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SOME ALTERNATIVES FOR THE LATTICE FERMIONS

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Some alternative schemes for the lattice fermion formulation are discussed, their pecularities and possible usefulness may serve the practical calculations better.

1. Lattice regularization for field theories is an invaluable tool in understanding field theories beyond perturbative regions. However how to put the fermionic fields on a lattice in a satisfactory way has long been recognized as an impasse [1]. The naive lattice free fermionic theory does not reproduce the correct continuum limit, the notorous species doubling problem. Various proposals were made to avoid this doubling problem [2–4]. Although they are not completely satisfactory, they are largely employed in practical calculations.

In this letter we would like to propose (or extend) still some alternatives to this problem. None of them will be the complete solution but they may be advantageous in certain circumstances as we shall see in what follows.

2. To start we want to reanalyze the second order formulation for the lattice fermions previously proposed by Banks and Casher [5]. They noted that the species doubling can be avoided by transforming the first order derivative prescription to a second order one. This is done by carrying out the bilinear fermionic integration and one obtains a determinant

$$\det(\mathrm{i}\not{D} + m)\,, \tag{1}$$

which can be formerly rewritten

$$[\det(-\not{D}^2 + m^2)]^2\,, \tag{2}$$

then the resulting theory can be effectively transcripted as a pseudofermion theory which is employed in many Monte Carlo simulations

$$S = S_0(A) - N_f\bar{\phi}(-\not{D}^2 + m^2)\phi\,, \tag{3}$$

where the global chiral symmetry (γ_5-symmetry) is not apparent and the theory will develop a mass term even we put the bare mass zero.

We notice that

$$-\not{D}^2 + m^2 = -\mathrm{D}^2 + \tfrac{1}{2}\sigma_{\mu\nu}F_{\mu\nu} + m^2\,. \tag{4}$$

This operator can be written using the standard procedure for D^2 on a lattice {i.e. $\bar{\phi}\mathrm{D}^2\phi = \Sigma_\mu\,[\bar{\phi}(i) - \bar{\phi}(i+\mu)]\,[\phi(i) - \phi(i+\mu)]$ for the free case} without doubling. The effective action for the gauge fields will be

$$S_{\mathrm{eff}}(A) = S_0(A) - \tfrac{1}{2}\,\mathrm{tr}\,\ln(-\mathrm{D}^2 + \tfrac{1}{2}\sigma_{\mu\nu}F_{\mu\nu} + m^2)\,, \tag{5}$$

with the lattice D^2.

The fermion propagator in the background gauge fields will be

$$G(\ell, 0|A) = \langle \ell|(\mathrm{i}\not{D} + m)(-\mathrm{D}^2 + \tfrac{1}{2}\sigma_{\mu\nu}F_{\mu\nu} + m^2)^{-1}|0\rangle\,, \tag{6}$$

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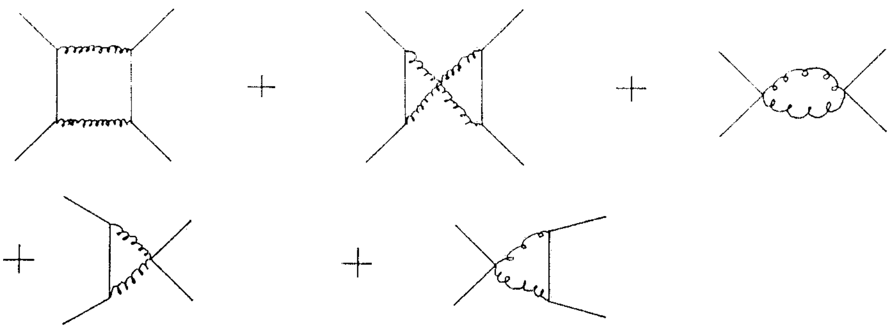

Fig. 1.

where $\not{D}$ on the lattice is the naive one: no doubling of fermions is present because it has no poles at the corners of the Brillouin zones.

To be precise we can write the physical quantities of interest, e.g.

$$\langle\bar{\psi}\gamma_5\psi(\ell)\bar{\psi}\gamma_5\psi(0)\rangle = \int \mathcal{D}A \exp[-S_{\text{eff}}(A)]|G(\ell,0|A)|^2 , \quad (7)$$

where $G(\ell, 0/A)$ is defined in eq. (6). A similar expression can be obtained for different operators. Formally it corresponds to substituting in the continuum

$$Z_{\text{cont}}(J,\bar{J}) = \int \mathcal{D}A\, \mathcal{D}\bar{\psi}\, \mathcal{D}\psi \exp[-S_0(A) + \bar{\psi}(\mathrm{i}\not{D}+m)\psi + \bar{J}\psi + \bar{\psi}J] , \quad (8)$$

with the following expression on the lattice

$$Z_{\text{latt}}(J,\bar{J}) = \int \mathcal{D}A\, \mathcal{D}\bar{\phi}\, \mathcal{D}\phi \exp[-S_0(A) - \bar{\phi}(-\not{D}^2+m^2)\phi + \bar{J}(\mathrm{i}\not{D}+m)\phi + \bar{\phi}(\mathrm{i}\not{D}+m)J] , \quad (9)$$

where ϕ is a "pseudofermion" of multiplicity 1/2 in the sense that

$$\int \mathcal{D}\bar{\phi}\, \mathcal{D}\phi \exp(-\bar{\phi}M\phi) = (\det M)^{1/2} . \quad (10)$$

Some care must be given here if we want to treat the pseudofermions seriously like a field theory. Naive dimensionality counting shows the quartic interactions are allowed

$$(\bar{\phi}\Gamma\phi)^2 , \quad (11)$$

where Γ is some spinor matrix. Even if we do not put such an interaction term in the lagrangian, a finite contribution proportional to g^4 will be generated by the one-loop calculation. In fig. 1 one can readily check that the individual diagram is logarithmic divergent when the lattice spacing goes to zero, but the sum is finite. It remains to show such an induced interaction does not spoil the correct continuum limit. The effective interaction is in the form

$$g^4(\bar{\phi}\Gamma\phi)^2(\log a)^{\gamma/b_0} \sim (\log a)^{-2+\gamma/b_0}(\bar{\phi}\Gamma\phi)^2 , \quad (12)$$

where γ is the anomalous dimension of the composite operator $(\phi\Gamma\phi)^2$ and can be calculated using the diagrams shown in fig. 2. It turns out that

$$\gamma/b_0 = 2/(33-2f) \ll 2 , \quad (13)$$

which implies that such induced interactions disappear fast enough when lattice spacing goes to zero [‡1].

One disadvantage of this formulation is that also for dimensionless quantities the corrections to the continuum results are proportional to ln a as a goes to zero. While in the conventional approach the corrections are proportional to a or a^2.

[‡1] We note that a similar analysis to that for $(\bar{\phi}\Gamma\phi)^2$ should be done also for $\bar{\phi}\sigma_{\mu\nu}F_{\mu\nu}\phi$.

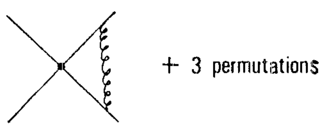

Fig. 2.

Now we want to point out a possibility of studying a pure "left-handed" theory. In the Dirac theory

$$S_\psi = \bar{\psi} i \not{D} \psi\,, \tag{14}$$

if we use the so-called chiral representation for the γ-matrices

$$\gamma_0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \boldsymbol{\gamma} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ \boldsymbol{\sigma} & 0 \end{pmatrix},$$

$$\gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{15}$$

and $\psi_L = \frac{1}{2}(1-\gamma_5)\psi$, $\psi_R = \frac{1}{2}(1+\gamma_5)\psi$ are two-component spinors, eq. (14) now becomes

$$S_\psi = \bar{\psi} \begin{pmatrix} 0 & D_0 + i\boldsymbol{\sigma}\cdot\mathbf{D} \\ -D_0 + i\boldsymbol{\sigma}\cdot\mathbf{D} & 0 \end{pmatrix} \psi$$

$$= \bar{\psi} \begin{pmatrix} 0 & -i\not{D}_R \\ i\not{D}_L & 0 \end{pmatrix} \psi\,, \tag{16}$$

where L and R denote the left-handed and right-handed, respectively. After integrating the fermionic variables we get

$$\det i\not{D} = \det \not{D}_L \det \not{D}_R$$

$$= |\det \not{D}_L|^2 = |\det \not{D}_R|^2\,, \tag{17}$$

the "left-handed" part and the "right-handed" part decouple and they are complex conjugates of each other. They may not be real themselves. Such a representation is known to be non-parity-conserving, which just fits our aim if we want to put neutrinolike particles on a lattice.

Now we want to concentrate on the left-handed part of the determinant.

$$\det i\not{D}_L = [\det(-\not{D}_L^2 + \delta m^2)]^{1/2}\,. \tag{18}$$

The effective pseudofermionic theory can be thought of describing only a "left-handed particle". Although the chirality is rather obscure since perturbatively the theory will develop a mass term on a lattice [5].

$-\not{D}_L^2$ is not a hermitian operator in the euclidean space,

$$-\not{D}_L^2 = -D_L^2 + \tfrac{1}{2} g \sigma_{\mu\nu} F_{\mu\nu}\,,$$

$$\sigma_{\mu\nu} = (1/2i)[\sigma_\mu, \sigma_\nu]\,, \quad \sigma_\mu = \{i, \boldsymbol{\tau}\}\,, \tag{19}$$

$$D_L^2 = \mathbf{D}^2 + i D_0 \boldsymbol{\tau}\cdot\mathbf{D}\,, \tag{20}$$

or, we can express it as

$$-\not{D}^2 = -\mathbf{D}^2 - \boldsymbol{\tau}\cdot(iD_0\mathbf{D} - \tfrac{1}{2} g \mathbf{B})\,, \tag{21}$$

where $B_k = \epsilon_{ijk} F_{ij}$ is the magnetic field. With the above definition we can see that still no pole for the doubling is obtained, e.g. for the free case,

$$\frac{1}{-\not{D}_L^2} \propto \frac{\Sigma_i \sin\frac{1}{2}p_i(\sin\frac{1}{2}p_i - \sin\frac{1}{2}p_0\tau_i)}{\Sigma_i \sin^2\frac{1}{2}p_i(\Sigma_\mu \sin^2\frac{1}{2}p_\mu)} \tag{22}$$

The last term can be regarded as Lorentz-invariance violating, therefore one should be careful in treating this term. Hopefully its effects vanish fast enough in the continuum limit.

As another possibility, we may think the chiral symmetry being broken spontaneously on the lattice. It was also discussed independently by Kawamoto and Shigemoto [6]. A simple example of which can be the following

$$S = \bar{\psi} i \not{\partial} \psi + a^2 g \bar{\psi}(\sigma + i\gamma_5\pi)\Delta\psi + (\partial\sigma)^2 + (\partial\pi)^2 + \lambda(\sigma^2 + \pi^2 - \mu^2)^2\,, \tag{23}$$

where σ and π are two scalar fields and Δ is the usual lattice Laplacian a is lattice spacing. This theory formally enjoys a U(1) symmetry and it is spontaneously broken since the potential is chosen such that the minimum occurs at, say

$$\langle\sigma\rangle = \sigma_0 = \mu\,. \tag{24}$$

Performing perturbation around this broken-symmetry vacuum we will have a mass term proportional to g^2 for the doubling fermions as well as the fermions with small momenta. This scheme is different from Wilson's one in that now we have a mass term for the doubling fermions proportional to g (tree level) while a mass term for the fermions with small momenta proportional to g^2 (one-loop level). If we assume g to be small, then we have a controllable way to distinguish the different masses.

This theory on a lattice will display in the continuum also a term proportional to $\bar{\psi}(\sigma + i\pi\gamma_5)\psi$ and we must add in a counterterm, which is the equivalent to the Wilson scheme of adjusting K so as to send the mass of the physical quark to the given value. The advantage of this choice is that the formal chiral symmetry is exact and no difficulty would be present to gauge it. This second proposal might be the best suited for writing the Weinberg–Salam model on a lattice.

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