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GLUEBALL MASS IN MIXED ACTION SU(2) LATTICE GAUGE THEORY AND UNIVERSALITY

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ABSTRACT

For the purpose of testing the universality directly, we compute the glueball mass in the fundamental-adjoint SU(2) lattice gauge theory by Monte Carlo simulations. We find a region where the ratio of two physical quantities is constant within error bars. This suggests that universality holds.

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Though one can define an infinite number of lattice actions which correspond to the same classical continuum field theory, it is expected that all these lattice theories will give the same results for physical quantities in the regime where the correlation length is large compared to the lattice spacing(1). Creutz has shown(2) that for the pure Wilson action there exists a region in β (or in the coupling constant), where one can observe scaling even at moderatly large values of the correlation length.

Bhanot and Dashen(3) recently measured the string tension in the fundamental-adjoint SU(2) lattice gauge theory(4), whose action is given by(9)

$$S = \beta_F \sum_{plaq.} \left[1 - \frac{1}{2} Tr_F(U_p)\right] + \beta_A \sum_{plaq.} \left[1 - \frac{1}{3} Tr_A(U_p)\right]. \qquad (1)$$

In Fig. 1 we report their result by showing the lines in the β_F - β_A plane along which $\sqrt{\sigma} \cdot a$ is constant (σ is the string tension and "a" is the lattice spacing).

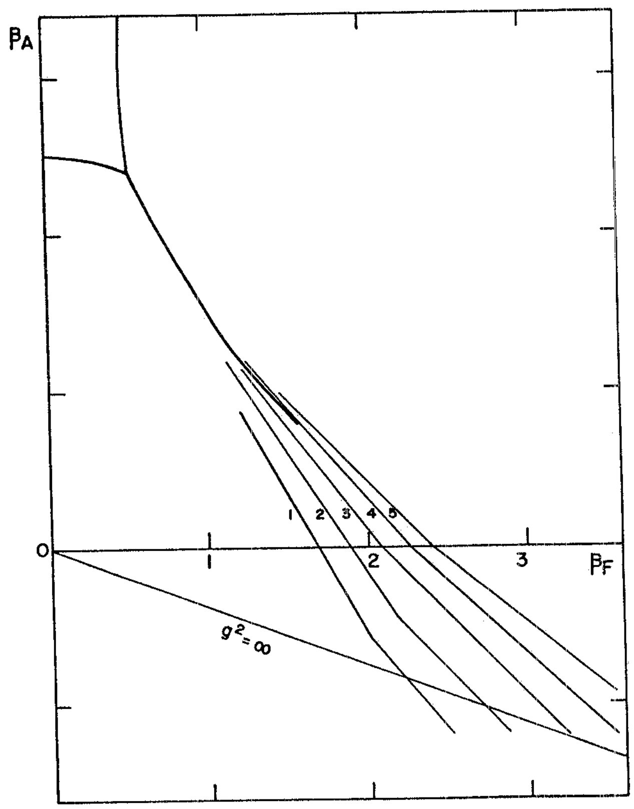


FIG. 1 - Lines of constant $\sqrt{\sigma} \cdot a$ obtained by G. Bhanot and R. Dashen in Ref. (3).

From Monte Carlo data for the string-tension, Bhanot and Dashen calculated the lattice Λ parameter and found that there is a discrepancy between the Monte Carlo results and theoretical expectations based on a weak coupling calculation.

After their observations, the universality problem in the mixed action has attracted a significant amount of attention(5) and the present situation is rather confused. Gonzalez-Arroyo et al. have shown that the string-tension data of Bhanot and Dashen can be fitted by the strong coupling expansion and they cast doubt on the applicability of the weak coupling formula to the data. Grossman and Samuel, and independently Makeenko et al. have obtained an effective formula in the large N limit. Using this formula they concluded that universality holds. On the contrary, Mütter et al. measured the mass gap at $\beta_A = 1.21$ and could not find a signal for asymptotic freedom. They declared that they close the window of scaling found by Creutz.

Here we present a direct test of universality. We measure the glueball mass $mg \cdot a$ in the mixed action (1) and investigate the behaviour of the ratio of the glueball mass over the string-tension

$$\frac{mg \cdot a}{\sqrt{\sigma} \cdot a} \quad . \tag{2}$$

If universality holds, this ratio must not depend upon β or β_A.

The glueball mass in this paper is evaluated in the following way. The time-sliced plaquette-plaquette correlation function is defined by

$$G(\tau) = N_s \left[\langle w(\tau)\, w(0) \rangle - \langle w(0) \rangle^2 \right], \tag{3}$$

where

$$w(\tau) = \frac{1}{N_s} \sum_{\substack{\text{all spacial} \\ \text{plaquettes}}} \frac{1}{2} \mathrm{Tr}_F(U_p)$$

and N_s is the number of spacial-like plaquettes. Here we have concentrated our attention only on O^+ glueballs and we will not consider the correlation between loops with more complicated topology than a simple plaquette. We define the glueball mass as

$$mg \cdot a = -\log\left[G(1)/G(0) \right]. \tag{4}$$

Real glueball mass should be calculated at large τ region. However: i) correlations at $\tau \geq 2$ are very small and suffer from severe statistical fluctuations, ii) the quantity evaluated by equation (4) is a function of the glueball mass spectrum, so in any case, it defines a physical quantity.

Before going ahead we shall point out two things. First, as a check, we compare in Fig. 2 $G(\tau = 1)$ on a 4^4 lattice with that of Ref. (6) on a $4^3 \times 16$ lattice as a function of β_F ($\beta_A = 0$). No significant difference can be seen. In the following we will analyse mainly Monte Carlo data on a 4^4 lattice and also will include complementary 8^4 data with less statistics.

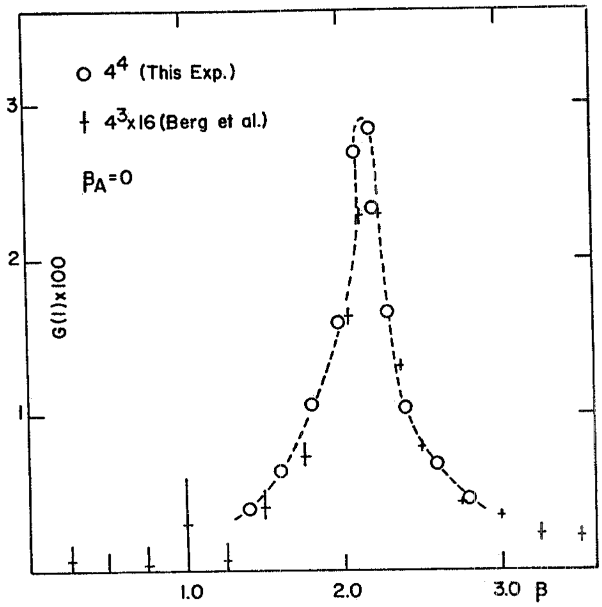


FIG. 2 - Comparison of our results for plaquette-plaquette correlation at $\tau = 1$ on a 4^4 lattice with that of Ref. (6) on a $4^3 \times 16$ lattice. The first 700 MC iterations are used for thermalization and 2700 MC iterations are performed to get results.

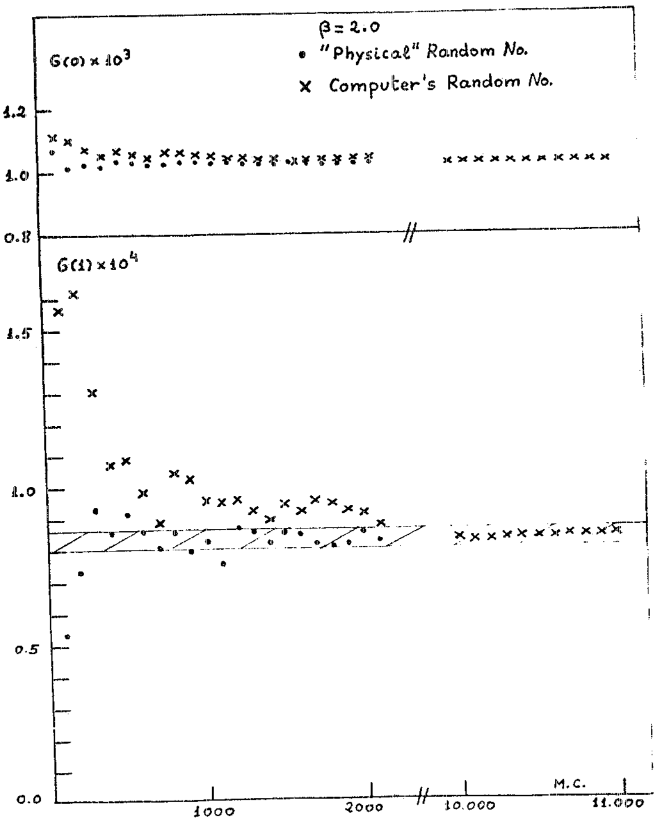


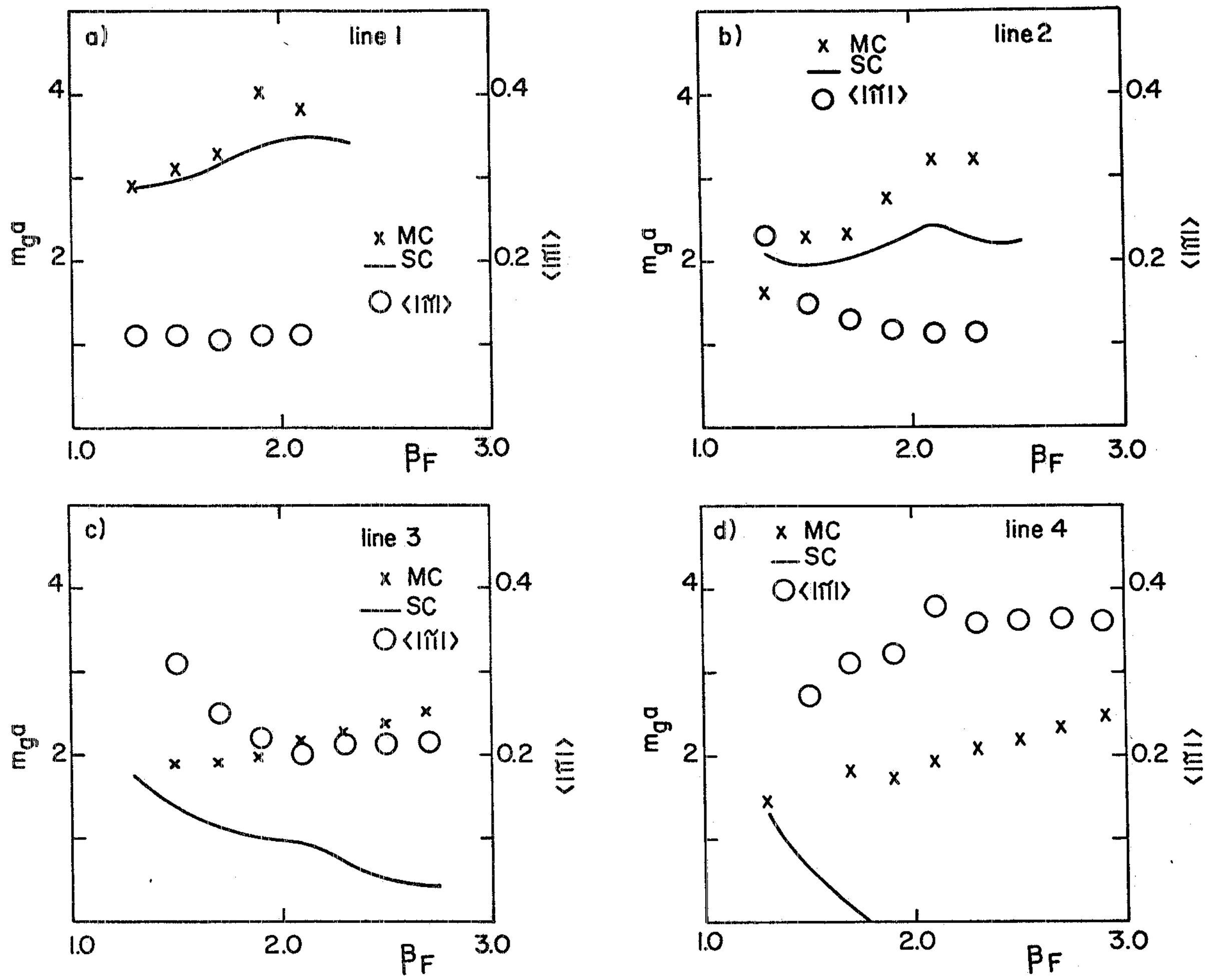
FIG. 3 - Numerical results of plaquette-plaquette correlation at $\tau = 0$ and $\tau = 1$ on a 4^4 lattice as a function of number of MC iterations. The crosses are obtained with VAX random numbers and the dotted lines are obtained with physical random number. The thermalization time is 700 MC iterations.

A random number generator is indispensable to a Monte Carlo algorithm. We study the evolution of the correlation functions as a function of the number of Monte Carlo iterations using the computer random number, a modified computer random number and a "physical" random number[10]. The hatched zone in Fig. 3 stands for the region upon which both hot-start and cold-start calculations using VAX random numbers converge after 5000 MC iterations. The physical random numbers are the first to reach equilibrium. The modified random numbers are the next. From this experiment we decide to use the physical random numbers in the following calculations[12].

We checked this point very carefully. We calculated the correlation functions using the same program that utilizes the "physical" random number, but with a different table of "Pseudo-Physical" random numbers generated by the VAX-random-number-generator. Although we have seen no significant difference between the numerical results obtained in this way, and the results of Fig. 3 for the computer random number.

A direct test of universality is to see whether the ratio of two physical quantities with the same dimensions is constant or not by varying the bare parameter. We investigate the ratio of the glueball mass defined in equation (4) over the square root of the string-tension. In Monte Carlo experiments they are measured as mg•a and $\sqrt{\sigma}$•a and depend on β_F and β_A.

In Fig. 4 we present the glueball mass defined in equation (4) and obtained in a 4^4 lattice, along the constant string-tension lines of Fig. 1.



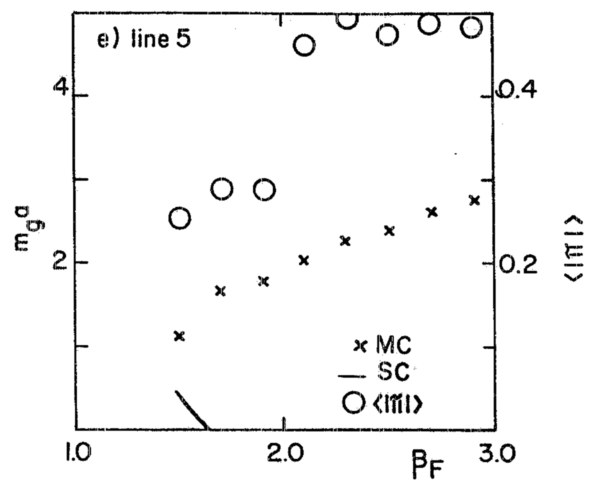


FIG. 4 - Numerical results of mg•a (x) on a 4^4 lattice along the constant string--tension lines in Fig. 1. We plot also the strong coupling expansion calculation (solid lines) and the expectation values of the absolute value of the Wilson line (Δ). The first 700 MC iterations are used for thermalization and 2700 MC iterations are performed to get results.

We plot also the strong coupling results and the expectation values of the absolute value of the Wilson line(7). The series of the strong coupling expansion for SU(2) has been completed by Münster(8) up to order g^{-16}. We can easily obtain the result for the mixed action by changing the character expansion. The Wilson line Π is defined as

$$\Pi = \frac{1}{N^3} \sum_x \mathrm{Tr}_F \prod_{i=1}^{N_T} U(\;_i, x)$$

and it is used as an order parameter of the deconfining phase. When the measured values of the Wilson line (shown in Fig. 4) becomes different from zero, we are out the deconfining point. This is the case near the line 5 in a 4^4 lattice. However, its value on a 8^4 lattice is about 0.05 at all the measured points.

In Fig. 5 we show the ratio of equation (1) along the lines of "constant physics". We also show the data obtained on a 8^4 lattice. Universality requires that all values should be the same when the lattice spacing "a" goes to zero. The ratio of equation (1) is constant within error bars in the region of β_F between 2.0 and 2.6, with the exception of line 5.

Out of this region there is a critical endpoint, at small β_F (see Fig. 1), that can produce unphysical effects. At large β_F we are near the $g^2 = \infty$ line of Fig. 1 and therefore far away from the continuum limit.

The ratio of equation (1) on the line 5 shows extraordinary behaviour both for 4^4 and 8^4 lattices. The origin of this anomalous behaviour is yet not clear to us.

In conclusion our measurement suggests that it is possible to find a window where universality holds. This does not contradict the conclusion of Mütter et al., i.e., "closing the window"; the region measured by them is really out of the region found here. We believe that Monte Carlo measurements of the ratio among physical quantities is the best way to check the universality at moderate values of β. More precise measurement of the glueball mass and string-tension will settle this problem decisively.

While we was writing this paper, we received a interesting paper(11) in which a similar conclusion about the universality problem is put forward.

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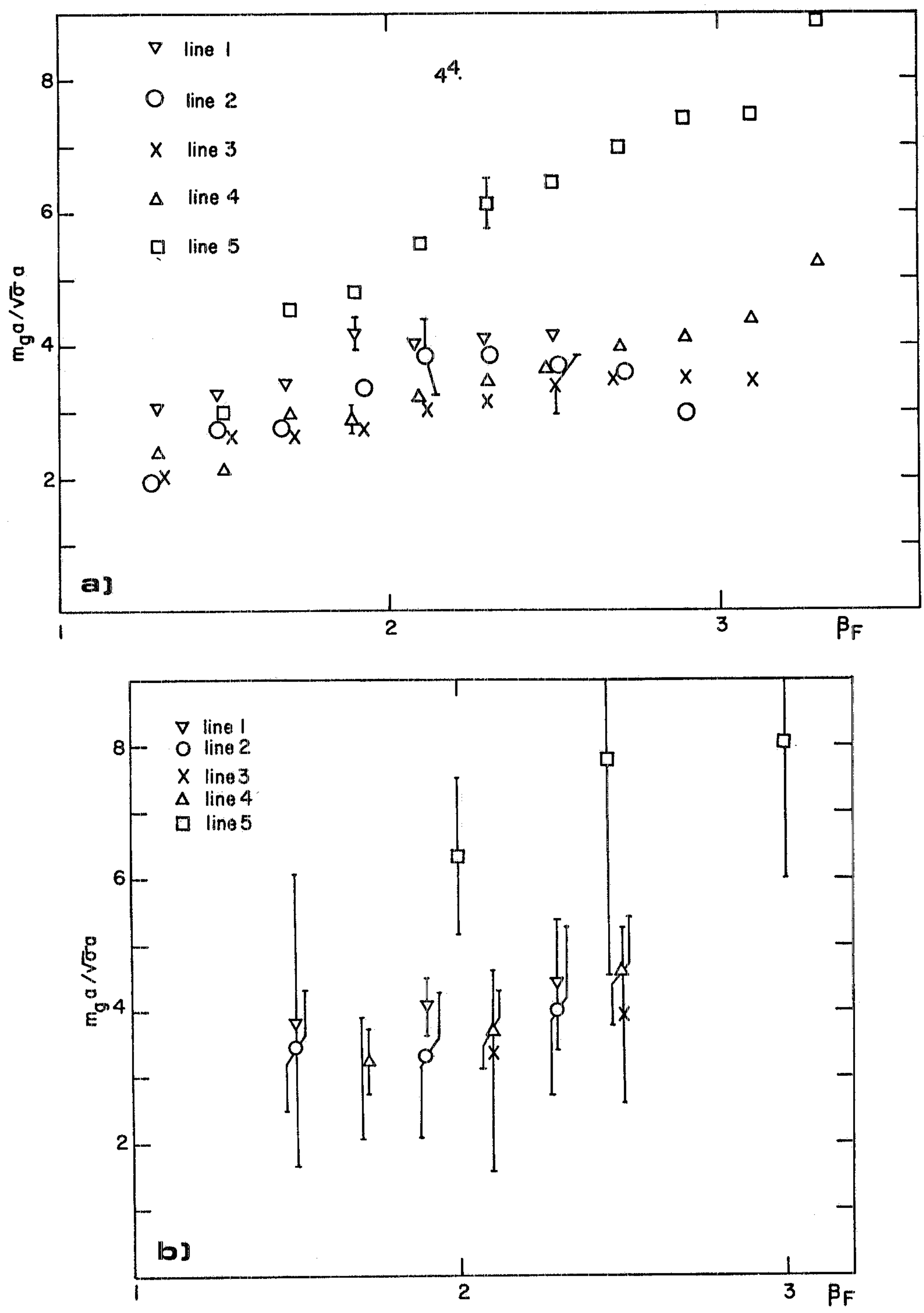

FIG. 5 - a) The ratios between glueball mass on a 4^4 lattice and square root of string-tension as a function of β_F along the constant string-tension lines in Fig. 1. The typical error bar for each line stands for the standard deviation obtained by averaging the plaquette-plaquette correlation over each 300 MC iterations. b) The same as a) but on a 8^4 lattice. The error bar is the standard deviation averaging the plaquette-plaquette correlation over each 28 MC iterations. Thermalization time is 987 MC iterations and the final results are obtained by averaging over 252 MC iterations.

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(8) - G. Münster, Nuclear Phys. B190 (FS3), 439 (1981); erratum B200 (FS), 536 (1982).

(9) - Our normalization for β_A is the same as in Ref. (4) and not Ref. (3). The bare coupling in the naive continuum limit is given by $(1/g)^2 = \beta_F/4 + 2\beta_A/3$.

(10) - a) The modified computer random numbers are generated by adding three sequential of computer random numbers (mod. 1). We thank G. Parisi for suggesting this method.

b) The "physical" random numbers, x_i, are made in the following way:

$$x_i = 0.1\left[n_i + (\text{computer random number})\right],$$

where n_i is the real physical integer random number between 0 and 9 (MIKY 1000) which was produced from the measurement of γ emission from radio--active nucleus.

(11) - E. Marinari, E. Rabinovici and P. Windey, SPHT Preprint SPHT-83-138 (1983).

(12) - We apologize to those who read the preliminary version of the paper where we showed a misleading figure.