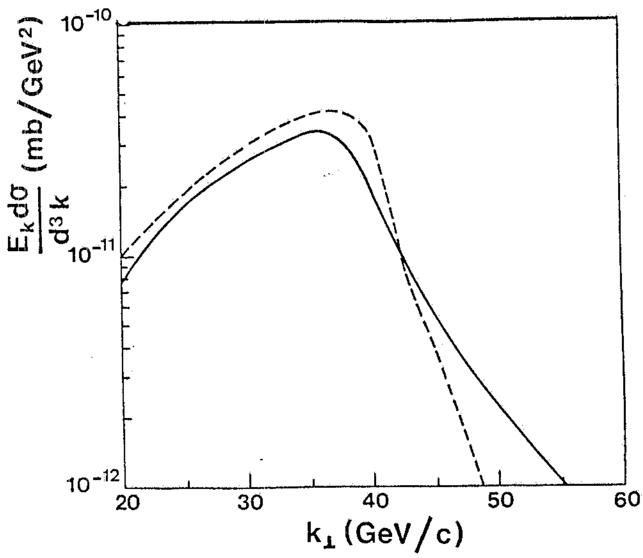

FIG. 2 - The same as in Fig. 1 using Ref. /11/ for the parton densities.

shows that our predictions are stable at the level of $\lesssim$ 15%, and for the remaining calculations we shall use the GHR set only. If we compare our results with those of Ref. (9) where both the leading approximation and the exponentiation of the full $O(\alpha_s)$ result are considered, we find important differences. In particular our cross-section is higher by about a factor three. In addition to the K factor which gives a correction of about 35%, this is due to the choice of the old parametrization of the structure function/12/, a large value Λ=0.5 GeV used and the evolution of the structure functions at the scale Q^2 instead of $1/b^2$. Indeed we have checked that using Ref. /12/ one obtains, for Λ=0.5 GeV and M_W^2 as evolution scale, a result smaller of a factor two than for Λ=0.2 GeV and $1/b^2$ as evolution scale. On the other hand our results are more in agreement, as far as the absolute magnitude of the cross section is concerned, with those of Ref. /13/, where a complete $O(\alpha_s)$ analysis has been performed. In Figs. 3 and 4 we give the distributions for θ=50° and θ=130° and we remark that the peak is very pronunced in this latter case. Our predictions seems to be consistent with the UA1/14/ data which show no event for k_T above 50 GeV/c.

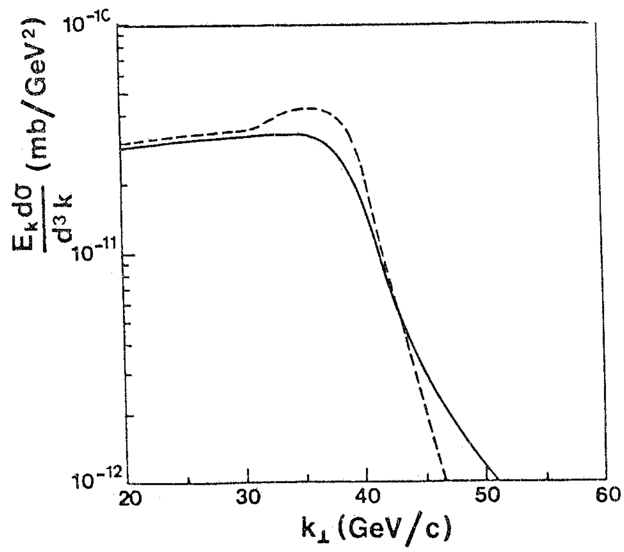

FIG. 3 - The same as in Fig. 1 at θ = 50°

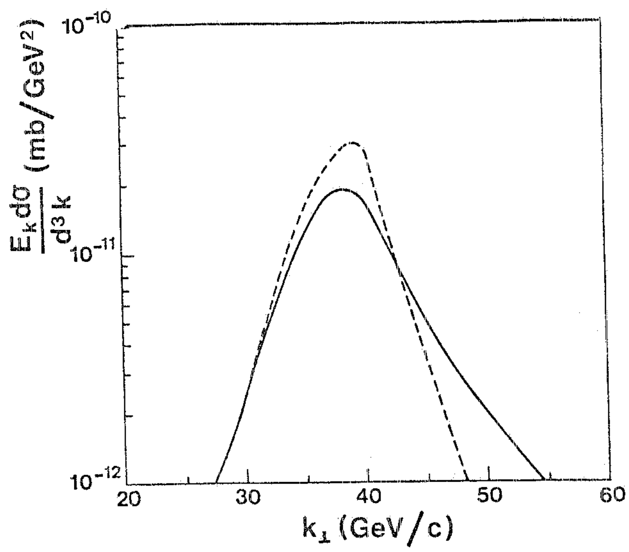

FIG. 4 - The same as in Fig. 1 at θ = 130°.

Let us now consider our predictions at $\sqrt{s}$=2000 GeV. We first show in Fig. 5 the distribution at 90° including only u, d and s quarks (see Eq. (14)). One observes an excess of events for small k_T compared to $\sqrt{s}$=540 GeV, due to the fact that the effect of the sea become much more relevant when the energy increases. There is no longer a peak and the cross section is about three times higher than at $\sqrt{s}$=540 GeV. This is again a major difference with the results of Ref. /9/ where the cross section increases by about one order of magnitude from $\sqrt{s}$=540 to $\sqrt{s}$=2000 GeV and keeps the same shape.

Finally in Fig. 6 we have used N_f=6, with all heavy flavours taken identical to s in the GHR parametrization, but the inclusion of the new flavour, increases the cross section by only 20%. This result also differs from Ref. /9/ where an increase of about 100% is expected going from N_f=2 to N_f=6. It is mainly due, again to the different choice of the evolution scale of the structure functions. The inclusion of the Compton contribution in the way discussed in Ref. /7/ does not change appreciably the result.

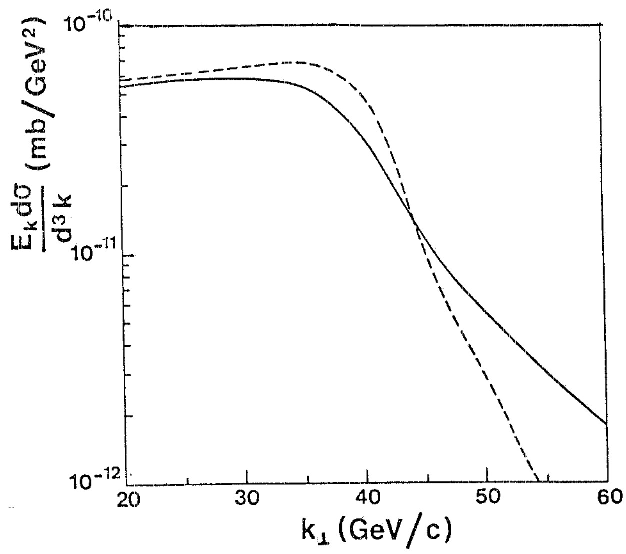

FIG. 5 - The same as in Fig. 1 at $\sqrt{s}$ = 2000 GeV and θ = 90° with N_f = 3.

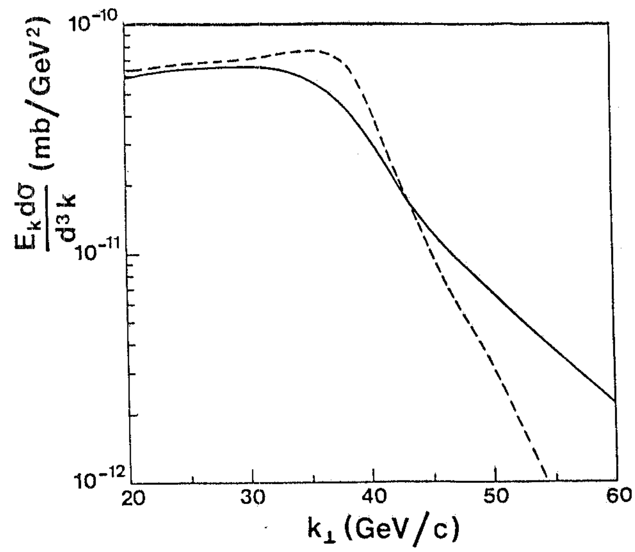

FIG. 6 - The same as in Fig. 5 with N_f = 6.

5. - CONCLUSIONS

In this paper we have performed an update analysis of the lepton k_T spectrum resulting from W production at collider energies. Recent results, concerning the p_T spectrum of the weak boson, have been used as input. They show the relevance of the subleading corrections for an accurate determination of the lepton spectrum. In particular the k_T spectrum decreases more rapidly for $k_T \gtrsim M_W/2$. Our results are affected by the theoretical uncertainties associated to the weak boson p_T distribution/7/, mainly coming from the value of Λ and the choice of the parton distributions. However they only induce a possible variation of the absolute cross section of order 20% at most, but do not change the shape of the lepton spectrum. We have also made a comparison with previous computations of the same quantity and given predictions at $\sqrt{s}$=2000 GeV.

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