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SOME NEGATIVE RESULTS FOR AVOIDING LATTICE FERMIONIC DOUBLING

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We show that it is impossible to solve the lattice fermion doubling problem via random fields in the perturbative framework.

The lattice approach to field theories is an invaluable tool in understanding them beyond perturbative regions. However how to put the fermions on a lattice has long been a headache to physists. The naive lattice free fermionic theory does not reproduce the correct continuum limit; the notorious species doubling problem. Various alternative proposals are made [1–3] and they are not completely satisfactory in the sense that a continuous chiral symmetry cannot be retained on a lattice. It is generally advised not to go to try some better schemes by the so called N–N no-go theorem [4]. However it may have seemed that some dynamical way could bypass the difficulty (which violates some presumption of the N–N theorem). Indeed an idea involving randomness has been proposed in ref. [5].

In their approach the lattice itself is random. However in this interesting case analytic computations are very difficult, if not impossible [6,7]. To bypass this difficulty and to shed some light on the problem, we propose to investigate the subject by using a regular lattice with slightly random coupling and to construct a perturbative expansion with randomness. In this way analytic results may be easily obtained. In this note we apply this procedure to a particular case (see also ref. [8]).

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It is known that on a lattice many actions having the same continuum limit can be different up to irrevant operators, moreover we may abandon the free field concept on the lattice by introducing "irrelevant" interactions! Consider for example the Wilson scheme for a free fermion on a lattice [1]

$$S = -\frac{a^3}{2} \sum_{n,\mu} \bar{\Psi}_n \gamma_\mu (\Psi_{n-\mu} - \Psi_{n+\mu}) + r\frac{a^3}{2} \sum_{n,\mu} \bar{\Psi}_n (2\Psi_n - \Psi_{n-\mu} - \Psi_{n+\mu}) , \qquad (1)$$

it reproduces the correct continuum limit with the price of chiral symmetry on the lattice. From lattice perturbative calculations we learn that there is always a perturbatively generated mass term proportional to r/a. The following reasoning may be plausible; if we do not prefer a particular choice of r, we may say it is better to have r randomly distributed around the origin. In this way for the effective theory the mass term might be cancelled. The annealed version looks like (using a gaussian random distribution)

$$S = -\frac{a^3}{2} \sum_{n,\mu} \bar{\psi}\gamma_\mu \nabla_\mu \psi + \lambda a^6 \sum_{n,\mu} (\bar{\psi}\Delta\psi)^2 . \qquad (2)$$

This action has a discrete chiral symmetry. The spectrum cannot be obtained in a straightforward way but

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we may hope (and we shall be deluded at the end) it reproduces the correct continuum limit. We emphasize that for a free theory our conclusion will be essentially the same both for annealed and quenched versions.

We propose to consider the following action on a lattice to describe the free fermion of the continuum

$$S = -\frac{a^3}{2} \sum_{n,\mu} \bar{\psi}\gamma_\mu \Delta_\mu \psi$$

$$+ a^6 \frac{\lambda}{4} \sum_n [(\bar{\psi}\Delta\psi)^2 - (\bar{\psi}\gamma_5\Delta\psi)^2] , \qquad (3)$$

where ∇_μ and Δ are the usual dimensionless lattice derivative and laplacian. The interacting part is Fierz equivalent to

$$a^6 \frac{\lambda}{8} \sum_{n,\mu} [(\psi\gamma_\mu\Delta\psi)^2 - (\psi\gamma_5\gamma_\mu\Delta\psi)^2] , \qquad (4)$$

it is easily seen that it is invariant under the full global chiral transformation

$$\psi \to \exp(\mathrm{i}\alpha\gamma_5)\psi, \quad \bar{\psi} \to \bar{\psi}\exp(\mathrm{i}\alpha\gamma_5) . \qquad (5)$$

The choice of the interaction is by no means unique. It is true that we cannot solve the spectrum of eq. (3) exactly (with the possible exception in two dimensions, it is very interesting to get the spectrum exactly there). We want to proceed perturbatively in the randomness λ.

The interaction vertex in momentum space is (see fig. 1)

$$a^2 \tfrac{1}{2}\lambda\Delta(k_2)\Delta(k_4)[\delta_{\alpha\beta}\delta_{\sigma\rho} - \delta_{\alpha\rho}\delta_{\sigma\beta}$$

$$+ (\gamma_5)_{\alpha\rho}(\gamma_5)_{\sigma\beta} - (\gamma_5)_{\alpha\beta}(\gamma_5)_{\sigma\rho}] \qquad (6)$$

where $\Delta(k) = 4\Sigma_\mu \sin^2 \tfrac{1}{2}ak_\mu$.

The full propagator is obtained by summing all the two-point diagrams. Here we make the approximation

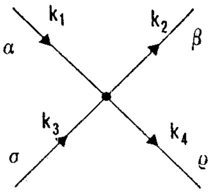


Fig. 1

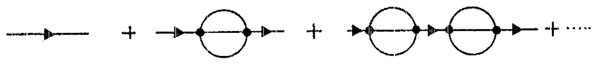


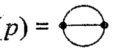
Fig. 2

by summing the chains of the lowest order diagram, that is:

Full propagator $\simeq$

$$\text{(fig. 2)} = \left(\sum_\mu \mathrm{i}\gamma_\mu \sin ap_\mu - \Sigma(p)\right)^{-1} , \qquad (7)$$

where

$\Sigma(p) =$  .

After some algebra we arrive we arrive at

$$\Sigma(p) = -\tfrac{1}{2}\lambda^2 a^7$$

$$\times \int_{-\pi/a}^{\pi/a} \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{\mathrm{d}^4l}{(2\pi)^4} \Delta(p)\Delta(l)\Delta(p-k)\Delta(k+l)$$

$$\times [S(p-k) \,\mathrm{tr}\, S(k+l)\, S(l)$$

$$+ S(k+l) \,\mathrm{tr}\, S(p-k)\, S(l)] , \qquad (8)$$

where

$$S(k) = \left(\sum_\mu \mathrm{i}\gamma_\mu \sin ap_\mu\right)^{-1} ,$$

However, as can be readily checked $\Sigma(p)$ is zero at the boundaries of the Brillouin zones, we still have the unwanted species degeneracy ‡1. What is modified is the spectrum in between of the two poles of the effective propagator.

We come to the conclusion that the dynamical approaches like the one we have presented here would not solve the doubling problem (it is also the case for the quenched version since when we calculate the effective propagator, we have the same type of effective interactions). Although we show this only in the perturbative region, we believe that this procedure is likely not to work any coupling.

‡1 It is not clear to us why the numerical simulations of ref. [8] seem to suggest an opposite conclusion.

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