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G. Pancheri and Y. N. Srivastava: COHERENT STATE
REGULARIZATION OF TRANSVERSE MOMENTUM
DISTRIBUTION IN QCD

Coherent State Regularization
of Transverse Momentum Distributions
in QCD

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Abstract

An expression is proposed for the transverse momentum distribution of produced virtual vector bosons, γ^* , W or Z_0 , which includes summed soft gluons, an 'intrinsic piece', asymptotes to the perturbative result for large Q_\perp and, upon integration over Q_\perp gives the Drell-Yan result for the cross-section.

1. Lepton pair production in hadronic processes (Drell-Yan mechanism) has long been recognized as providing a testing ground for QCD calculations. In particular, the transverse momentum (Q_\perp) dependence of the pair, which is a Dirac δ -function in the collinear parton model, is converted to a smooth distribution through QCD radiative corrections. In concert with increasing sophistication of experimental results, theoretical endeavours in this context have also produced major innovations^[1]. Indeed, deviations from the expected Q_\perp -spectrum of weak vector boson decays have been suggested

as possible signatures of squarks, sleptons and such^[2]. In order to be able to accomplish such a program successfully, it is important to have a theoretical formula for the Q_{\perp} -distribution which has as many ingredients from QCD and the parton model as possible - without extra parameters.

In this paper, we propose a formula which should be reliable both for small as well as large Q_{\perp} . The small Q_{\perp} behaviour is built through a summation of soft gluons. An IR singular coupling constant is responsible for the growth of an 'intrinsic' transverse momentum (how this occurs, is explained in detail in ref.(3)). For large Q_{\perp} , the proposed distribution asymptotes to the first order perturbative result, as demanded by asymptotic freedom. The distribution is integrable, so that one recovers the differential cross-section (in the left over variables) upon integration over Q_{\perp} . Also, the mean value $\langle Q_{\perp}^2 \rangle$ as given by the formula we propose agrees with the first order result augmented by the dynamically generated intrinsic piece. Since we do not need any intrinsic piece for the regularization, the only parameter is now Λ_{QCD} , the QCD scale factor.

2. The sought for expression should include :

(i) soft gluons : as in QED, we need to sum the soft gluons

(ii) intrinsic Fermi motion : unlike QED, quarks (and gluons) do not occur freely, but must be confined (at least at low temperature and low densities). Thus for quarks e.g. we must have some Fermi motion and consequently a $\langle Q_{\perp} \rangle$ intrinsic.

(iii) Perturbative limit : due to asymptotic freedom, we expect the differential cross section for $q\bar{q} \rightarrow \gamma^* + g$ at, say large $\hat{s}, \hat{t},$ and Q^2 to be given by first order hard scattering, i.e.

$$\frac{d\sigma}{dQ_{\perp}^2 dQ^2} \rightarrow \frac{d\sigma_{hard}^{(1)}}{dQ_{\perp}^2 dQ^2}, \quad \text{for } Q_{\perp}^2 \text{ large}$$

For massless quarks and gluons, the right hand side of this cross-section has a $\frac{1}{Q_{\perp}}$ singularity. Thus the cross section integrated in Q_{\perp}^2 is logarithmically divergent (and hence does not give us the Drell-Yan limit). So, a regularization scheme has to be found which incorporates all the three properties (i), (ii) and (iii).

In ref.(3) we have given explicit formulas to incorporate (i) and (ii). They are also quite successful in describing the phenomenology of Drell-Yan processes for small to medium Q_{\perp}^2 from very low up to the collider energies. In this paper, we present a regularization scheme which includes the hard spectrum and in addition does not jeopardize the features of soft gluons and intrinsic transverse

momentum. Present regularization schemes, in fact, either do not include the soft gluon summation feature, or introduce energy scales to separate the hard from the soft region, with consequent loss of control on the natural (Λ -type) scales. We present here explicit formulas only for the Drell-Yan process, but a straightforward modification of our expressions can be made to consider Compton scattering as well.

Let the differential probability of occurrence of a general process

$$i \rightarrow f + (\text{soft gluons})$$

be given by $\mathcal{F}_{soft}(Q_{\perp})$, where we have exhibited explicitly only the dependence on Q_{\perp} , the total transverse momentum carried by all the soft gluons. Now, we wish to compute a correction, to order α_s , due to a hard scattering. This contribution may be written as

$$\int d^2\vec{K}_{\perp} L^{(1)}(\vec{K}_{\perp})(\mathcal{F}_{soft}(\vec{Q}_{\perp} - \vec{K}_{\perp}) - \mathcal{F}_{soft}(\vec{Q}_{\perp}))$$

where $L^{(1)}(\vec{K}_{\perp})$ is the first order transition probability of having a hard scattering at \vec{K}_{\perp} . We show this pictorially in fig.1. So, the complete distribution $F(\vec{Q}_{\perp})$ including terms of order α_s becomes

$$\frac{d^2P}{d^2\vec{Q}_{\perp}} = F(\vec{Q}_{\perp}) = \mathcal{F}_{soft}(\vec{Q}_{\perp}) + \frac{1}{2\pi} \int d^2\vec{K}_{\perp} L^{(1)}(\vec{K}_{\perp})(\mathcal{F}_{soft}(\vec{Q}_{\perp} - \vec{K}_{\perp}) - \mathcal{F}_{soft}(\vec{Q}_{\perp})) \quad (1)$$

In Eq.(1), $L^{(1)}$ is multiplied by $\mathcal{F}_{soft}(\vec{Q}_{\perp} - \vec{K}_{\perp}) - \mathcal{F}_{soft}(\vec{Q}_{\perp})$ to insure that there is no double counting. As K_{\perp} becomes 'soft', the contribution of $L^{(1)}$ is reduced. Otherwise said, hard scatterings (quantum corrections) are computed using not bare, but soft-gluon corrected states. (For a derivation of the preceding statement in QED, see ref.(4)). This approach, which is physically transparent is an alternative to that presented in ref.(5), where the needed subtractions to the hard terms are made perturbatively. For notational clarity, in the following we shall present only the leading order corrections. However, the method is quite general.

It is interesting to note here the similarity of eq.(1) with the Boltzmann equation. The distribution \mathcal{F}_{soft} in our equation plays the role of the Fermi distribution in the transport equation. In both cases, the contribution from small angles (small Q_{\perp}) are considerably reduced.

Now we discuss two important features of eq.(1). The first concerns normalization and the second concerns the mean value $\langle Q_{\perp}^2 \rangle$.

(a) Normalization : The normalization of $\mathcal{F}_{soft}(Q_{\perp}^2)$ is maintained. That is

$$\int d^2\vec{Q}_{\perp} F(\vec{Q}_{\perp}) = \int d^2\vec{Q}_{\perp} \mathcal{F}_{soft}(\vec{Q}_{\perp}) = 1 \quad (2)$$

(b) Mean Value of Q_{\perp}^2 : From eq.(1) we readily find that

$$\langle Q_{\perp}^2 \rangle = \int d^2\vec{Q}_{\perp} Q_{\perp}^2 F(\vec{Q}_{\perp}) = \langle Q_{\perp}^2 \rangle_{soft} + \langle Q_{\perp}^2 \rangle_{hard}^{(1)} \quad (3)$$

Eq.(3) allows to obtain the perturbative result for the mean value, as well as the intrinsic term, which arises through our $\langle Q_{\perp}^2 \rangle_{soft}$ as we are now going to describe.

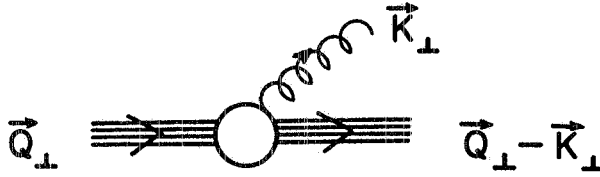


FIG.1. Hard gluon emission in the scattering between two coherent states

For the scattering $q\bar{q} \rightarrow \gamma^* + gluons$ we present explicit formulas for \mathcal{F}_{soft} and $L^{(1)}$. We have

$$\mathcal{F}_{soft}(\vec{Q}_{\perp}) = \frac{d^2 P_{soft}}{d^2 \vec{Q}_{\perp}} = \int \frac{d^2 \vec{b}}{(2\pi)^2} e^{i\vec{b} \cdot \vec{Q}_{\perp} - h_s(b)} \quad (4)$$

where

$$h_s(b) = \frac{2c_F}{\pi} \int_0^{E^2} \frac{dk^2}{k^2} \alpha_s(k^2) [1 - J_0(bk)] \ln \left(\frac{E}{k} + \sqrt{\left(\frac{E}{k}\right)^2 - 1} \right) \quad (5)$$

where $c_F = \frac{4}{3}$ for SU(3) and $E = \frac{s-Q^2}{2\sqrt{s}}$, \sqrt{s} being the C.M. energy of the $q\bar{q}$ system. As discussed in detail in ref. (3), the above soft gluon distribution can give rise to a constant, intrinsic transverse momentum piece, if one uses in eqs.(4) and (5), the singular, but integrable, expression for α_s

$$\alpha_s(k^2) = \frac{12\pi}{25} \frac{p}{\ln[1 + p(\frac{k^2}{\Lambda^2})^p]} \quad (6)$$

where, following an argument due to Polyakov^[6], $p = \frac{5}{8}$.

This can be seen in short, if, as in Parisi and Petronzio^[7], we divide the integration interval in eq.(5) into a 'non perturbative' region, for which $k_{\perp} \leq \Lambda$, and a region for which $k_{\perp} \geq \Lambda$, i.e. we

write

$$h_s(b) = h_{intrinsic}(b) + \Delta(b, E)$$

with

$$h_{intrinsic} = \frac{16}{25} \int_0^{\Lambda^2} \frac{dq^2}{q^2} \frac{p}{\ln(1 + p(\frac{q^2}{\Lambda^2})^p)} \ln \frac{E + \sqrt{E^2 - q^2}}{E - \sqrt{E^2 - q^2}} (1 - J_0(bq)) \quad (7)$$

From eq.(10), one gets

$$h_{intrinsic} \approx Ab^2$$

with

$$A = \frac{48}{25} (3 + \ln \frac{2E}{\Lambda}) \Lambda^2$$

Using $\Lambda = 0.1 GeV$, and for $2E = 1 \div 10 GeV$, we obtain for the intrinsic transverse momentum per parton

$$\langle p_{\perp} \rangle \approx 0.4 \div 0.5 GeV$$

which is of the order of magnitude of the phenomenologically introduced intrinsic contribution [8].

$L^{(1)}$ is computed by subtracting from the first order [9] the contribution due to the soft gluons.

We obtain :

$$L^{(1)}(K_{\perp}) = \frac{4c_F}{\pi} \alpha_s(K_{\perp}^2) \frac{1}{K_{\perp}^2} \frac{E^2}{Q^2} \sqrt{1 - \frac{K_{\perp}^2}{E^2}} \quad (8)$$

Through eqs.(4 ÷ 7) we have a complete specification of eq.(1) without any parameters. Due to the angular correlations in the integral in eq.(1), it is useful to rewrite it in an alternative form :

$$\frac{dP}{dQ_{\perp}^2} = \frac{1}{2} \int b db J_0(Q_{\perp} b) e^{-h_s(b)} [1 + g^{(1)}(b)] \quad (9)$$

where the regularized Bessel transform of the perturbative contribution is given by :

$$g^{(1)}(b) = \int K_{\perp} dK_{\perp} [J_0(K_{\perp} b) - 1] L^{(1)}(K_{\perp}) \quad (10)$$

Comparing eqs.(8) and (9) with (4) and (5), we notice that the same regularization procedure is implied for both soft and hard contribution (as it should be), but only the soft part is exponentiated. The hard corrections are treated perturbatively over the soft background. Through eqs.(8) and (9), we obtain $F(Q_{\perp})$ vs Q_{\perp} for fixed values of Q^2 and E or (3).

Since $\bar{s} = sy_1y_2$, where $y_{1,2}$ are the fractional momenta of quarks and antiquarks in the hadron, we now integrate eq.(9) over the parton densities, writing

$$\frac{s d\sigma}{dx_1 dx_2 d^2\vec{Q}_\perp} = \int_{x_1}^1 dy_1 \int_{x_2}^1 dy_2 \mathcal{G}(y_1, y_2) F(\vec{Q}_\perp; Q^2, sy_1y_2) \quad (11)$$

which now leads to a finite Drell-Yan cross section, i.e.

$$\frac{s d\sigma}{dx_1 dx_2} = \int_{x_1}^1 dy_1 \int_{x_2}^1 dy_2 \mathcal{G}(y_1, y_2) \quad (12)$$

As stated earlier a similar expression can be derived for Compton graphs as well^[10].

The above algorithm for the transverse momentum distribution of produced γ^* (or W or Z_0) in the Drell-Yan processes preserves the normalization (of the integrated cross-section), includes an indefinite number of soft gluons as well as an intrinsic transverse momentum, and asymptotes to the perturbative result for large Q_\perp . No parameters beyond the QCD scale Λ appear in the resulting expression.

In the near future, through the collider and other facilities, high statistics data would become available for Q_\perp distributions of W and Z_0 ^[11,12]. Even though Q^2 for these processes are quite large, expressions of the type presented here, containig both soft and hard gluons, are needed for comparison with data. Also, we need expressions which do not contain arbitrary parameters, so that possible new signatures, e.g. production of squarks etc., may reliably be checked for.

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