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Infrared Radiative Corrections
And The
Zero Momentum Mode of Abelian Gauge Fields

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Abstract

The contribution to radiative corrections of the zero momentum mode of abelian gauge fields is evaluated in analogy with Bose-Einstein condensation in statistical mechanics. This contribution, present only with periodic boundary conditions, yields a non-vanishing effect which depends on the infrared regularization.

1 - In this paper we evaluate the contribution to radiative corrections of the zero momentum mode of abelian gauge fields. This mode is usually not taken into account, but this is consistent only if the gauge field is required to vanish at the surface of the quantization volume. We will refer to this condition as vanishing boundary condition(v.b.c.). If periodic boundary conditions (p.b.c.) are assumed,however, the zero momentum mode should be included. Now it has been argued that this single mode could give rise to a finite effect due to a mechanism analogous to Bose-Einstein

condensation in statistical mechanics. This possibility has been suggested by the exact solution of the infrared sector in Galileian Gauge Theories, showing that radiative corrections depend on b.c.[1]. This is one among the physical effects of b.c.

Phenomenology requires v.b.c. in Q.E.D., but abelian gauge theories might be relevant in physics beyond QED and in order to know the appropriate b.c. one must study their physical consequences. In addition to the interest that such study can have per se, it has other motivations, related to QCD and Galileian Gauge Theories.

It is not known which are the b.c. appropriate to QCD. Twisted boundary conditions have been investigated and related to flux tubes[2], but the possible effects of p.b.c. with respect to v.b.c. have been ignored in the continuum as well as in lattice calculations. The only exception,as far as we know, is ref.(1) where p.b.c. are related to the admissible color states. One of our motivations is to show that p.b.c. allow a freedom in the definition of the infrared behaviour of abelian gauge theories, which could have its counterpart in the non abelian ones. The other motivation is related to galilean gauge theories. We will in fact show that the $c \to \infty$ limit of radiative corrections evaluated in relativistic gauge theories agrees with the result from galilean gauge theories. This gives support to galilean theories as a tool to investigate in a simplified form non perturbative features of quantum gauge theories.

2 - Any reaction with charged particles is accompanied by radiation of soft photons according to the Poisson distribution[3]

$$P(n_{\vec{k}}) = \prod_{\vec{k}} \frac{(\bar{n}_{\vec{k}})^{n_{\vec{k}}}}{n_{\vec{k}}!} e^{-\bar{n}_{\vec{k}}} \tag{1}$$

where $n_{\vec{k}}$ is the number of photons with momentum $\vec{k}$, and $\bar{n}_{\vec{k}}$ is their average. The momentum spectrum for the photons is discrete, according to quantization in a cubic box of volum L^3. Assuming that the infrared regularization has been done by giving the photon a small mass μ, the energy of the photons is $\omega_k = \sqrt{\mu^2 + k^2}$ (we put $h = c = 1$, unless otherwise specified). The probability [4] that soft photons carry a four momentum K is

$$d^4P(K) = \sum P(n_{\vec{k}})\delta^4(\sum_{k'} k' n_{\vec{k}'} - K)d^4K \tag{2}$$

where the sum is carried out over all the $n_{\vec{k}'}$. We are interested in the probability that the soft photons carry a total energy K_0, irrespective of their total momentum. We therefore integrate eq.(2) over $\vec{K}$. By introducing [5] the integral representation for the δ-function we obtain

$$dP(K_0) = \frac{1}{2\pi} dK_0 \int_{-\infty}^{+\infty} dt e^{iK_0 t - h(t)}, \tag{3}$$

where

$$h(t) = \sum_{\vec k} \bar n_{\vec k} (1 - e^{-i\omega_k t}) \tag{4}$$

We must eventually take the limit $\mu \to 0, L \to \infty$. The second limit allows us to approximate the sum with an integral in eq.(4). Before doing so, however, we must single out the zero momentum term, which is present with p.b.c. and would otherwise be killed by the measure in the integral. This is done in analogy with the phenomenon of Bose-Einstein condensation in statistical mechanics. To do so, we write :

$$h(t) = \bar n_0 (1 - e^{-i\omega_0 t}) + \bar h(t) \tag{5}$$

with

$$\bar h(t) = \left(\frac{L}{2\pi}\right)^3 \int d^3k \bar n_{\vec k} (1 - e^{-i\omega_k t}). \tag{6}$$

and $\bar n_{\vec k}$ given by

$$\bar n_{\vec k} = \frac{2\pi e^2}{\omega_k L^3} \left|\sum_i \frac{\epsilon_i \vec v_{i\perp}}{\vec k_i \cdot \vec v_i - \omega_k}\right|^2 \tag{7}$$

where ϵ is positive for the creation of a positive or the destruction of a negative particle, it is negative for the creation of a negative and destruction of a positive particle. and $\vec v_{i\perp}$ is the component of the velocity of the i-th particle perpendicular to $\vec K$. Eq. (7) holds for $\vec K = 0$. For $\vec K = 0$, $\vec v_{i\perp}$ must be replaced by $\vec v_i$. This can be checked by a careful quantization of the radiation field coupled to a classical source, and can be intuitively understood from the fact that the current conservation condition $k^\nu j_\nu = 0$ does not put any constraint on $\vec j$ for $\vec k = 0$. In this case it only requires $j_0 = 0$, which is in fact a consequence of p.b.c. [1].

We can now take the $\mu \to 0$ limit in $\bar h(t)$, obtaining

$$\bar h(t) = \beta \int_0^E \frac{dk}{k} (1 - e^{-ikt}) \tag{8}$$

with

$$\beta = \frac{\alpha}{(2\pi)^2} \int d^2\hat n \sum_{\hat e} \left|\sum_i \frac{(p_i \cdot \hat e)\epsilon_i}{(\vec p_i \cdot \hat n) - p_{0i}}\right|^2 \tag{9}$$

The cut-off energy E in eq.(8) represents the maximum energy allowed to a single <u>soft</u> photon. Its value is defined by a comparison with first order perturbation theory[6]. Following ref.(5), where a discussion of this point can be found, we take it to be of the order of the c.m. energy of the process under consideration. The $\mu \to 0$ limit in the first term of eq.(5) is however more delicate. We get

$$\bar{n}_0(1 - e^{-i\omega_0 t}) = \frac{2\pi e^2}{L^3\mu^3}|\sum_i \epsilon_i \vec{v}_i|^2(1 - e^{-i\mu t})$$

$$\simeq i\frac{2\pi e^2}{L^3\mu^2}|\sum_i \epsilon_i \vec{v}_i|^2 t \qquad (10)$$

The theory is therefore defined by our prescription for $lim_{\mu\to 0_{L\to\infty}} \frac{1}{\mu^2 L^3}$. Let us define for later convenience the dimensionless parameter

$$\eta = \frac{4\pi e^2}{L^3\mu^2 m} \qquad (11)$$

where we have introduced the mass m of the charged particles. This parameter represents all the arbitrariness inherent to the infrared behaviour of the theory. Let us also define the energy

$$\bar{\omega} = \frac{1}{2}m|\sum_i \epsilon_i \vec{v}_i|^2 \qquad (12)$$

The probability $dP(K_0)$ can now be expressed as

$$dP(K_0) = \frac{1}{2\pi}dK_0 \int dt e^{i(K_0 - \eta\bar{\omega})t - h(t)} \qquad (13)$$

The above integral has been evaluated in ref.(4) with the result

$$dP(K_0) = \frac{1}{\gamma^\beta \Gamma(1+\beta)}\beta \frac{dK_0}{K_0 - W}\left(\frac{K_0 - W}{E}\right)^\beta \Theta(K_0 - W) \qquad (14)$$

where $\gamma = 1.781$ is Euler's constant, $W = \eta\bar{\omega}$, and the above expression holds only for $K_0 - W \leq E$. Cross sections are proportional to the integral of $dP(K_0)$ over all undetectable energy losses due to soft photons up to the experimental resolution ΔE. We then obtain

$$\int_0^{\Delta E} dP(K_0) = \frac{1}{\gamma^\beta \Gamma(1+\beta)}\left(\frac{\Delta E - W}{E}\right)^\beta \Theta(\Delta E - W) \qquad (15)$$

With v.b.c. one obtains the above result with $\eta = 0$, i.e. W=0. The parameter η entering eq.(13) is a new parameter of abelian gauge theories with p.b.c.. This parameter is related to the infrared regularization and it is quite possible that other regularization schemes would preclude its appearance. It would mean that such regularizations correspond to the present one with $\eta = 0$. But since there is no a priori reason to choose one regularization scheme rather than another, it is always possible to define the theory so as to give radiative corrections as in eq.(15).

We have emphasized the above point, having in mind that an analogous freedom might be present in the definition of the infrared behaviour of QCD.

It may be observed that the presence of the parameter η amounts to the introduction of a new energy scale in the theory. This energy scale is related to the threshold of disintegration of neutral bound states into their charged constituents. A large value of η implies that the disintegration would occur only at high energies.

3 - If we reinstate $\hbar$ and c in eq.(15), we see that β is $O(\frac{1}{c^2})$ so that the limit $c \to \infty$ in eq.(15) yields

$$lim_{c\to\infty} \int_0^{\Delta E} dP(K_0) = \Theta(\Delta E - \eta\bar{\omega}) \tag{16}$$

This result can be compared to radiative corrections in galilean gauge theories with p.b.c.. Such theories can be formulated in two ways, both yielding radiative corrections consistent with eq.(16). Taking the $c \to \infty$ limit in QED[7], one obtains a theory in which the regulator μ is present, and evaluation of radiative corrections reproduces exactly eq.(16). On the other hand, performing the $c \to \infty$ limit on classical gauge theories and then quantizing, one obtains an infrared finite theory with no need for the regulator μ. In this case the radiative corrections are given [1] by eq.(16) with $\eta = 1$.

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