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ONE-TIME AND TWO-SPACE DIMENSIONS
QUANTUM ELECTRODYNAMIC HALL EFFECT IN ONE-TIME AND TWO-SPACE DIMENSIONS

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ABSTRACT

The topological features of quantum electrodynamics in two-space and one-time dimensions are shown to lead to the quantization of the Hall effect conductance.

There has been considerable recent interest in the precision determination of the quantum electrodynamic coupling strength

$$\alpha = (e^2/\hbar c) \tag{1}$$

using the Hall conductance of the gate region of a field effect transistor(1). Our purpose is to discuss the theory of this measurement using recent progress made on the topological features of quantum gauge field theories in two space plus one time dimensions(2).

Recall that the Hall conductance g, in two-spatial dimensions, is defined by the axial vector part of the current induced by small electric fields[3]

$$\delta J_x = g \ \delta E_y , \quad \delta J_y = - g \ \delta E_x , \tag{2a}$$

and that this equation must be supplemented by a thermodynamic law relating the charge density to magnetic field changes[4]

$$c \, \delta \varrho = g \delta \ B. \tag{2b}$$

Although the original derivation of Eq. (2b) is not overly long[5], the utility of using quantum electrodynamics in two plus one dimensions becomes evident when it is realized that the three vector potential a^{μ} yields a description of the electromagnetic field[2] as the axial vector

$$f_{\mu} = \varepsilon_{\mu\nu\lambda} \partial^{\nu} a^{\lambda}. \qquad (\mu, \nu, \lambda = 0, 1, 2) \tag{3}$$

Eqs. (2) then yield a (Lorentz) relativistic Hall effect equation for the linear response current

$$\delta J^{\mu} = - g \ \delta f^{\mu} , \tag{4}$$

wherein both electric charge conservation and "magnetic flux conservation" become clear,

$$\partial_{\mu} J^{\mu} = 0 , \tag{5a}$$

$$\partial_{\mu} f^{\mu} = 0. \tag{5b}$$

Eq. (5b) follows from Eq. (3) and represents Faraday's voltage law in thinly disguised form.

The nature of the "Hall step" is simply that there is a region wherein g is truely constant. For a constant g, Eq. (4) follows from the action

$$\Delta S = - (g/2c^2) \int (d^3x) \, f_{\mu} \, a^{\mu} . \tag{6}$$

For example, with a uniform f_{μ} and a two component Dirac spinor in two plus one dimensional quantum electrodynamics, the Landau levels[6] yield the expected result for a quantum Hall effect[1]; the exact answer is

$$g\ (\text{Dirac Vacuum}) = (e^2/\ 2\pi\hbar), \tag{7}$$

and Eq. (7) follows from the results thus far proved. (A longer route for computing the one electron loop action ΔS uses the Schwinger proper time method[7]).

The important topological feature of the Hall effect action ΔS, in Eq. (6), is that for gauge transformations of the form ($U^{*}=U^{-1}$),

$$a_\mu \longrightarrow a_\mu - i\left(\frac{\hbar c}{e}\right) U^{-1}\, \partial_\mu\, U\,, \tag{8}$$

the integral is proportional to the Prontryagin index (in a pure gauge configuration) describing the winding numbers of the vortex "flux quanta". This is the case whereby the existence of such quanta prevent a smooth deformation of U to unity[8].

The essential magnetic flux quantum unit for a winding number of unity is given by

$$\varphi_o = (2\pi\hbar c/e)\,. \tag{9}$$

For a single vortex whose charge is integral, n φ_o, the pure gauge action due to the Hall conductance is given by

$$S_n = (g\ \varphi_o^2\ /\ c^2)\ n^2. \tag{10}$$

Suppose that some vortices have winding number n and some vortices have winding number n+1. The condition that the two types of vortices are in chemical potential equilibrium (at zero temperature) is that the action difference be an integer multiple of planck's constant[9], i.e.

$$(S_{n+1} - S_n) = 2\pi\ N\hbar\,. \tag{11}$$

With

$$R = (4\pi / c) \tag{12}$$

as the vacuum radiation impedance, Eqs. (1), and (9) – (11) yield the rational quantization condition

$$\frac{1}{2}(g R) = \left(\frac{N}{2n+1}\right)\cdot\alpha , \tag{13}$$

with odd integer denominators. Whether or not particular rational quantization mechanisms are present, depends on the total quantum electrodynamic field action for the experimental problem at hand, i.e. on the minimum total action configurations.

A previously studied quantum electrodynamic model appears to be fairly realistic for a Hall surface whose physical thickness is determined by a length scale ℓ. The model[2] is given by the action in (2+1) dimensions

$$S = (\ell / 2 R c^2)\int d^3x \ \ (f_\mu f^\mu) - (g / 2c^2)\int d^3x \ \ f_\mu a^\mu . \tag{14}$$

The rational quantizations of the Hall conductance g (for the model) in principle follow from (i) the minimum action configurations of S, and (ii) the Pontryagin indices of the vortex flux quanta (instantons) present in the stable configurations.

From Eq. (14) one can deduce the following: (i) An external charge of integral strength n_1 e induces a magnetic flux of integral strength $(c/g)n_1$ e. This strongly suggests Dyon solutions " with $n_2\,\varphi_o$" magnetic charge and n_1 e electric charge[10],

$$|\mathrm{Dyon}\rangle = |n_1 e, \ n_2\,\varphi_o\rangle , \tag{15}$$

provided g is rationally quantized to $\frac{1}{2}(gR)=\alpha\,(n_1/n_2)$. (ii) Since the photon has a mass, the interaction between Dyons is screened. The inverse screening length k is related to the photon mass m by

$$k = (mc/\hbar) = \left| \ R\,g/\ell \ \right| . \tag{16}$$

(iii) In the analogous superconducting case, such screening in two spatial dimensions yields a triangular flux lattice[11]. Thus a periodic array of Dyons would appear reasonable here as well.

In conclusion, it appears that in one[12] and three spatial dimensions[13], respectively, the "γ_5-anomaly" is merely Ohms law for charges and chirally charged particles. In two spatial dimensions the "γ_5-anomaly" is the Hall effect.

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